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by

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# NEUTRAL CURRENT CONSTRAINTS IN THE $SU(2)\otimes U(1)$ MODELS

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## ABSTRACT:

We present a systematic analysis of hadronic neutral currents in low energy neutrino reactions applicable to $SU(2)\otimes U(1)$ models with any number of quarks. For the case $z = m_Z^2 \cos^2\theta_W / m_W^2 \simeq 1$, only two classes of models are consistent with the data: (1) the Weinberg-Salam model, (2) models with more than four quarks, but with the representations of the right-handed up and down quarks suppressed by fixed mixing angles.

New data on various neutrino reactions induced by neutral currents are recently reported.[1] While the structure of neutral currents is not yet established, it is worth noting that the simple Weinberg-Salam model[2] is consistent with the data within experimental errors.[1] More heavy quarks have also been proposed within the $SU(2)\otimes U(1)$ framework with the aim to explain the new particles and the rising anti-neutrino charged-current cross section and the associated y anomaly.[3] Comparison of several $SU(2)\otimes U(1)$ models for the hadronic neutral currents (ie. the Weinberg-Salam(W-S), the six quark vector(V),[4] the Gürsey-Sikivie(G-S) models,[5] and the five-quark model due to Achiman, Koller and Walsh[6]) with the low energy inclusive neutrino cross section data and the elastic $\nu p$ scattering can be found in the recent work of Albright et al[7] assuming the Weinberg-Salam model for the ordinary leptons and Higgs scalars (ie. $z = m_Z^2 \cos^2\theta_W / m_W^2 = 1$). For the experimental limits on z and a more general discussion on the various models, see for example, Achiman and Walsh.[8]

We present here an analysis similiar to that in ref.(7) but <u>general</u> enough to be applicable to any $SU(2)\otimes U(1)$ model with an arbitrary number of quarks, (in which the up and down quarks appear in doublet and/or singlet representations.) We confine ourselves also to the discussion of the hadronic neutral currents below heavy quark thresholds with the same assumptions as in ref.(7). (The fact we assume $z \simeq 1$ is crucial for the conclusions drawn in this note[9], but our formulas hold for the more general case.) We also incorporate the preliminary result of a neutral current experiment in atomic Bi,[10] which also places a useful constraint on the analysis. As already noted, the Weinberg-Salam model is consistent with the data within experimental errors. If, however, the right-handed currents couple to the ordinary up and down quarks in order to give rise to the y anomaly and the rising anti-neutrino charged-current cross section, we find that the right-handed neutral diagonal currents of the up and down quarks cannot be arbitrary but are already specified by the data.

Since the analysis is limited to low energy region where no new particle is excited, one can parametrize the neutral currents of the up and down quarks in a very general way. For the left-handed up and down quarks we take the GIM scheme[11] which is fixed by the Cabibbo angle. For the right-handed up and down quarks in the $SU(2)\otimes U(1)$ model, only the pieces belonging to the doublets (but not the singlets) contribute to the $I = 1$ neutral currents. We introduce two mixing angles $\theta$ and $\phi$ such that $u_R\cos\theta + \ldots$ and $d_R\cos\phi + \ldots$ belong to different doublets, (further mixing of these states with other heavy quarks does not change the diagonal neutral currents.) The orthogonal components $u_R \sin\theta + \ldots$ and $d_R\sin\phi + \ldots$ belong to singlets. The values of $\cos\theta$ and $\cos\phi$ in several models are given in Table 1. The neutral currents can be written in the following general form,

$$\begin{aligned} J_\mu^{neut} &= J_\mu^3 - 2X_w J_\mu^{em} \\ &= \frac{1}{2}\{\bar{u}_L\gamma_\mu u_L + \cos^2\theta\, \bar{u}_R\gamma_\mu u_R - \bar{d}_L\gamma_\mu d_L - \cos^2\phi\, \bar{d}_R\gamma_\mu d_R \\ &\quad - 4X_w(\tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d)\} + \ldots\ldots \end{aligned} \tag{1}$$

