

# DEUTSCHES ELEKTRONEN-SYNCHROTRON **DESY**

DESY 75/39  
October 1975



Target Asymmetry in Inclusive Photoproduction

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Target Asymmetry in Inclusive Photoproduction of Pions.

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Abstract: We study the target asymmetry in inclusive pion photoproduction in the photon fragmentation region using a helicity dependent Mueller-Regge model, in which the Regge cut contributions are also included. We obtain predictions for the  $t$  - dependence and the magnitude of the target asymmetry.

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## 1. Introduction

In a previous work we carried out a Mueller-Regge analysis of the inclusive photo- and electroproduction of charged and neutral pions on protons in the photon fragmentation region <sup>1)</sup>. It was found there that the available data at intermediate energies can not be explained in terms of Regge poles alone. Subsequently it was shown that the inadequacies of a pure Regge pole expansion can be remedied if certain types of Regge cut contributions are included. In particular the Regge cuts filled in the forward dips present in the pure Regge pole model. In addition they had the effect that the dependence of the cross section on the photon polarization is completely changed compared to a pure Regge pole model. Therefore Regge cuts are needed to explain essential features of the experimental data.<sup>+</sup>)

It is well known that in the triple - Regge limit any factorizing Regge pole contributions give zero target asymmetry in the beam fragmentation region to leading order in  $s$ .<sup>3)</sup> Hence the main contribution to the target asymmetry for this kinematic region must come from a Regge cut.<sup>4)</sup> Therefore the target asymmetry seems to be a phenomenon to learn further about the Regge cut mechanism in inclusive reactions. A comparison with such a theoretical analysis may become possible in the near future as experimental data for a polarized target in the reaction  $\gamma + p \rightarrow \pi^{\pm} + \text{anything}$  is expected to become available for medium energies in the beam fragmentation region.<sup>5)</sup> The size of polarization for the process  $\pi^{\pm} p \rightarrow \pi^{\pm} + \text{anything}$  has been studied theoretically by Soffer and Wray<sup>6)</sup> and compared to experimental data in ref. 7.

Since the exchange mechanisms for inclusive pion production induced by pions and photons are very different it is not possible to apply the results of Soffer and Wray directly to  $\gamma + p \rightarrow \pi^\pm + \text{anything}$ . Furthermore Soffer and Wray considered only the cut effects in connection with the incoming proton and not those coupled to the ingoing pion, whereas in I it was found that the Regge cuts originating from rescattering corrections to the ingoing particles are indispensable in order to introduce non-factorizing contributions in the  $\gamma\pi^-$ -channel. In section 2 we set up the basic formulas for calculating the target asymmetry for unobserved photon helicities and calculate the Regge cut contributions in a helicity dependent framework. In this part we rely heavily on our earlier work referred to as I. The results and further discussion is given in sect. 3.

## 2. Calculation of the Target Asymmetry.

In analogy to I we work with the s - channel helicity inclusive structure functions defined by

$$H_{\lambda'_p, \lambda_p}^{\lambda', \lambda} (p', q'; p, q) = \sum_{\chi(n, \xi)} \int \prod_{i=1}^n \frac{d^3 k_i}{(2\pi)^3} (2\pi)^4 \delta^{(4)} \left( \sum_{i=1}^n k_i + k - p - q \right) \times \langle k, k_1, \dots, k_n | T | p, \lambda_p; q, \lambda \rangle \langle k, k_1, \dots, k_n | T | p', \lambda'_p; q', \lambda' \rangle^* \quad (2.1)$$

where in the summation over  $\chi(n, \xi)$ ,  $\xi$  denotes all the unobserved discrete labels involved in the missing mass state X.

In (2.1)  $q$ ,  $k$  and  $p$  are respectively the momenta of the incoming virtual photon, the outgoing pion and the target proton;  $q'$ ,  $p'$

refer to the complex conjugate matrix element and in order that we can write separate partial wave expansions for both of the above matrix elements, we allow  $(q, p)$  and  $(q', p')$  to be different, but keep the constraint  $q + p = p' + q'$ . For the inclusive polarized cross section we have  $p' = p$  and  $q' = q$ .  $\lambda, \lambda'$  and  $\lambda_p, \lambda'_p$  are the helicities of the ingoing photon and proton, respectively.

