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Abstract

Because of the suitability of $\gamma N \rightarrow \phi N$ for studying the Pomeron, we systematically investigate the tests for Pomeron factorisation possible in this rather clean reaction, particularly from the more feasible experiment which measures the ϕ-density-matrix, and also an experiment measuring the recoil nucleon polarisation; the complete set of initial polarisation configurations has been considered.

For any two-body parity-conserving process, a simple consequence of factorisation is M-purity which asymptotically corresponds to purely natural or purely unnatural parity in the crossed channel. Factorisation tests, therefore, include M-purity tests, but M-purity does not necessarily imply factorisation.

For the ϕ-decay density-matrix we give all the possible factorisation tests, and show that our tests are exhaustive. A separate measurement of the recoil nucleon polarisation is shown to complement adequately the information obtained from the ϕ-decay density-matrix in the factorising case.

For the ϕ-density matrix, some of the M-purity tests refer to dominant amplitudes and persist even if s-channel meson-helicity-conservation (which is experimentally true approximately) holds exactly. These tests should be easy to perform. The tests which invoke factorisation more crucially than only M-purity do not persist in that manner; these refer to the helicity nonconserving amplitudes. However, factorisation for such small amplitudes could be advantageously tested here, because of their being masked by the large amplitudes elsewhere.

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The factorisation tests for the ϕ-density-matrix can be used to distinguish a pure Regge pole type Pomeron from a) an M-pure 'cut-pole mixture' type Pomeron or an M-impure (hence nonfactorising) 'cut-pole mixture' type Pomeron and also b) a factorising 'cut-pole mixture' type Pomeron or a nonfactorising 'cut-pole mixture' type Pomeron. Such tests would require polarised photons and/or targets.

Present $\gamma N \to \phi N$ data are not adequate enough to allow firm conclusions about Pomeron factorisation, though they do indicate M-purity for the Pomeron, corresponding to pure natural parity. This is consistent with Pomeron factorisation, but M-purity is only a necessary consequence of factorisation. Better and more $\gamma N \to \phi N$ data are needed to get a more complete picture of Pomeron factorisation.

1. Introduction

The Pomeron is not yet fully understood. In particular, Pomeron factorisation has not been experimentally well established, though there are indications for it [1]. If the Pomeron is some mixture of a Regge cut and a pure Regge pole, it may or not factorise; in general, it would not. If it is a pure Regge pole, Pomeron factorisation would hold.

The present work is encouraged by the importance of the question of Pomeron factorisation, by the hope that $\gamma N \rightarrow \phi N$ is a good laboratory to study the Pomeron, and by the feasibility of the appropriate $\gamma N \rightarrow \phi N$ experiments in the near future. Whatever information one may obtain from other sources, it seems very natural to appeal to $\gamma N \rightarrow \phi N$ before one gets a complete picture of the Pomeron.

A. Some Existing Tests of Pomeron Factorisation

Pomeron factorisation has been found to be good, within experimental errors, in single particle inclusive distributions [2]. Using Mueller's generalised optical theorem, these tests refer to zero momentum transfer where, in general, Regge cut-effects are expected to be the smallest.

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[1] For reviews, see for example refs. [1], [2], [3]

[2] For a review, see for example ref. [4]

A model analysis of proton-proton scattering up to a laboratory momentum of 500 GeV/c in and near the forward direction indicates [5] only minor (at the 0.1% level) deviations from factorisation. A direct experimental answer would require polarised protons, and this has been done [6] so far at relatively low energies (in the few GeV region) where non-Pomeron contributions are expected to be significant.

At nonzero momentum transfers[3], Pomeron factorisation tests have been attempted for ratios of differential cross sections [1],[2],[8] of the type

$$\frac{d\sigma(Ap \to Ap)}{d\sigma(Ap \to Ap^*)} = \text{Independent of } A \qquad (1.1)$$

for a given s and t where s is the total c.m. energy squared and t, the squared momentum transfer variable; A is some projectile ($\pi^\pm$, p, for example) and p^* is some nucleon resonance supposed to be a diffractive excitation of the proton p. These tests, done in the energy region below 30 GeV, are generally consistent with factorisation being satisfied though the experimental errors are often large[4].

Unfortunately, some difficulty lies in unambiguously identifying the resonance p^* taken to be a pure state in Eq. (1.1). Another point is

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[3] The importance of Pomeron factorisation tests for large momentum transfers has been emphasized in ref. [7].

[4] For a review of the corresponding tests at higher energies, see, for example, ref. [3].

that in the cases tested so far, other (non-Pomeron) contributions are not apriori negligible. Also, results from very different experiments may have to be used in certain cases; this brings in further difficulties. A somewhat formal point is that though Eq. (1.1) is a necessary test of factorisation, it does not test the full implications of factorisation for the reactions in question: the test in Eq. (1.1) is sensitive to the factorisation property of primarily those helicity amplitudes which dominate the cross sections in (1.1); even large nonfactorising contributions in the weaker amplitudes[5] would not significantly affect (1.1). This calls for factorisation tests for separate amplitudes; that may be too ambitious, especially for resonances p^* which are not even uniquely identifiable. A similar remark about insensitivity to contributions from t-values where $d\sigma$ is relatively small would apply to tests of the type of Eq. (1.1) integrated over t.