where $X_w = \sin^2\theta_w$, $\theta_w$ is the Weinberg angle, and $L,R = 1 \pm \gamma_5$. In terms of isospin indices,

$$J_\mu^{neut} = \alpha V_\mu^3 + \beta A_\mu^3 + \frac{1}{3}\gamma V_\mu^o + \delta A_\mu^o \tag{2}$$

one has

$$\begin{aligned} \alpha &= 1 - 2X_w + \frac{1}{2}(\cos^2\theta + \cos^2\phi) \\ \beta &= 1 - \frac{1}{2}(\cos^2\theta + \cos^2\phi) \\ \gamma &= \frac{3}{2}(\cos^2\theta - \cos^2\phi) - 2X_w \\ \delta &= \frac{1}{2}(\cos^2\phi - \cos^2\theta) \end{aligned} \tag{3}$$

Using the parton model with eq.(1), one can calculate the ratio of the neutral-

current and charged-current cross sections for the neutrino and anti-neutrino beams on an isoscalar target,

$$\begin{aligned} R^\nu &= \frac{\sigma_{nc}^\nu}{\sigma_{cc}^\nu} = \frac{1}{4z^2}\{A + \frac{B}{3}\} + O \\ R^{\bar{\nu}} &= \frac{\sigma_{nc}^{\bar{\nu}}}{\sigma_{cc}^{\bar{\nu}}} = \frac{1}{4z^2}\{A + 3B\} + O \end{aligned} \tag{4}$$

where

$$z = \frac{m_Z^2\cos^2\theta_w}{m_w^2}$$

$$\begin{aligned} A &= (1 - \frac{4}{3}X_w)^2 + (1 - \frac{2}{3}X_w)^2 \\ B &= (\cos^2\theta - \frac{4}{3}X_w)^2 + (\cos^2\phi - \frac{2}{3}X_w)^2 \end{aligned} \tag{5}$$

and $O$ denotes the corrections due to neglecting the $\sin^2\theta_c$ and the strange sea contribution which are (experimentally) estimated to be less than 10% for low neutrino reactions.[1]

We find from eqs.(4) a general relation

$$9R^\nu - R^{\bar{\nu}} = z^{-2}\{4(1 - X_w)^2 + \frac{4}{9}X_w^2\} + O \tag{6}$$

which is independent of the structure of the heavy quarks and may be used to determine the Weinberg angle. The ratios $R^\nu$, $R^{\bar{\nu}}$ are plotted in Fig. 1 as functions of $X_w = \sin^2\theta_w$ for the W-S, vector, AKW models and one variant of the G-S model. Different model predictions for the same value of $X_w$ are seen to lie approximately on a straight line according to eq.(6). We also show the data point corresponding to the world average values[12]

$$R^\nu = 0.26 \pm 0.04, \qquad R^{\bar{\nu}} = 0.37 \pm 0.10 \tag{7}$$

From Fig. 1, one immediately reads off the corresponding value of $X_w$ for eq. (7),

$$X_w = \sin^2\theta_w = 0.37 \pm 0.06 \qquad (8)$$

One notes that the value of $X_w$ is more sensitive to the error of $R^{\nu}$ than $R^{\bar{\nu}}$.

