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Narrow Resonances by Strong Coupling

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Abstract

It is shown in a two-channel model, that a resonance which is mainly a bound state in one channel can be made very narrow by a strong attractive force in the other channel. This is done by solving the Bethe-Salpeter equation in a kinematical situation similar to the J(3.1 GeV) particle, adopting essentially the bound charm-anticharm picture. The coupling of the charm-anticharm state to normal hadrons (in terms of a transition potential) does not have to be very small, in contrast to the asymptotically free gluon annihilation picture. Though we consider an interaction of rather short range, we obtain the correct order of magnitude for the leptonic width.

The "new particles" J(3.1) and ψ'(3.7) have hadronic width which are about 10^{-4} times normal hadronic ones. In a bound state picture of two constituents (we shall adopt the charm-anticharm or $c\bar{c}$ picture) this would naively imply that a transition potential between a $c\bar{c}$ state and conventional hadron states, given for instance by charmed meson exchange, is reduced by 10^{-2} with respect to normal meson exchange. This is hard to understand. Of course, this estimate rests on the assumption that the outgoing hadrons are essentially in a plane wave state. We shall show that the transition potential need not be so small when the effect of a fairly strong interaction in the outgoing hadron channel is taken into account. What happens is roughly that the scattering wave function of the hadrons becomes small and/or oscillates rapidly inside the range of the interaction such that the overlap with the bound state wave function becomes very small.

We shall demonstrate this in the following framework. We consider a two-channel problem at an energy where channel 1 is closed and has an interaction which would lead to a stable bound state, if there were no coupling to channel 2. The latter is open, and its constituents have a very strong attractive interaction. For demonstrative purposes we first treat the case of separable interactions. Then we investigate a P -wave Bethe-Salpeter model with ladder approximation. This model enables us also to calculate the leptonic width. The strength of the attraction in channel 2 is of central importance. We know that it must be strong enough to yield a resonance at the ω - or φ - mass. For an interaction given by a scalar particle exchange with the coupling constant fixed by the above constraint, we don't find a sufficient reduction of the width of the resonance in channel 1. We thus are led to consider interactions which depend on the

CM-energy $\sqrt{s}$, such that the attraction increases when going from the ω -mass to the J-mass. Such interactions are, for a spin 1 state, provided either by the spin factor $\sqrt{s}$ in the transition amplitude $K\bar{K} \longrightarrow K\bar{K}^*$ or by chiral invariant fermion loops for the pseudoscalar-pseudoscalar interaction [1] which lead to a linear increase of the interaction in s. For the ensuing separable case we shall treat an explicitly s-dependent potential, whereas in the Bethe-Salpeter case we shall investigate the dependence of the width in channel 1 on the coupling constant in channel 2. We shall find out that the extreme case of a linear s-dependence is not required.

For a separable interaction

$$V_{ba} = \lambda_{ba}(s)\,|f_b\rangle\langle f_a| \qquad a,b = 1,2 \tag{1}$$

the scattering equation (G_c is the free Greens function in channel c)

$$T_{ba} = V_{ba} + \sum_{c=1,2} V_{bc}\, G_c\, T_{ca} \tag{2}$$

can be solved explicitly giving

$$\det\left(\delta_{ba} - \lambda_{ba}(s)\langle f_a|G_a(s)|f_a\rangle\right) = 0 \tag{3}$$

as condition for resonances. Guided by the above considerations we shall assume the somewhat extreme linear s-dependence +)

$$\lambda_{ba}(s)\langle f_a|G_a(s)|f_a\rangle = s\,\gamma_{ba}\, e^{-i\varphi_a} \tag{4}$$

+) For a very short range potential the Greens functions introduce little s-dependence.

One has $\varphi_1 = 0$ since channel 1 is closed. Then to lowest order in γ_{12}^2 one gets for $\gamma_{11} < \gamma_{22}$

$$\frac{\Gamma_1}{M_1} \approx \left(\frac{\gamma_{12}}{\gamma_{22}}\right)^2 \sin\varphi_2 \approx \left(\frac{\gamma_{12}}{\gamma_{11}}\right)^2 \left(\frac{M_2}{M_1}\right)^4 \sin\varphi_2 \qquad (5)$$

Identifying $M_1 = M_J$ and $M_2 = M_\omega$ one sees that the value [2] $\Gamma_J/M_J = 2\cdot 10^{-5}$ does not require an excessively small value for $\gamma_{12}^2/\gamma_{11}^2$. The fact that Γ_1 decreases as $1/\gamma_{22}^2$ follows independently of the s-dependence (4), from the separable ansatz (1). For local interactions we expect a different behaviour. To lowest order in the transition potential V_{12} one has [3]

$$\Gamma_1 \approx 2\pi \left|\langle\psi_1 | V_{12} | \psi_2\rangle\right|^2 \qquad (6)$$

where ψ_1 is the bound state wave function due to V_{11} and ψ_2 the scattering state wave function in channel 2 determined by V_{22}. For large attractive V_{22} the WKB method becomes exact [4] and gives rapid oscillations for ψ_2. As a consequence one gets, provided that $\langle\psi_1|V_{11}$ behaves sufficiently smooth, a strong decrease of Γ_1.