The asymmetry of a polarized target proton with polarization normal to the production plane summed over photon helicities is then given by<sup>8)</sup>

$$A(s, t, M^2) = \frac{2 \sum_{\lambda} \text{Im} H_{-\frac{1}{2}, \frac{1}{2}}^{\lambda, \lambda} (p, q; p, q)}{\sum_{\lambda, \lambda_p} H_{\lambda_p, \lambda_p}^{\lambda, \lambda} (p, q; p, q)} \quad (2.2)$$

The Regge cuts are obtained as absorption corrections to a pure Regge pole expansion of  $\gamma p \pi \rightarrow \gamma' p' \pi'$ . The detailed structure of such an expansion for the forward direction ( $p=p', q=q'$ ) was discussed in ref. 1. For charged pion production the main contribution of the unpolarized cross section comes from pion exchange in the  $t$ -channel ( $t = (k - q)^2$ ). Other possible exchanges in the  $t$ -channel are  $A_2, \rho$  and  $B$ -exchange. We have two types of absorption or rescattering corrections: (i) additional Regge exchanges (mostly Pomeron) between the ingoing photon and proton as shown in fig. 1 and (ii) additional Regge exchanges between the  $t$ -channel Regge poles and the incoming proton (see fig. 2). For unpolarized cross sections the second category is already included essentially by using phenomenological expressions for the forward Reggeon-proton scattering amplitude. But for polarized cross

sections the category (ii) leads to a nonvanishing contribution as well as the contributions of category (i). In fact Soffer and Wray considered only category (ii) for the process  $\pi^\pm p \rightarrow \pi^\pm X$ . 6)7)

First let us calculate the contribution of category (i) as shown in fig. 1. The calculations proceed in close similarity to the evaluations of the unpolarized structure functions as described in detail in I.

In the following we shall use the same notation as in I.

Then  $H_{\lambda_p, \lambda_p}^{\lambda', \lambda}(\vec{c}', \vec{c}; R)$  is the nonforward structure function in a pure Regge pole model with Regge poles in the  $t$ ,  $t'$  and  $t_0$  channel and  $\vec{c}$  is the two-dimensional transverse momentum of the observed pion. For the absorption corrected structure function  $H_{\lambda_p, \lambda_p}^{\lambda', \lambda}(\vec{c}', \vec{c})$  we obtain the same formula as in I except that the structure functions now depend also on the proton helicities.

The final result is:

$$H_{\lambda_p, \lambda_p}^{\lambda', \lambda}(\vec{c}', \vec{c}) = \int \frac{d^2 \tau_1'}{2\pi} \int \frac{d^2 \tau_1}{2\pi} S^*(\vec{c}' - \tau_1') H_{\lambda_p, \lambda_p}^{\lambda', \lambda}(\vec{\tau}_1', \vec{\tau}_1; R) S(\vec{c} - \vec{\tau}_1) \quad (2.3)$$

where  $^{++}$ )

$$S(\vec{c}) = 2\pi \delta^{(2)}(\vec{c}) - ca e^{-a\vec{c}^2} \quad (2.4)$$

is the correction factor for pure Pomernanchuk exchange between photon and proton. This result is presented pictorially in fig. 3, where one distinguishes four terms: the pure Regge term, two terms with additional Regge exchanges in one of the ingoing  $\gamma p$  channels and, lastly, additional Regge exchanges in both ingoing  $\gamma p$  channels.

As contributions of category (i) we consider only pion exchange in the



t - channel. All other contributions,  $A_2$ ,  $\rho$  and B are small for  $\gamma p \rightarrow \pi^\pm X$  and  $k_\perp^2$  below  $0.5 \text{ GeV}^2$ . Then we need only  $H_{\lambda'_p, \lambda_p}^{\lambda', \lambda}(\vec{z}', \vec{z}, \pi)$ . Since pion exchange is not explicitly gauge invariant, we must introduce a gauge invariant extension.

We take an extension equivalent to the one used in I. Then the contribution with pion exchange in the t- and t' - channels is:

$$H_{-\frac{1}{2}, \frac{1}{2}}^{\lambda', \lambda}(\tau', \phi', \tau, \phi, \pi) = x^2 \tau' \tau e^{-i(\phi' - \phi)} (\tau' e^{i\phi'} - \tau e^{i\phi}) \times \zeta_{\alpha_\pi}^*(t') \zeta_{\alpha_\pi}(t) (S, M^2)^{\alpha_\pi(t') + \alpha_\pi(t)} \text{Im } A(M^2, t_0) \quad (2.5)$$

In (2.5) the off-shell pion-proton helicity flip scattering amplitude is described, to leading order in  $M^2$ , by the usual invariant amplitude A. We see that in the forward direction ( $\tau = \tau', \phi = \phi'$ )  $H_{-\frac{1}{2}, \frac{1}{2}}^{\lambda', \lambda}$  vanishes already in the double-Regge approximation without going to the triple-Regge limit. Apart from the kinematical factor in eq.(2.5) vanishing, the double pion Regge exchange without absorption does not contribute to the polarization asymmetry, as usual, since it has no imaginary part in the forward direction.

In the following we shall approximate Im A by the  $\rho$  exchange term

$$\text{Im } A(M^2, t_0) = g_\rho M^{2\alpha_\rho(t_0)} \quad (2.6)$$

where one assumes, as usual, the decoupling of the Pomanchuk - and  $f$ -exchanges from the helicity flip amplitude.