Parity violation of neutrino currents in atoms can be tested by measuring the optical rotation of laser light tuned to frequences close to the resonant one. The preliminary result for the Bi target is consistent with the Weinberg-Salam model[13], but also sets a constraint on the general case which we are considering. The angle of rotation is proportional to the weak charge[14]

$$\begin{aligned} Q_w &= Z\langle p|\, J_o^{\text{neut}}\, |p\rangle + N\, \langle n|\, J_o^{\text{neut}}\, |n\rangle \\ &= Z\,\{1 + 2\cos^2\theta - \cos^2\phi - 4X_w\} + N\{-1 + \cos^2\theta - 2\cos^2\phi\} \end{aligned} \qquad (9)$$

where Z and N are the numbers of the protons and neutrons in the target. The proportionality constant depends on the atomic physics and we refer to the literature for detailed analysis.[14] What seems signigicant from the experimental data is the sign of the angle of rotation which already disagrees with the simplest versions of some of the models. For our analysis, we plot in Fig. 2 the eq.(9) for the following values of $Q_w$.

$$Q_w = -144 \sim -193 \qquad (10)$$

(corresponding to the Weinberg-Salam model with $X_w = 0.3 \sim 0.45$) which represents typically the order of magnitude of the preliminary data. Our conclusions will not depend crucially on the exact value of the data.

From eqs.(4) and (9), and the data (7) and (10), we have enough constraints to narrow down the models which are consistent with the data. Our analysis can be carried out for a general value of $z$, the result with $z \simeq 1$ is as follows: For $z \simeq 1$, not only the Weinberg angle is determined in eq.(8), the mixing angles $\theta$ and $\phi$ are also unigue. We present the solutions in the plane of $\cos^2\theta$ and $\cos^2\phi$ (Fig.2), where the locations of several models are also shown. The constraint eqs.(4) and (9), being quadratic functions of $\cos\theta$ and $\cos\phi$, we find two non-

trivial solutions within the experimental errors as shown in Fig. 2. The implications of these solutions are discussed below.

(1) For the class of models where heavy quarks are introduced in order to explain the y anomaly and the rising cross section in anti-neutrino charged-current, one must take the solution with nonzero $\cos\theta$ and $\cos\phi$. The difference $1-\cos\theta$ and $1-\cos\phi$ correspond to the amount of parity violation in the diagonal neutral currents. The assignments of the right-handed quarks are restricted: One cannot allow $u_R$ and $d_R$ (or $s_R$) in the same doublet since low energy meson and hyperon decay via V-A durrents. One must also avoid right-handed strangeness-changing neutral currents in lowest order. With in addition the constraint on $\cos\theta$ and $\cos\phi$, one is forced to have a minimum of seven quarks using only doublet and/or singlet representations. We suggest below a eight-quark vector-like (but non-vector) model with lepton-quark symmetry. We assign the quarks and leptons in the following doublets[15]

$$\left(\begin{array}{c|c} u & \nu_e \\ d(\theta_c) & e^- \\ \hdashline s(\theta_c) & \mu^- \\ c & \nu_\mu \end{array}\right)_L \qquad \left(\begin{array}{c|c} t(\theta) & \nu_L \\ b & L^- \\ \hdashline b'(\phi) & M^- \\ t' & \nu_M \end{array}\right)_R$$

and singlets: $e^-_R$, $\mu^-_R$, $L^-_L$, $M^-_L$; $t_L$, $b_L$, $b'_L$, $t'_L$; $u(\theta)_R$, $d(\phi)_R$, $s_R$, $c_R$. where $L^-$, $M^-$ are two sequential heavy leptons, (one of them may be responsible for the anomalous $\mu e$ events[16],) and $\nu_L$, $\nu_M$ are two massless neutrinos. t, b, t', b' are four heavy quarks[17] and $d(\theta_c) = d\cos\theta_c + u\sin\theta_c$, $s(\theta_c) = -d\sin\theta_c + s\cos\theta_c$ $t(\theta) = t\sin\theta + u\cos\theta$, $u(\theta) = t\cos\theta - u\sin\theta$, $b'(\phi) = b'\sin\phi + d\cos\phi$, $d(\phi) = b'\cos\phi - d\sin\phi$, where $\theta_c$ is the Cabibbo angle, $\theta$, $\phi$ the two mixing angle discussed above.