If the Pomeron factorises, it would asymptotically have a purely natural or a purely unnatural parity (see sec.3). The results of polarised proton-proton scattering [6] in the few GeV range are consistent [9] with a dominant natural parity exchange which could be due to Pomeron factorisation, but the assumption of Pomeron dominance may be safe only at much higher energies for this process.

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[5] Such amplitudes may become relatively important at "dips" of the differential cross-section; see also ref. [7].

B. Why Test Pomeron Factorisation in $\gamma N \rightarrow \phi N$?

It is desirable to have Pomeron factorisation tests which are comparatively free from the preceding worries. The reaction $\gamma N \rightarrow \phi N$ may provide the needed possibility because of the following reasons:

a) The only resonance in question (the ϕ meson) is relatively easy to identify.

b) It is believed [10],[11] that $\gamma N \rightarrow \phi N$ gets contributions from only Pomeron exchange. This is empirically indicated [10],[12] because of a decoupling of the ϕ meson from systems (in particular, the secondary trajectories like f^o, A_2, ρ, ----, N, Δ,----) which are commonly regarded as being built up from only nonstrange quarks. Within the quark model, this decoupling is natural [11] because the ϕ is built up as a $\lambda\bar{\lambda}$ system from only the strange quarks.

Though this decoupling[6] is not mathematically rigorous[7], it is experimentically supported. Thus, one expects the other contributions (like f^o exchange) to be

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[6] An extension of this decoupling to other ϕ-like systems ϕ' is interesting. For ϕ' systems having even charge conjugation (e.g., $\phi' = f'(1514)$, $J^P = 2^+$, $I^G = 0^+$) the cross-section for $\gamma N \rightarrow \phi' N$ should be very small because (unlike the case $\phi' = \phi$) even the Pomeron coupling is forbidden here by C-invariance, others being forbidden as for $\gamma N \rightarrow \phi N$.

[7] There is, for example, a nonzero branching ratio for $\phi \rightarrow 3\pi$ decay.

very much weaker here than elsewhere in hadronic reactions. In that sense, one needs not go [13] to extremely high energies to perform Pomeron factorisation tests in $\gamma N \to \phi N$. In fact, this reaction has been previously suggested [1] to [3],[10],[11],[13] as a very good place to study the Pomeron.

c) One can perform Pomeron factorisation tests within the single reaction $\gamma N \to \phi N$, so that problems due to data coming from very different experiments do not arise.

d) Since all the external particles are now well defined, factorisation tests in $\gamma N \to \phi N$ are feasible for separate amplitudes; one can therefore test Pomeron factorisation also for the nondominant amplitudes. In fact, existing data on $\gamma N \to \phi N$ already indicate several features for the Pomeron, but as reviewed in ref. [2], more and better data are needed to allow firm conclusions about, for example, i) its slope, ii) its being s-channel helicity conserving, iii) its being a purely natural parity system, and iv) the phase of the forward $\gamma N \to \phi N$ amplitude.

C. Our Attempt, and Plan of the Paper

The total number of independent functions needed to know the complete set of $\gamma N \to \phi N$ amplitudes decreases if factorisation holds. This puts constraints on the final state density matrix; we are interested in these constraints for various initial and final polarisation configurations because this density matrix contains all the experimental information.

At present, ϕ-meson decay density matrix data [14] are available at 9.3 GeV/c and (2.8 & 4.7) GeV/c as an average over the $-t$ region (0.02-0.8) $(GeV/c)^2$ for unpolarised target and linearly polarised photons; the errors are rather large. In the energy range 4.6 to 6.7 GeV/c these data are available [15] for unpolarised photons and nucleons in the range $|t| < 0.3$ $(GeV/c)^2$. It would be interesting to have all the Pomeron factorisation tests for the general case of the final joint density matrix with the initial photons and nucleons also polarised in complete generality. The measurement of this joint density-matrix seems to be a remote experimental possibility; it requires target and recoil nucleon polarisation information. However, we shall see (subsec. 4,B) that in the factorising case, the joint density matrix does not carry information beyond the ϕ-density matrix and (separately) the recoil nucleon polarisation. Since polarised targets are already being used for other reactions, we shall give Pomeron factorisation tests for the situation when the ϕ-meson decay angular distribution is observed and the polarisation of the final nucleons is summed over, the photons and the target being polarised in generality. We shall also discuss the consequences of Pomeron factorisation for recoil nucleon polarisation, the photons and the target nucleons being polarised in generality.