As in I we shall use the following approximation for the pion Regge exchange

$$\zeta_{\alpha_\pi}(t) (S, M^2)^{\alpha_\pi(t)} = \frac{1}{m_\pi^2 - t} e^{b_\pi t} \quad (2.7)$$

with

$$b_{\pi} = d_{\pi}' \left( \ln(S/M^2) - i\pi/2 \right) \quad (2.8)$$

For the pion pole term the integral representation (eq. (3.10) of I) will be used.

The result is written in the form:

$$\begin{aligned} H_{-\frac{1}{2}, \frac{1}{2}}^{1,1}(\bar{z}', \bar{z}) &= H_{-\frac{1}{2}, \frac{1}{2}}^{1,1}(\bar{z}', \bar{z}; \pi) \\ &- ca \left( G_{-\frac{1}{2}, \frac{1}{2}}^{1,1} + \bar{G}_{-\frac{1}{2}, \frac{1}{2}}^{1,1} \right) + (ca)^2 D_{-\frac{1}{2}, \frac{1}{2}}^{1,1} \end{aligned} \quad (2.9)$$

G and  $\bar{G}$  stand for the single absorption terms (second and third diagram of fig. 3) and D for the double absorption contribution (fourth diagram of fig. 3). The evaluation of G,  $\bar{G}$  and D in the forward limit gives:

$$\begin{aligned} G_{-\frac{1}{2}, \frac{1}{2}}^{1,1} &= G_0 x^2 \tau e^{-i\phi} \int_0^{\infty} dz' \int_0^{\infty} dz e^{-(z'+z)} J_1(z', z) \\ \bar{G}_{-\frac{1}{2}, \frac{1}{2}}^{1,1} &= G_0 x^2 \tau^3 e^{-i\phi} \int_0^{\infty} dz' \int_0^{\infty} dz e^{-(z'+z)} \bar{J}_1(z', z) \end{aligned} \quad (2.10)$$

with

$$J_1(z', z) = e^{-\beta_1^*(z', z) \tau^2} \frac{1}{2(a + B(z) + B_0)^2} \left( 1 - \frac{B(z)(a + B_0) \tau^2}{a + B(z) + B_0} \right) \quad (2.11)$$

$$\bar{J}_1(z', z) = e^{-\beta_1(z, z') \tau^2} \frac{1}{2(a + B^*(z) + B_0)^2} \frac{B^*(z')(a + B_0)}{a + B^*(z') + B_0} \quad (2.12)$$

where

$$G_g = g_s^0 M^{2\alpha_g(0)} \frac{1}{(m_\pi^2 - t_{\min})^2} e^{t_{\min}(b_\pi + b_\pi^*)} \quad (2.13)$$

and

$$\beta_1(z, z') = B(z) + \frac{B^*(z')(a + B_g)}{a + B^*(z') + B_g} \quad (2.14)$$

The expressions  $B(z)$ ,  $B_g$  and  $t_{\min}$  are defined in I.

Up to the two integrations originating from the integral representation of the pion pole (eq. (3.10) of I), the integrations in (2.3) have been done analytically.

The double absorption term is somewhat more complicated and has the following form:

$$D_{-\frac{1}{2}, \frac{1}{2}}^{1,1} = G_g x^2 \tau e^{-i\phi} \int_0^\infty dz' \int_0^\infty dz e^{-(z'+z)} J_2(z', z) \quad (2.15)$$

with

$$J_2(z', z) = e^{-\beta_2 \tau^2} \frac{a}{4(D(z, z'))^3} \left\{ D(z, z') - 2B_g (B(z) - B^*(z')) \right. \\ \left. - \frac{a^2 \tau^2}{D(z, z')} (B(z) - B^*(z')) (D(z, z') + B_g (2a + 4B_g + B(z) + B^*(z'))) \right\} \quad (2.16)$$

and  $\beta_2$  and  $D(z, z')$  are defined in I.

In order to calculate the numerator of (2.2) we need also  $H_{-\frac{1}{2}, \frac{1}{2}}^{-1, -1}(\frac{z'}{z})$ .

The single and double absorption terms of this structure function are related to the above terms. In the forward direction  $\tau = \tau'$   $\phi = \phi'$  and  $\phi = 0$  we have:

$$G_{\frac{1}{2}, \frac{1}{2}}^{-1, -1} = - G_{\frac{1}{2}, \frac{1}{2}}^{1, 1} \quad * \quad (2.17)$$

$$\bar{G}_{\frac{1}{2}, \frac{1}{2}}^{-1, -1} = - G_{\frac{1}{2}, \frac{1}{2}}^{1, 1} \quad *$$

and

$$D_{\frac{1}{2}, \frac{1}{2}}^{-1, -1} = - D_{\frac{1}{2}, \frac{1}{2}}^{1, 1} \quad * \quad (2.18)$$

so that the total contribution to the numerator in (2.2) is:

$$\begin{aligned} 2 \operatorname{Im} \left( H_{-\frac{1}{2}, \frac{1}{2}}^{1, 1} + H_{-\frac{1}{2}, \frac{1}{2}}^{-1, -1} \right) &= 4 \operatorname{Im} H_{-\frac{1}{2}, \frac{1}{2}}^{1, 1} \\ &= + \operatorname{Im} \left[ ca_1^2 \left( D_{-\frac{1}{2}, \frac{1}{2}}^{1, 1} - ca_1 \left( G_{-\frac{1}{2}, \frac{1}{2}}^{1, 1} + \bar{G}_{-\frac{1}{2}, \frac{1}{2}}^{-1, -1} \right) \right) \right] \quad (2.19) \end{aligned}$$