(2) The other unique solution corresponds to the Weinberg-Salam model. It is indeed amazing that the Weinberg-Salam model is consistent with all the data of neutral current neutrino reactions within experimental errors. For the y anomaly

and the rising anti-neutrino charged-current cross sections, one could take the alternative of introducing more heavy leptons to the Weinberg-Salam model[18] (in constrast to additional quarks as in the first class of models).
The production of neutral heavy leptons in neutrino scattering will give rise to the y anomaly and the rising anti-neutrino charged-current cross sections in a natural way[19,20]. (Note that the Pais-Treiman bound[21] for the neutral heavy lepton decays cannot be tested here for practical reasons). We have pointed out in ref.(20) how to test the hypothesis of neutral heavy leptons versus heavy quarks in neutrino reactions when charm is presumably also produced.

For elastic $\nu p$ scattering, we refer the reader to the recent analysis by Albright et al[7] who concluded that the five quark model[6] and the Gürsey-Sikivie model[5] satisfactorily account for the observed data in shape and magnitude, and the Weinberg-Salam model predicts a cross section one and one-half standard deviations smaller than observed. Our analysis above indicates, however, that for the choice of $z \simeq 1$, we must require nonzero values for both the mixing angles of the up and down quarks in order to agree with the atomic neutral current data. Using the analysis of Albright et al[7] and the decomposition eq.(2), we calculate for the first class of models with $\cos^2\theta \simeq 0.8$, $\cos^2\phi \simeq 0.7$ and $X_W \simeq 0.37$ (Fig. 2). We find a cross section for $\nu p$ scattering in between the Weinberg-Salam model and the Gürsey-Sikivie(B ) model, and a $q^2$ dependence only slightly flatter than the Weinberg-Salam model. We wish to note the theoretical assumptions in the analysis and possible np charge exchange corrections to the data. We conclude that the elastic $\nu p$ scattering data is compatible with the data of the inclusive neutrino reactions via neutral currents and the neutral current data for atomic Bi, and does not impose an additional constraint on the mixing angles.

I thank K. Fujikawa and T. F. Walsh for discussions and K. Fujikawa for pointing out an error in the preliminary version.

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   I am indebted to K. Fujikawa for the remarks.
10. D.C. Soreide, Phys. Rev. Lett. 36 (1976) 352.
11. S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D2 (1970) 1285.
12. Recent experimental values of $R^\nu$ and $R^{\bar\nu}$ are:
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15. One can also assign $(t_L, b_L)$, and $(c_R, s_R)$ in doublets.
I thank K. Fujikawa for the remarks.

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See also G. Snow, Phys. Rev. Lett. 36 (1+76) 766.

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18. We present here two gauge models which incorporate the neutral heavy lepton $M^o$. The first model uses the $SU(2) \otimes U(1)$ gauge symmetry but introduces a leptonic mixing angle in the leptonic sector i.e. $(\nu_\mu \cos\delta + M^o \sin\delta, \mu^-)_L$. The leptonic mixing angle is limited by experiments (see for example, K. Fujikawa and N. Kawamoto, to be published) and may not be enough to account for the y anomaly. The second model uses a larger gauge group, SU(4) U(1) and the representations
$(u, d(\theta_c), s(\theta_c), c)_L$, $(\nu_\mu, \mu^-, M^-, M^o)_L$ etc.
where $d(\theta_c) = d \cos\theta_c + s \sin\theta_c$, $s(\theta_c) = -d \sin\theta_c + s \cos\theta_c$.
The SU(2) subgroup of the SU(4) group and the U(1) group, which couple to the doublets $(u, d(\theta_c))$, $(c, s(\theta_c))$, $(\nu_\mu, \mu^-)$ etc., give masses to the gauge gluons after spontaneous symmetry breaking as in the Weinberg-Salam model (except that z is no longer constrainted to be 1 if more than one set of Higgs scalars are needed to break the SU(4) symmetry). One of the SU(4) gluons is used to excite the neutral heavy lepton $M^o$.