To get quantitative results for the case of local interactions we turn to a two-channel Bethe-Salpeter model for scalar particles and shall study the dependence of Γ_1 on g_{22}, the coupling constant in channel 2. In contrast to the Schrödinger equation this will enable us also to calculate the leptonic width in a satisfactory way. In the CMS the P-wave scattering amplitudes satisfy

$$t_{ba}(k_1^2, k_2^2, s) = v_{ba}(k) + \frac{i}{4\pi^3} \sum_{c=1,2} \int dq_0 \, |\vec{q}|^2 d|\vec{q}| \, \frac{v_{bc}(k,q)\, t_{ca}(q_1^2, q_2^2, s)}{(q_1^2 - m_c^2 + i\varepsilon)(q_2^2 - m_c^2 + i\varepsilon)} \qquad (7)$$

$$a, b = 1, 2$$

where k_1, k_2 are the particle momenta in the final state, and $q = \frac{1}{2}(q_1 - q_2)$, $k = \frac{1}{2}(k_1 - k_2)$. The momentum dependence of v_{ba} was taken to be independent of a and b. As the first case we considered a super-renormalizable interaction:

Version I: $$v_{ba}(k,q) = g_{ba} \, \tilde{Q}_1(k,q) \qquad (8)$$

where $$\tilde{Q}_\ell(k,q) = \frac{1}{2} \int_{-1}^{+1} d\cos\vartheta_{kq} \; P_\ell(\cos\vartheta_{kq}) \frac{1}{(k-q)^2 - \mu^2} \qquad (9)$$

A more singular interaction [+)] was taken as case II, which may arise by spin 1 exchange:

Version II: $$v_{ba}(k,q) = g_{ba} \, |\vec{k}||\vec{q}| \, \tilde{Q}_0(k,q) \Big/ \left(1 + (|\vec{k}|^2 + |\vec{q}|^2)/\Lambda^2\right) \qquad (10)$$

The technique of solution was to iterate (7) and Padéize the Born series. In order to cope with its bad divergence for large g_{22}, we subtracted a separable term from the potential matrix [5)]. The physical input for the masses and coupling constants was the following: We used $m_1 = 2.1$ GeV (giving a reasonable $c\bar{c}$ threshold) and $m_2 = 0.48$ GeV (corresponding to $K\bar{K}$ as constituents of channel 2). We have taken a rather large exchanged

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+) A regularization with a cut-off mass Λ was necessary to calculate Γ_{ee} .

mass μ = 2.7 GeV since we believe that the main forces in the P-state are given by fermion loops [6]. The coupling constant g_{11} was adjusted to give a resonance at $\sqrt{s} = M_1 = 3.1$ GeV. For the off-diagonal coupling we took $g_{12} = g_{21} = \frac{1}{10} g_{11}$. For this guess we have three motivations: (i) The coupling of J to the pomeron is 1/20 times that of normal mesons. (ii) The decay rates [7] $\psi' \to J\pi\pi$ and $\psi' \to J\eta$ which do not suffer from a wave function reduction are suppressed by 10^{-2} -10^{-3}. (iii) A suppression by 10^{-1} in the potential can be understood in terms of (masses)^2 of exchanged charmed particles vs. normal hadrons. Finally we have defined a reference coupling $\bar{g}_{22}$ by the requirement that for $g_{22} = \bar{g}_{22}$ there is a resonance in channel 2 at $M_2 = m_\omega$.

The results for Γ_1/M_1 as a function of $g_{22}/\bar{g}_{22}$ are shown in Fig. 1. It is seen that for the singular interaction the experimental value $\Gamma_1/M_1 = 2.3 \cdot 10^{-5}$ requires $g_{22}/\bar{g}_{22} \approx 3.4$ (if we extrapolate our curve a bit beyond the point were our method of solution ceased to work). This is much less than the value 16 corresponding to a linear increase in s of the coupling.
It is of course rather unrealistic to replace the whole hadronic continuum by a two particle state. If we simulate many particle states by adding to the bare propagators in eq. (7) a large spectrum of continuum states, this would have increased the net attraction in channel 2 considerably, requiring a reduction of $\bar{g}_{22}$. We do not think that the curves in Fig. 1 would change qualitatively, but the decay of J into the discrete 2 particle state would be much suppressed for the same $g_{22}/\bar{g}_{22}$.