As already remarked earlier the unabsorbed amplitude (2.5) does not contribute to (2.19) in the forward direction. Since in (2.19) the imaginary part has to be taken, the target asymmetry would vanish for a pion trajectory with zero slope, because, then  $B(z)$  ( defined in (I(3.18) ) and all other quantities appearing in  $G$ ,  $\bar{G}$  and  $D$  are real. Therefore the contribution of terms (i) to the target asymmetry are proportional to the slope of the pion trajectory. Furthermore,

as it should be, the target asymmetry is proportional to  $k_{\perp}$ . The denominator of (2.2) has been calculated in I for the unpolarized cross section. It will be used in this form also for the calculation of the target asymmetry.

Concerning category (ii) we do not expect any sizable contributions to the target asymmetry as long as  $k_{\perp}^2$  is not too large (below  $0.5 \text{ GeV}^2$ ). The general formula for  $A(s, t, M^2)$  in case of category (ii) is:

$$A(s, t, M^2) \sigma = 2 \sum_{i, j, \lambda} \beta_i(\lambda, t) \beta_j(\lambda, t) S^{\alpha_i(t) + \alpha_j(t)} \times \text{Im}(\beta_{\alpha_i(t)} \beta_{\alpha_j(t)}^*) \times \text{disc } T_{i, P, \frac{1}{2}; j, P, -\frac{1}{2}}(M^2, t) \quad (2.20)$$

In (2.20) disc T stands for the discontinuity of the Reggeon-proton scattering amplitude with Reggeon i and j in the initial and final state respectively. This amplitude is calculated in a model with Regge poles and cuts in the  $M^2$  channel. It is clear that in (2.20) only nondiagonal terms with  $i \neq j$  contribute. Furthermore, all nondiagonal terms with opposite normality (parity times signature) give also zero contribution in (2.20). This latter result follows quite generally from the selection rules concerning the normality of exchanged Regge poles in the t - channel proven in ref.9, (see in particular section 4.2 of this reference). So we are left in (2.20) only with interference terms, where the Regge poles i and j have the same normality. Candidates are  $\pi^-B$ ,  $\pi^-A_1$  and  $\rho^-A_2$  interferences. All three contributions do not have the double pion pole as the contribution of the first category. Apart from this fact it is well known that the exchange degeneracy of  $\pi^-B$  and  $\rho^-A_2$  give zero target asymmetry in the case of exclusive pion photoproduction. Using very mild assumptions about the phases of production amplitudes

the EXD - arguments can also be applied to the inclusive case implying absence of  $\pi$ -B and  $\rho$ -A<sub>2</sub> contributions. Even if it should turn out that exchange degeneracy is strongly broken for inclusive processes one should note that the  $\rho$ -A<sub>2</sub> term does not have any enhancement for small  $k_{\perp}^2$  and therefore can be neglected. Compared to the  $\rho$ -A<sub>2</sub> term the interference terms  $\pi$ -B and  $\pi$ -A<sub>1</sub> could be enhanced by one pion pole. Neither the coupling of the B nor the A<sub>1</sub> to the  $\gamma\pi$ -system is known. Analysis of exclusive pion photoproduction indicate that B and A<sub>1</sub> do not couple strongly if compared with the pion-exchange contribution. Therefore we shall neglect these interference terms also.

Altogether we conclude that in a calculation of the target asymmetry for small  $k_{\perp}^2$  the main contribution comes from terms of the first category and all terms of the second category should be small.

Using only  $\pi$ - exchange the predicted asymmetry A for a photon energy  $E_{\gamma} = 6$  GeV is shown in fig. 4 a,b,c, for  $x = 0.6, 0.7$  and  $0.8$ . The value of the absorption  $c$  is the same as that required to explain the

$k_{\perp}^2$ -dependence of the unpolarized inclusive distribution for  $\gamma p \rightarrow \pi^{\pm} X$  which was given in I. We notice that a significant target asymmetry is obtained for  $k_{\perp}^2 > 0.2 \text{ GeV}^2$  and that it increases considerably as  $x \rightarrow 1$ . The corresponding unpolarized cross sections are given also in Fig. 4 a,b,c. The difference in the absolute value of the asymmetry for  $\pi^+$  and  $\pi^-$  production comes from the different  $\pi^+$  and  $\pi^-$  cross sections in the denominator. Only for  $x = 0.7$  the forward spike originating from the pion pole has been calculated. For the other two  $x$  values the unpolarized cross section is plotted only for  $k_{\perp}^2 > 0.05 \text{ GeV}^2$ .