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TABLE I: Mixing angles for the four models of interest.

| Model | $\cos\theta$ | $\cos\phi$ |
|---|---|---|
| W-S | 0 | 0 |
| Vector | 1 | 1 |
| AKW | $\cos\alpha$ | 0 |
| G-S(B) | 1 | 0 |

## FIGURE CAPTIONS

Fig. 1: The ratios $R^{\nu}$ and $R^{\bar{\nu}}$ as functions of $\sin^2\theta_W$ for the Weinberg-Salam (W-S), vector, Gürsey-Sikivie (G-S) model (B), and the five-quark model of Achiman et al. The curves are for E below heavy quark thresholds (see Albright et al., ref. (7)). The tick marks on each curve denote the value of $\sin^2\theta_W$. The dashed lines are given by equation (6) as compared with the model predictions. The data point corresponds to the world average values given in eq. (7). See ref. (11) for details.

Fig. 2: The domain of $\cos\theta$ and $\cos\phi$ (the crossed area) consistent with the limits $0.22 \leq R^{\nu} \leq 0.30$, $0.37 \leq R^{\bar{\nu}} \leq 0.47$ (the cross hatched area), and $-144 \geq Q_W \geq -193$. We assume $\sin^2\theta_W = 0.37$ (see eq. (8)). The predictions of the Weinberg-Salam (W-S), vector, and Gürsey-Sikivie (G-S(B)) models are also shown (see Table I).

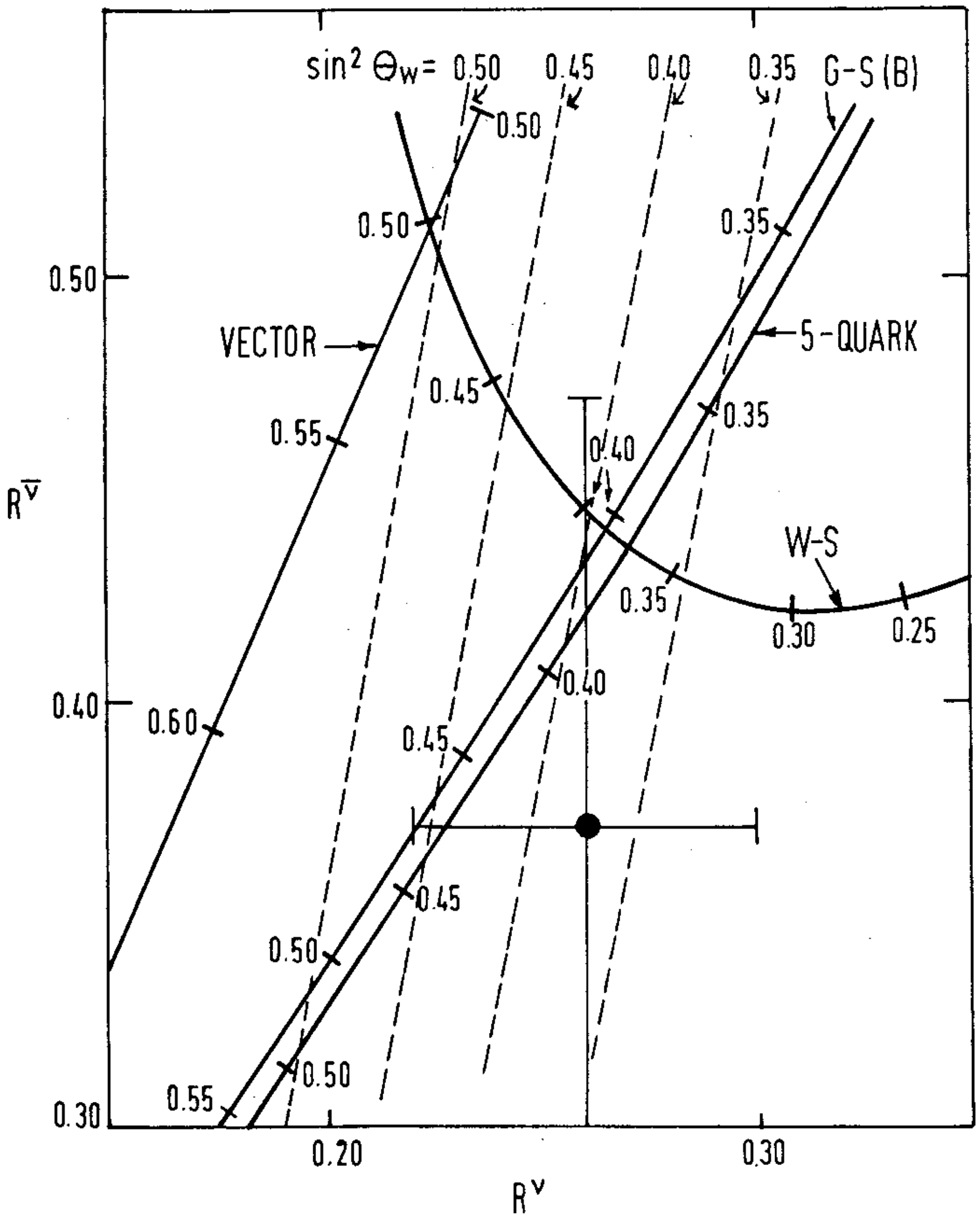
$\sin^2\Theta_W=$ 0.50
0.45
0.40
0.35
G-S (B)
0.50
0.50
VECTOR
5-QUARK
0.45
0.55
0.35
0.35
0.40
0.60
W-S
0.35
0.30
0.25
0.40
0.45
0.45
0.50
0.50
0.55
0.50
0.40
0.30
0.20
0.30
$R^{\bar{\nu}}$
$R^{\nu}$