We mention four classes of factorisation tests. The first two classes, (A) and (B) follow because factorisation and parity conservation imply M-purity [16] which relates amplitudes having reversed meson (or nucleon) helicities (see sec. 3). Asymptotically (to leading order in s), M-purity means pure normality in the t-channel. The type (A) and (B) follow also if only M-purity holds, but no separability of the helicity amplitudes into meson and nucleon vertices. In that sense, the classes (C) and (D) may be regarded as stronger tests of factorisation. The type (C) results from the separability of the joint density matrix into a meson and a nucleon part, a lack of correlation between the two types of particles. This relates the density matrices for different polarisation configurations. The type (D) results from a decrease (due to factorisation) of the number of independent functions needed to describe a given polarisation configuration, and relates different density matrix elements (otherwise independent) within that configuration. Tests of all the four types (A), (B), (C) and (D) occur when the ϕ -decay distribution is measured and the initial state is completely general; Pomeron factorisation tests for this configuration are given in subsec. 4.A where we also prove that these tests are exhaustive for that configuration. Subsection 4.B shows that if Pomeron factorisation holds, a measurement of the recoil nucleon polarisation adequately complements the information obtainable from the configuration discussed in subsec. 4.A. A knowledge of the complete set of meson and of nucleon vertex functions does not, therefore, require a measurement of the correlations between the final nucleon and the ϕ -meson. In that sense, a measurement of the joint density matrix of the ϕ N final state is not obligatory.

Section 5 is devoted to the "practical meaning" of the tests of sec. 4: which of the tests are easy, which ones refer to small amplitudes, how can the tests help to distinguish between a pure Regge pole and a mixture of a Regge cut and a Regge pole.

Though the errors on the relevant density matrix elements [14] are rather large, one may regard the Pomeron in γ N $\to \phi$ N as s-channel helicity conserving to a zeroth approximation [17]; this refers to the meson vertex, see also ref. [15]. There are indications for approximate s-channel helicity conservation also at the nucleon vertex in πN-scattering [17],[18]. In subsec. 5.A is treated the case of a purely s-channel helicity conserving Pomeron, to see which of the Pomeron factorisation tests of subsec. 4.A are for small amplitudes. Since s-channel helicity conservation is experimentally only approximate at the meson as well as at the nucleon vertex, we consider separately the cases of helicity conservation at only the meson vertex, at only the nucleon vertex, and at both the vertices.

Tests in sec. 4 hold when the helicity amplitudes factorise, but no restriction is placed on the phases of the amplitudes. The corresponding case of the relative reality of all the helicity amplitudes arises for a pure Regge pole, and is considered in subsec. 5.B. The results of sec. 4 are more general, and apply when factorising cut contributions may also be present. Several of the tests of subsec. 5.B allow one to make a distinction between a pure pole and a cut-pole mixture of different types. The case of a pure Regge pole when the beam and target are unpolarised has been considered also in ref. [19].

In sect. 6 an attempt is made to confront the tests of sec. 4.A with experiment. As the tables below show, tests of the types (A) and (C) require polarised targets. Half of the type (B) tests and the type (D) can be applied for unpolarised targets, but the class (B) tests only M-purity and the class (D) requires photons to be polarised in generality. Since the available [8] ϕ-meson density matrix data are for only unpolarised

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[8] There are no recoil nucleon polarisation measurements at present.

targets and linearly polarised photons, one can hope to confront only the class (B) with experiment. Unfortunately, the errors are rather large, but one test of this class shows that the Pomeron is unlikely to have unnatural parity, if it has pure normality; this conclusion is not[9] surprising [16],[20]. Better and more data are needed to allow more useful statements about Pomeron factorisation.

A convenient summary of our main points is in sec. 7 taken together with the tables. The following sec. 2 is devoted to our definitions and notation. The factorisation tests of subsec. (4.A.1), (4.A.2) and (5.A) for the ϕ-meson density-matrix are collected in tables.

2. Notation and Definitions

We shall work with s-channel helicity amplitudes $f_{i\alpha}^{i'\alpha'}(s,t)$ throughout. The symbols p_μ, i are the four momentum and the helicity of the target nucleons, p'_μ, i' those of the recoil nucleons; for the photons (vector mesons) we write $k_\mu, \alpha(k'_\mu, \alpha')$. The invariants s and t are defined by $s = -(p+k)^2$, $t = -(p'-p)^2$. As[10] in ref. [16], [21], [22] the polarisation of the photon beam and that of the target are described by the conventional helicity representation of the spin density matrices:

$$\rho_\gamma = \frac{1}{2}(1 + \vec{P}.\vec{\sigma}) = \frac{1}{2}\sum_{\mu=0}^{3} P_\mu \sigma_\mu \tag{2.1}$$

[10] These references may be consulted for further information about our general formalism.

[9] See also footnote 19.

and

$$\rho_N = \frac{1}{2}(1 + \vec{\zeta}\cdot\vec{\sigma}) = \frac{1}{2}\sum_{\mu=0}^{3} \zeta_\mu \sigma_\mu \tag{2.2}$$

where the "four-vector" notation implies $P_o = \zeta_o = 1$, is the unit matrix and $\vec{\sigma}$ represent the three Pauli matrices. The vector

$$\vec{P} = |\vec{P}| \{-\cos 2\phi, -\sin 2\phi, 0\}$$

describes linearly polarized photons with an angle ϕ between the polarisation vector $\vec{\varepsilon} = (\cos\phi, \sin\phi, 0)$ of the photons and the production (xz) plane; P_3 corresponds to circular polarisation. For the target, the parameter ζ_1 (ζ_2) is transverse polarisation in (normal to) the production plane and ζ_3 is the longitudinal polarisation. The unnormalized joint density matrix of the vector meson-nucleon final state is

$$\rho_{N',V}^{i'j',\alpha'\beta'} = \sum_{i,j,\alpha,\beta} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_\gamma^{\alpha\beta} f_{j\beta}^{j'\beta'*} . \tag{2.3a}$$