Footnotes

- +) There has been an attempt by J. Pumplin <sup>2)</sup> to derive the form Regge cuts take in inclusive distributions. In his work he makes some claims regarding the relationship between our and his work which should be disregarded since the argument used is ill founded.
- ++) In I there was an error in formula I (2.13), which is corrected in eq. (2.4). As a consequence the value of  $c$  quoted in I, which is required to fit the  $\gamma p \rightarrow \pi^\pm X$  inclusive data should be multiplied by  $2\pi$ .
- +++) The  $t_0$  - dependence in the  $\rho$  - Regge residue  $g_\rho(t_0)$  in eq. (2.6) is absorbed in the exponential factor  $B_\rho$  as described in I. Then  $g_\rho^\circ$  is the remaining  $t_0$  - independent residue factor. For the relative normalization of helicity flip and nonflip contributions we use the VDM model.

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Figure Captions.

Fig. 1: Direct channel rescattering corrections to the  
Regge expansion of  $\gamma + p \rightarrow \pi + X$

Fig. 2: Rescattering correction between t-channel Regge pole  
and proton for  $\gamma + p \rightarrow \pi + X$

Fig. 3: Reggeon diagram expansion for the tripe Regge limit  
of  $\gamma + p \rightarrow \pi + X$  (The double dotted line represents the  $\rho^0$ )

Fig. 4: The predicted target asymmetry and the unpolarized cross section  
for  $E_\gamma = 6 \text{ GeV}^2$  for  $x = 0.6, 0.7$  and  $0.8$  respectively plotted as  
a function of  $k_{\perp}^2$ .

Acknowledgement

One of us (K.A.) would like to thank the Alexander-von-Humboldt Foundation for a fellowship and H. Joos, H. Schopper, G. Weber and K. Symanzik for the generous hospitality at DESY.