We now turn to the calculation of the leptonic width Γ_{ee} of our spin 1 state, assuming that the constituents have charge 1. From the nonrelativistic formula for spin 1/2 constituents [8)]

$$\Gamma_{e^+e^-} = \frac{16\pi}{3}\,\alpha^2\,\frac{|\chi(0)|^2}{M_1^2} \tag{11}$$

where $\chi(0)$ is the bound state wave function at the origin, one might expect a rather large width for the singular potential II with the large value of μ (eq. (10)). This is, however, not the case due to strong vertex renormalization effects which reduce $\Gamma_{e^+e^-}$ as compared to (11). Thus one has the welcome possibility to describe the new particles as tightly bound systems, which is advantageous in view of the small slopes in photoproduction [9)] (around 2-3 GeV^{-2}) and the small photon transition rates [10)] $\psi' \to \{\chi(3.41),\ P_c(3.51)$ and $\chi(3.53)\}$, which are proportional to the radius squared of the wavefunction if they are electric dipole transitions [11)].

We first calculate the constituent form factor F(s) both at s = o and at $s = M_1^2$ from

$$F(s) = Z_1 + Z_1\,\frac{i}{4\pi^3}\int dq_0\, d|\vec{q}|\,\frac{|\vec{q}|^3}{\sqrt{s/4 - m_1^2}}\,\frac{t_{11}(q_1^2, q_2^2, s)}{(q_1^2 - m_1^2 + i\varepsilon)(q_2^2 - m_1^2 + i\varepsilon)} \tag{12}$$

where Z_1 is determined by the requirement F(o) = 1. Of course, Z_1 depends on Λ. We have $Z_1 \approx 0.02$, which means that the nonrelativistic approximation is not adequate. The leptonic coupling constant [+)] f_1 is now derived from

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[+)] Note that $\Gamma_{e^+e^-} = \frac{4\pi}{3}\,\frac{\alpha^2}{f_1^2}\,M_1$

the Breit-Wigner formulas (with the constituents on-shell)

$$F(s) \approx \frac{g}{f_1} \frac{M_1^2}{M_1^2 - s - i\Gamma_1 M_1} \tag{13}$$

and

$$t_{11}(s) \approx g^2 \frac{M_1^2 - 4m_1^2}{3} \frac{1}{s - M_1^2 + i\Gamma_1 M_1} \tag{14}$$

where g is the coupling constant of the J to the constituents.
We find

$$\left.\begin{array}{lcl} f_1^2/4\pi & \approx & 37 \\ \Gamma_{e^+e^-} & \approx & 1.5\ \text{keV} \end{array}\right\} \tag{15}$$

which is too small compared to the experimental value [2] $\Gamma_{e^+e^-} = 4.8 \pm 0.6$ keV
In our calculation we have assumed that the potential (10) is s-independent. A slight reduction of the potential at s = o would have increased the renormalization constant Z_1 and reduced f_1 accordingly.
Our dynamical explanation of the OZI-rule [12] by the very strong interaction of normal hadrons (h) has consequences which are only in part similar to that of the asymptotically free gluon scheme [13] (AFS). Since the final state interaction between psions and h's is not large our mechanism gives no suppression for the decays $\psi' \to J h$, the rates of which are determined by the magnitude of the transition potential. Thus the fact that the rate for $\psi' \to \eta J$ is relatively large does not conflict with the fact that $\psi' \to \eta h$ is not

large since only the latter is suppressed. Analogous conclusions hold for radiative decays $J \to h\gamma$, if the γ can originate [14] from an intermediate J. In AFS the above transitions can proceed via two gluon exchange, and important differences in rates can arise due to suspected differences in the gluon coupling to η, η', f and f' from mass and wave function effects which would suppress $J \to f'\gamma$ drastically as compared to $J \to \eta'\gamma$. We have no a priori reasons for such suppressions.