The unnormalized density matrix $\rho_V^{\alpha'\beta'}$ of the vector meson and $\rho_{N'}^{i'j'}$ of the final nucleon are obtained by summing over respectively the recoil nucleon helicities and the vector meson helicities:

$$\rho_V^{\alpha'\beta'} = \sum_{i,j,\alpha,\beta,i'} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_\gamma^{\alpha\beta} f_{j\beta}^{i'\beta'*} , \tag{2.3b}$$

$$\rho_{N'}^{i'j'} = \sum_{i,j,\alpha,\beta,\alpha'} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_\gamma^{\alpha\beta} f_{j\beta}^{j'\alpha'*} . \tag{2.3c}$$

The normalisation of the helicity amplitudes is provided by the differential cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{2\pi}{E^*}\right)^2 \operatorname{tr} \rho_{N',V} , \tag{2.4}$$

E^* being the photon energy in the c.m. system. The polarisation of the recoil nucleons is described by polarisation parameters ζ_i' $(i = 1, 2, 3)$ analoguous to ζ_i of Eq. (2.2):

$$\begin{aligned} \zeta_1' &= \frac{2\mathrm{Re}\rho_{N'}^{+-}}{\operatorname{tr} \rho_{N'}} , \\ \zeta_2' &= \frac{-2 \,\mathrm{Im}\, \rho_{N'}^{+-}}{\operatorname{tr} \rho_{N'}} , \\ \zeta_3' &= \frac{\rho_{N'}^{++} - \rho_{N'}^{--}}{\operatorname{tr} \rho_{N'}} . \end{aligned} \tag{2.5}$$

We shall expand the density matrices in terms of the polarisation parameters of photons $(P_m, m \neq o)$ and target nucleons $(\zeta_n, n \neq o)$; for example,

$$\rho_V(P_m, \zeta_n) = \rho_V(0,0) + P_m \rho_V(m,0) + \zeta_n \rho_V(0,n) + P_m \zeta_n \rho_V(m,n) \tag{2.6}$$

where $\rho_V(0,0) \equiv \rho_V(P_0, \zeta_0)$ is the vector meson density matrix for the unpolarised case. The indices m and n will always refer to photon and nucleon polarisation components respectively. It is not necessary to consider polarisation mixtures since all the relevant information is contained in coefficients like $\rho_V(m,n)$ (m = 0, 1, 2, 3 and n = 0, 1, 2, 3) of Eq. (2.6). Only the unnormalised density matrix can be written in this form; the normalised density matrix[11]

$$\rho_V(P_m, \zeta_n) \; / \; \mathrm{tr}\, \rho_V(P_m, \zeta_n)$$

is not a simple polynomial in the initial polarisation parameters; similarly the recoil polarisation ζ_i' of Eq. (2.5) is not a simple polynomial in P_m and ζ_n. . We shall, therefore, use unnormalised polarisation parameters

$$\tilde{\zeta}_i'(P_m, \zeta_n) = \zeta_i'(P_m, \zeta_n) \cdot \mathrm{tr}\, \rho_{N'}(P_m, \zeta_n) \tag{2.7}$$

which can be expanded as

$$\tilde{\zeta}_i'(P_m, \zeta_n) = \tilde{\zeta}_i'(0,0) + P_m \tilde{\zeta}_i'(m,0) + \zeta_n \tilde{\zeta}_i'(0,n) + P_m \zeta_n \tilde{\zeta}_i'(m,n) \quad . \tag{2.8}$$

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[11] Schilling et al. [22] use a normalisation independent of the initial polarisations

$$\rho^m_{\alpha\beta} \Big|_{\text{Theirs}} = \rho_V^{\alpha\beta}(m,0)/\mathrm{tr}\, \rho_V(0,0) \Big|_{\text{Ours}}$$

The quantity $\tilde{\zeta}_i'(P_m, \zeta_n)$ is the experimentally measured i-th component of the recoil nucleon polarisation,

$$\zeta_i' \frac{d\sigma}{d\Omega} = \left(\frac{2\pi}{E^*}\right)^2 \tilde{\zeta}_i' \quad , \tag{2.9}$$

where, as on both sides of Eq. (2.4), the arguments (P_m, ζ_n) of ζ_i', of $\frac{d\sigma}{d\Omega}$ and of ζ_i' are implied.