Fig.1

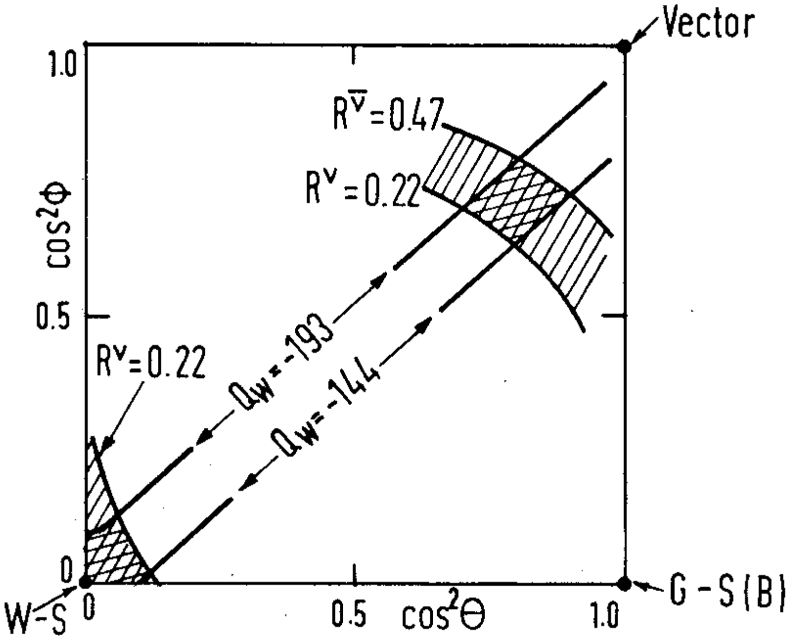
Vector
1.0
$R^{\bar{\nu}}=0.47$
$R^{\nu}=0.22$
$\cos^2\Phi$
0.5
$R^{\nu}=0.22$
$Q_W=-193$
$Q_W=-144$
0
W-S
0
0.5
$\cos^2\Theta$
1.0
G-S (B)

Fig.2