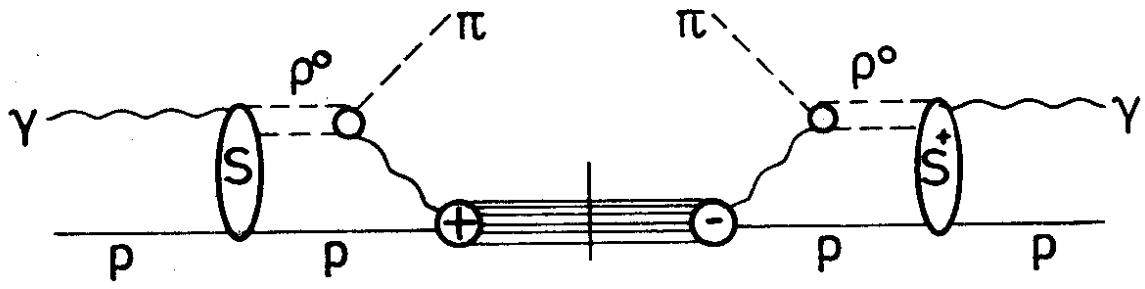


Fig.1

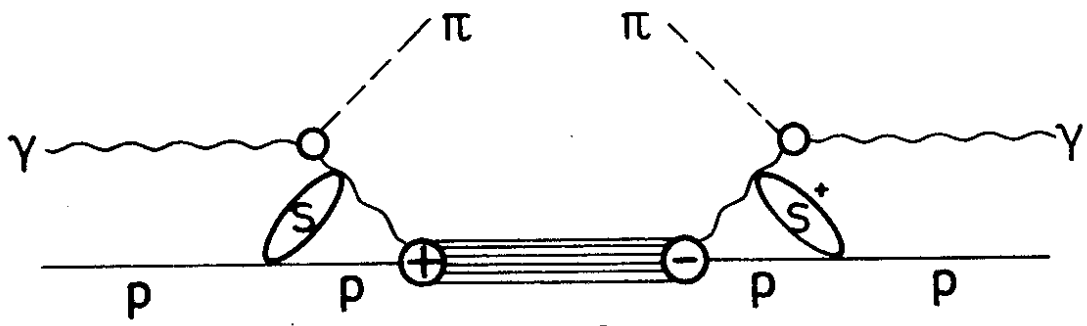


Fig.2