Experimentally, the angular distribution W of one of the pseudoscalar mesons from the decay of the vector meson is [21]

$$\begin{aligned} W(\Theta^*,\phi^*) \frac{d\sigma}{d\Omega} (\Theta,\phi) &= \frac{3}{4\pi} \{\tfrac{1}{2}(\rho_v^{++} + \rho_v^{--})\sin^2\Theta^* + \rho_v^{oo}\cos^2\Theta^* - \\ &- \sin^2\Theta^*[\mathrm{Re}\rho_v^{+-}\cos2\phi^* - \mathrm{Im}\rho_v^{+-}\sin2\phi^*] - \\ &- \frac{1}{\sqrt{2}} \sin2\Theta^*[\mathrm{Re}(\rho_v^{+o}-\rho_v^{o-})\cos\phi^*-\mathrm{Im}(\rho_v^{+o}-\rho_v^{o-})\sin\phi^*]\} \end{aligned} \tag{2.10}$$

where the angles (Θ^*, ϕ^*) of the decay product and $\Omega = (\Theta,\phi)$ are measured in the vector meson rest system and the overall C.M. frame, respectively [21]; the arguments (P_m, ζ_n) for W, $\frac{d\sigma}{d\Omega}$ and $\rho_v^{\alpha\beta}$ are again implied. Eq. (7) of ref. [21] is another way of writing (2.10).

3. Asymptotic Purity of Normality as a Consequence of Factorisation and Parity Invariance

Factorisation of s-channel helicity amplitudes follows [23] asymptotically from the more conventional t-channel factorisation, or may be postulated [24] separately. The s-channel factorisation

$$f_{i\alpha}^{i'\alpha'}(s,t) = \gamma_{\alpha'\alpha}\,(s,t)\;\Gamma_{i'i}\,(s,t) \qquad (3.1)$$

implies that asymptotically one has either purely natural or purely unnatural parity contributions in the t-channel. This holds for any factorising contribution in any two body process, but we shall indicate the proof for only the elastic-like reaction $\gamma N \rightarrow \phi N$. . This statement becomes interesting if the exchanged system is more general than a pure Regge pole.

Parity invariance gives [25]

$$f_{-i-\alpha}^{-i'-\alpha'} = (-)^{i'-i-\alpha'+\alpha}\; f_{i\alpha}^{i'\alpha'} \qquad (3.2)$$

Using (3.1) in (3.2) for amplitudes with the same meson helicities but different nucleon helicities, and taking ratios, one gets

$$\frac{\Gamma_{-i'-i}}{(-)^{i'-i}\Gamma_{i'i}} = M = \text{Independent of } (i',i) \qquad (3.3a)$$

and similarly,

$$\frac{\gamma_{-\alpha'-\alpha}}{(-)^{\alpha'-\alpha}\gamma_{\alpha'\alpha}} = N = \text{Independent of } (\alpha',\alpha) \tag{3.3b}$$

Changing $i \to -i$ and $i' \to -i'$, (3.3a) gives $M = {}^1/M$, $M = \pm 1$. Similarly $N = {}^1/N = \pm 1$. Using (3.3a) and (3.3b) in (3.2) gives $MN = 1$, $M = N = \pm 1$. Writing

$$\Gamma^{\pm}_{i'i} = \Gamma_{i'i} \pm (-)^{i'-i}\,\Gamma_{-i'-i} \quad , \tag{3.4a}$$

Eq. (3.3a) gives

$$\frac{(\Gamma^{+}_{i'i}-\Gamma^{-}_{i'i})/(-)^{i'-i}}{(-)^{i'-i}(\Gamma^{+}_{i'i}+\Gamma^{-}_{i'i})} = M = +1 \text{ or } -1 \tag{3.3c}$$

which holds only if either $\Gamma^{-}_{i'i}$ or $\Gamma^{+}_{i'i}$ vanishes. Similarly, either $\gamma^{+}_{\alpha'\alpha}$ or $\gamma^{-}_{\alpha'\alpha}$ vanishes, where

$$\gamma^{\pm}_{\alpha'\alpha} = \gamma_{\alpha'\alpha} \pm (-)^{\alpha'-\alpha}\,\gamma_{-\alpha'-\alpha} \quad . \tag{3.4b}$$

Moreover, because $M = N$,

both $\Gamma^{+}_{i'i}$ and $\gamma^{+}_{\alpha'\alpha}$ vanish for $M = -1$, (3.5a)

and

both $\Gamma^{-}_{i'i}$ and $\gamma^{-}_{\alpha'\alpha}$ vanish for $M = +1$. (3.5b)

This M-purity [16] implies that the amplitudes[12]

$$n_{i\alpha}^{i'\alpha'} = f_{i\alpha}^{i'\alpha'} + (-)^{i'-i} f_{-i\alpha}^{-i'\alpha'} \tag{3.6a}$$

and

$$u_{i\alpha}^{i'\alpha'} = f_{i\alpha}^{i'\alpha'} - (-)^{i'-i} f_{-i\alpha}^{-i'\alpha'} \tag{3.6b}$$

vanish for M = -1 and M = +1 respectively, and are the surviving ones for M = +1 and M = -1 respectively. In the factorising case, Eq. (3.6) becomes

$$n_{i\alpha}^{i'\alpha'} \equiv \gamma_{\alpha'\alpha} \Gamma^{+}_{i'i} = \frac{1}{2} \gamma^{+}_{\alpha'\alpha} \Gamma^{+}_{i'i} \quad , \quad M = +1 \tag{3.7a}$$

and

$$u_{i\alpha}^{i'\alpha'} \equiv \gamma_{\alpha'\alpha} \Gamma^{-}_{i'i} = \frac{1}{2} \gamma^{-}_{\alpha'\alpha} \Gamma^{-}_{i'i} \quad , \quad M = -1 \tag{3.7b}$$

where the second equalities in (3.7a,b) have used Eq. (3.5). From their definitions, (3.6a,b), the n- and u-amplitudes correspond [26] asymptotically (to leading order in s) to pure normality in the t-channel. For $\gamma N \to \phi N$, M = +1 (-1) means natural (unnatural) parity exchanges.