From the fact that the strong interactions are approximately SU(6) invariant we conclude that the interaction of normal hadrons in the $J^{PC} = 0^{-+}$ state is roughly the same as in the 1^{--} state. Therefore a pseudoscalar $c\bar{c}$ state (with the X(2.8 GeV) as a candidate [16]) should be as narrow as the vector states are [+)], in contrast to AFS. An indirect check for this is possible by combining a theoretical estimate of the width $\Gamma_{\gamma\gamma}^{(0^{-+})}$ with the branching ratio $B\left(\frac{0^{-+} \to \gamma\gamma}{0^{-+} \to all}\right)$, for which stringent lower limits should be obtainable. Similar remarks apply to the ortho-charmonium P-wave states [11].

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+) The same conclusion is reached in the dual model with cylinder corrections [15].

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Figure Caption

Fig. 1: The resonance width in channel 1 as function of the coupling strength in channel 2 for versions I and II (see eqs. (8) and (10)).

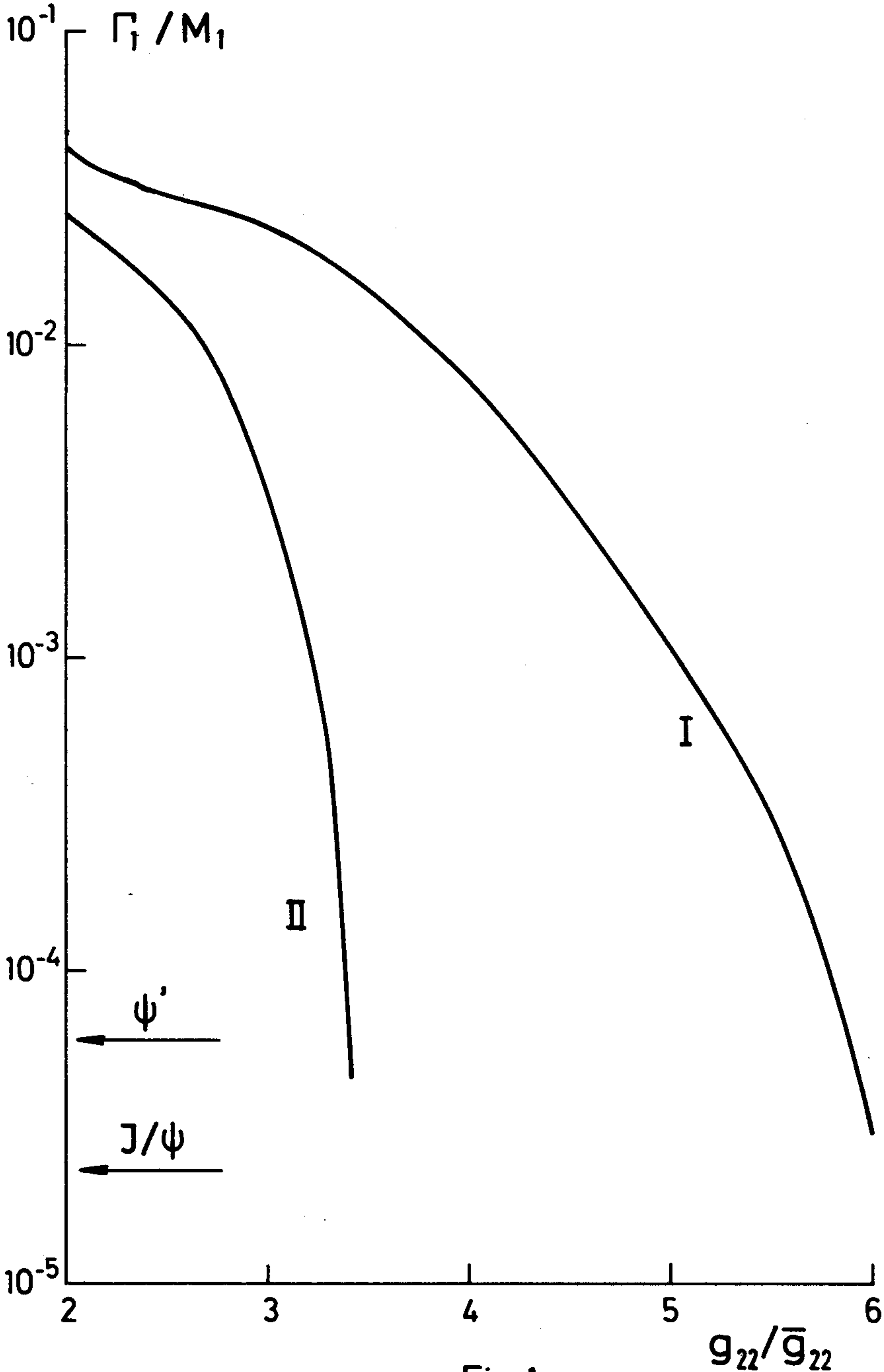

Fig.1