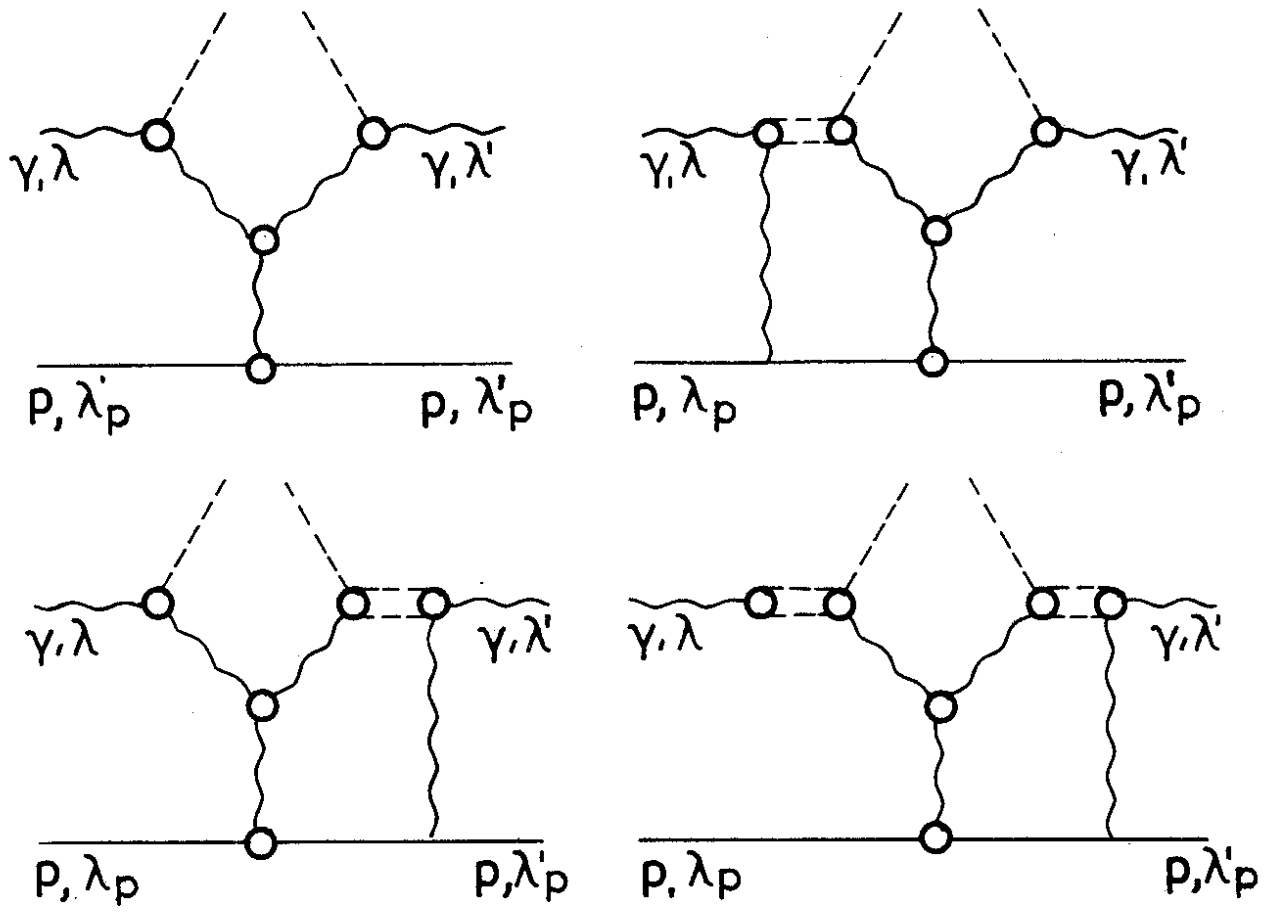
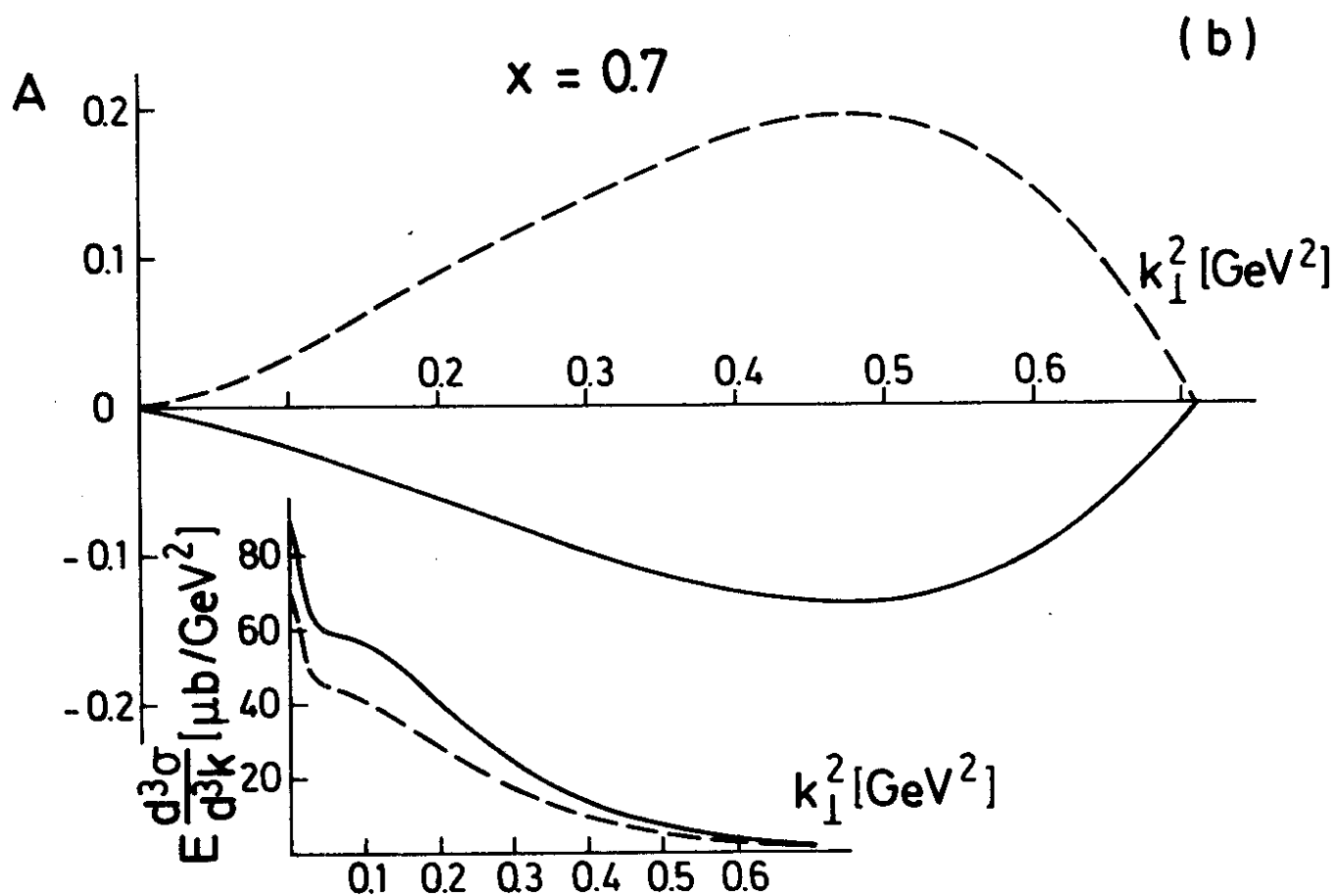
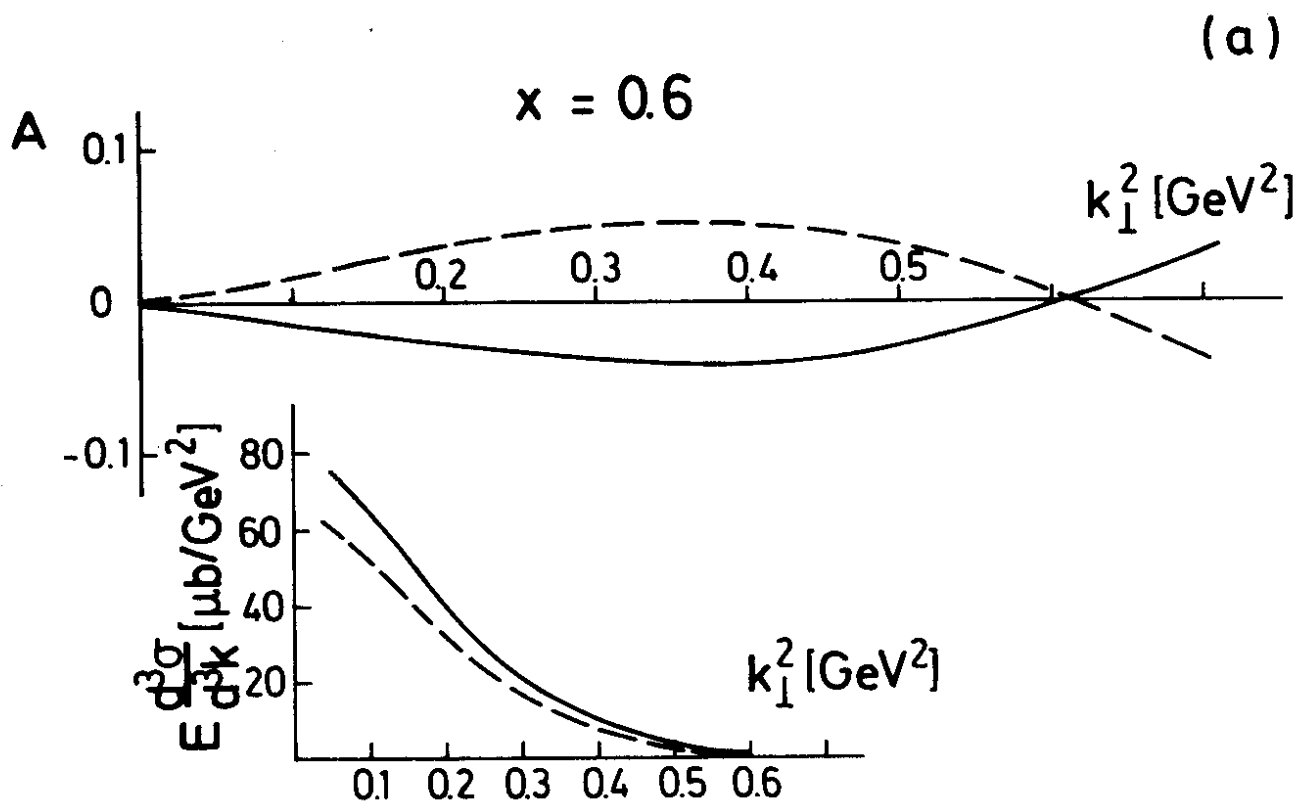