While factorisation leads to M-purity (which asymptotically is purity of

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[12]Using parity invariance (3.2), one can replace $(-)^{i'-i} f_{-i\alpha}^{-i'\alpha'}$ in (3.6) by $(-)^{\alpha'-\alpha} f_{i-\alpha}^{i'-\alpha'}$ so that one can reverse either the nucleon or the meson helicities.

t-channel normality), the converse is not true. Also, of course, testing M-purity does not test the full content of factorisation.

4. Consequences of Factorisation for the Vector Meson Density Matrix and for the Recoil Nucleon Polarisation

A. The Unnormalised Density Matrix of the ϕ-Meson

1. Consequences of M-Purity

As shown above, factorisation implies M-purity which corresponds, asymptotically, to purity of normality in the t-channel. The first class (A) of factorisation tests follows from the vanishing of interference terms between the M = +1 and the M = −1 amplitudes. For the expansion coefficients of the unnormalised density matrix of the ϕ-meson, this gives

$$\rho_V^{\alpha\beta}(m,n) = 0 \qquad \text{for } n = 1,3 \tag{4.1}$$

This means that the density matrix $\rho_V(P_m, \zeta_n)$ is independent of the target polarisations $\zeta_{1,3}$ in the production plane. Since some of these coefficients vanish due to parity-invariance, and some others are not measurable from the ϕ-decay angular distribution [21], this gives only 24 conditions for the measurable coefficients:

$$\rho_v^{oo}(m,n) = \rho_v^{++}(m,n) = \mathrm{Re}\rho_v^{+-}(m,n) = \mathrm{Re}(\rho_v^{+o}-\rho_v^{o-})(m,n) = 0 \quad ,$$

(A) (4.2)

$$\mathrm{Im}\rho_v^{+-}(m',n) = \mathrm{Im}(\rho_v^{+o}-\rho_v^{o-})(m',n) = 0$$

for m = 2 or 3 and n = 1 or 3,

m' = 0 or 1 .

The other coefficients are linear (incoherent) mixtures [21],[22] of M = +1 and M = −1 contributions. This gives [16] the class (B) of factorisation tests

$$\text{(B)} \; -\frac{\rho_v^{oo}(1,n)}{\rho_v^{oo}(0,n)} = \frac{\rho_v^{++}(1,n)}{\mathrm{Re}\rho_v^{+-}(0,n)} = \frac{\mathrm{Re}\rho_v^{+-}(1,n)}{\rho_v^{++}(0,n)} = -\frac{\mathrm{Re}\rho_v^{+o}(1,n)}{\mathrm{Re}\rho_v^{+o}(0,n)} = M = 1/M \qquad (4.3)$$

for n = 0,2.

These eight relations refer to unpolarised (m=0) <u>and</u> linearly polarised (m=1) photons, and to unpolarised (n=0) targets <u>or</u> targets polarised perpendicular to the production plane (n=2). In fact, one may determine M from the (B) type tests.

<u>2. Further Consequences of Factorisation</u>

The separability (3.1) of the helicity amplitudes into a mesonic part and a nucleonic part makes the density matrix also separable:

$$\rho_{N',V}^{i'j',\alpha'\beta'}(P_m,\zeta_n) = \sum_{i,j}(\Gamma_{i'i}\rho_N^{ij}(\zeta_n)\Gamma^*_{j'j})\sum_{\alpha,\beta}(\gamma_{\alpha'\alpha}\rho_\gamma^{\alpha\beta}(P_m)\gamma^*_{\beta'\beta}) \; ; \tag{4.4}$$

the same property holds for the matrices $\rho_{N'}$ and ρ_V . This has the consequence that

$$\rho_V^{\alpha'\beta'}(m,2)/\rho_V^{\alpha'\beta'}(m,0) = \text{Independent of } (\alpha';\beta') \text{ and of the photon polarisation m,} \tag{4.5}$$

giving rise to the class (C) of factorisation tests:

$$\text{(C)} \quad \frac{\rho_V^{++}(m,2)}{\rho_V^{++}(m,0)} = \frac{\rho_V^{oo}(m,2)}{\rho_V^{oo}(m,0)} = \frac{\mathrm{Re}\rho_V^{+-}(m,2)}{\mathrm{Re}\rho_V^{+-}(m,0)} = \frac{\mathrm{Re}\rho_V^{+o}(m,2)}{\mathrm{Re}\rho_V^{+o}(m,0)} = \frac{\mathrm{Im}\rho_V^{+-}(m',2)}{\mathrm{Im}\rho_V^{+-}(m',0)} = \frac{\mathrm{Im}\rho_V^{+o}(m',2)}{\mathrm{Im}\rho_V^{+o}(m',0)} \text{ for m = 0 (or 1) and m' = 2 or 3.} \tag{4.6}$$

Combining (4.6) for m = 0(1) with (4.3) gives (4.6) for m = 1(0) so that only one of the two m values in (4.6) gives independent tests; this is indicated by the brackets around m = 1 in (4.6) which, in all, provides 7 independent tests for unpolarised targets and those polarised normal to the production plane, for appropriate photon polarisations (m and m').
We shall take m = 0 in (4.6).