Fig.3



—  $\pi^+$   
 - - -  $\pi^-$

$E_{\gamma} = 6 \text{ GeV}$

Fig. 4 a,b

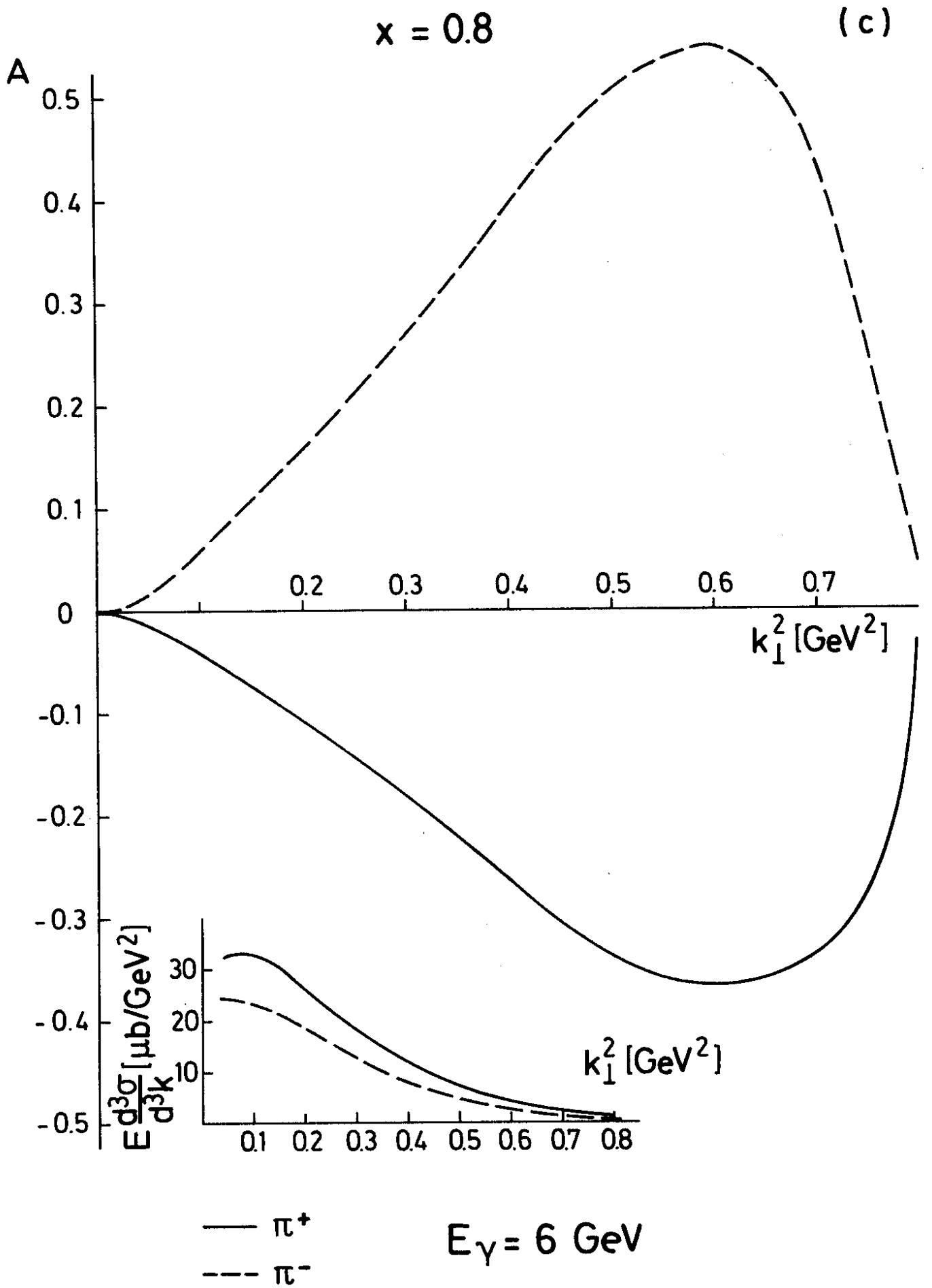


Fig. 4c