The tests (4.2,3,6) are summarised in tables (I a,b,c).
The last class (D) of factorisation tests results because within a given polarisation configuration, factorisation decreases the number of the necessary independent functions. We list in table (I d) these three independent tests for the simplest configuration, i.e., for unpolarised targets. These tests require the coefficients $\rho_V^{\alpha\beta}(m,0)$ for all m. If data for one (say m = 3) photon polarisation do not exist, one may use the (D) type tests to estimate the values of the corresponding coefficients assuming factorisation and using data for all the other three m values.

3. Completeness of the Above Tests

In order to prove that the above tests (4.2,3,6) and those of the class (D) are exhaustive, one recalls that one starts [21] with 48 measurable coefficients $\rho_V^{\alpha\bar{\beta}}(m,n)$, and there are, in all, 42 factorisation tests. Out of these, there are 32 tests for M-purity (types (A) and (B)) and 10 further tests for factorisation (types (C) and (D)). There are, therefore, only 6 independent coefficients which may be taken to be, for example,

$$\begin{aligned} &\mathrm{Re}\rho_V^{+-}(0,0),\ \mathrm{Im}\rho_V^{+-}(2,0),\ \mathrm{Im}\rho_V^{+-}(3,0)\ , \\ &\mathrm{Im}\rho_V^{+o}(2,0),\ \mathrm{Im}\rho_V^{+o}(3,0)\ \text{and}\ \mathrm{Im}\rho_V^{+-}(3,2) \end{aligned} \tag{4.7}$$

which allow, for the factorising case, a reconstruction of the full ρ_V matrix using the tests given above.

The above empirical counting has to be matched by the corresponding dynamical counting in terms of the available amplitude parameters. The necessary independent meson vertex functions are γ_{++}, γ_{+-} and $\gamma_{\dot{o}+}$ and the corresponding nucleon functions are Γ_{++} and Γ_{+-}. For an unpolarised target, only the combination $G = |\Gamma_{++}|^2 + |\Gamma_{+-}|^2$ is relevant, corresponding to a sum over all nucleon helicities. One, then, obtains 5 independent amplitudes corresponding to the five[13] independent real bilinears (each multiplied by G) formed out of the meson vertex functions. One can check that the five coefficients in (4.7) for an unpolarised target correspond exactly to these 5 independent bilinears multiplied by G. Going over to a polarised target, the only[14] additional information needed is the combination

$$H = \mathrm{Im}\ (\Gamma_{++}\Gamma_{+-}^{*}) \tag{4.8}$$

of nucleon vertex functions needed for n = 2 (target polarisation normal to the reaction plane), the other values of n (= 1,3) being eliminated by (4.1). Since in the factorising case,

$$\rho_v^{\alpha\beta}(m,2)/\rho_v^{\alpha\beta}(m,0) = 2H/G \quad , \tag{4.9}$$

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[13] The three magnitudes $|\gamma_{++}|, |\gamma_{+-}|$ and $|\gamma_{\dot{o}+}|$ and the corresponding two relative phases between the three meson vertex functions provide the relevant 5 significant quantities out of which the 5 independent meson bilinears are formed.

[14] The remaining nucleonic bilinears $|\Gamma_{++}|^2 - |\Gamma_{+-}|^2$ and $\mathrm{Re}(\Gamma_{++}\Gamma_{+-}^{*})$ do not appear in ρ_v and are obtainable from the recoil nucleon polarisation, sec. 4.5.

one needs only one further coefficient to provide the necessary dynamical parameter H ; this is the sixth (n=2) coefficient in (4.7). Thus the empirical and the dynamical countings agree.

In the above discussion, we took M to be known from outside the system. Otherwise, the additional parameter M should be included among the dynamically independent ones. The two countings would still match because (4.3) provides eight relations minus the knowledge of M; a similar remark would (equivalently) apply to the tests (D2), (D3). In fact, if the Pomeron factorises, present data indicate that $M = +1$ should hold [2], see also sec. 6. One may also determine M from (4.3) at a particular kinematical(s,t) point, and use it elsewhere.

B. The Recoil Nucleon Polarisation

Recoil nucleon polarisation provides the remaining (nucleon) vertex functions, and together with ρ_V givesthe complete set of independent amplitudes in the factorising case. We show below that there are, in the factorising case, only two independent components of this polarisation providing information beyond ρ_V ; these components correspond to the two nucleon vertex function $(|\Gamma_{++}|^2 - |\Gamma_{+-}|^2)$and $\mathrm{Re}(\Gamma_{++}\Gamma_{+-}^*)$. A measurement of recoil nucleon polarisation is much harder than that of ρ_V ; we have included the present subsection mainly for completeness.

Out of the 48 expansion coefficients $\tilde{\zeta}_i'(m,n)$, $i = 1,2,3$ of the unnormalised recoil polarisation, Eq. (2.8), parity invariance leaves only the following 24 nonvanishing:

$$i = 1,3: \quad m = 0 \text{ or } 1,\ n = 1 \text{ or } 3; \quad m = 2 \text{ or } 3,\ n = 0 \text{ or } 2$$
$$i = 2\ : \quad m = 2 \text{ or } 3,\ n = 1 \text{ or } 3; \quad m = 0 \text{ or } 1,\ n = 0 \text{ or } 2 \qquad (4.10)$$

The 4 relations [21], due to parity invariance,

$$\tilde{\zeta}_2'(m',n') = 2\mathrm{Re}\rho_v^{+-}(m,n) - \rho_v^{oo}(m,n) \tag{4.11}$$

$m' \neq m$, $n' \neq n$; m' or $m = 0$ or 1, n' or $n = 0$ or 2
further reduce the number of $\tilde{\zeta}_1'(m,n)$ which go beyond $\rho_v^{\alpha\beta}$ to 20.

To consider implications of factorisation for the remaining coefficients $\tilde{\zeta}_1'(m,n)$, one notes the equivalent of the tests (4.1) of the type (A)

$$\tilde{\zeta}_i'(m,n) = 0 \text{ for } m = 2 \text{ or } 3 \tag{4.12}$$

which makes 12 further coefficients vanish leaving only $\tilde{\zeta}_{1 \text{ or } 3}'(m,n)$ where $m = 0$ or 1, $n = 1$ or 3 for examination. Out of these 8 coefficients, the equivalent of the tests (4.3) of the type (B) gives, for $m = 0$ or 1,

$$\tilde{\zeta}_3'(m,3) = M\,\tilde{\zeta}_1'(m,1) \tag{4.13a}$$

$$\tilde{\zeta}_3'(m,1) = -M\,\tilde{\zeta}_1'(m,3) \tag{4.13b}$$

further reducing the independent coefficients for consideration to only $\tilde{\zeta}_i'(m,n)$ where $m = 0$ or 1, $n = 1$ or 3, $i = 1$ (or equivalently, 3). Thus given ρ_v and M purity (which is only one consequence of factorisation), one has only 4 coefficients to test factorisation with. These 4 coefficients require target polarisation in the production plane. Factorisation, Eq. (3.1), further relates the two m values:

$$\tilde{\zeta}_1'(1,n) = \tilde{\zeta}_1'(0,n)[\mathrm{tr}\rho_V(1,0)/\mathrm{tr}\rho_V(0,0)] \qquad (4.14)$$

or

$$\zeta_1'(1,n) = \zeta_1'(0,n) \text{ for } n = 1 \text{ or } 3$$

(and similarly for i = 3) leaving only two independent nonvanishing

coefficients which may be taken as $\tilde{\zeta}_i'(0,n)$, n = 1 or 3, i = 1

(or, equivalently, 3) requiring (final and initial) nucleon polarisations in the production plane, for unpolarised photons. These coefficients

$$\tilde{\zeta}_1'(0,1) = M\,(|\Gamma_{++}|^2 - |\Gamma_{+-}|^2).\,L$$

$$\tilde{\zeta}_1'(0,3) = -\,2M.\,\mathrm{Re}(\Gamma_{++}\Gamma_{+-}^{+}).\,L \qquad (4.15)$$

$$L = |\gamma_{++}|^2 + |\gamma_{+-}|^2 + |\gamma_{o+}|^2$$

provide the two remaining nucleon vertex functions which did not appear in ρ_V, the factor L corresponding to a sum over mesonic helicities.

The fact that the recoil nucleon polarisation and ρ_V cover all the meson and nucleon vertex functions is not surprising because, in the factorising case, the most general observable (the joint density matrix of the ϕ-nucleon final state) is given in terms of ρ_V and $\rho_{N'}$. The expansion coefficients of this joint density matrix are

$$\rho_{N',V}^{i'j',\alpha'\beta'}(m,n) = \frac{1}{4}\,J_m^{\alpha'\beta'}\,.\,K_n^{i'j'} \quad ,$$

$$J_m^{\alpha'\beta'} = \gamma_{\alpha'\alpha}(\sigma_m)^{\alpha\beta}\,\gamma_{\beta'\beta}^{*} \quad , \qquad (4.16a)$$

$$K_n^{i'j'} = \Gamma_{i'i}(\sigma_n)^{ij}\,\Gamma_{j'j}^{*} \quad ;$$

while the density matrices $\rho_{N'}$ and ρ_V are given by the expansion coefficients

$$\rho_{N'}^{i'j'}(m,n) = \frac{1}{4} K_n^{i'j'} \sum_\alpha J_m^{\alpha\alpha} \quad ,$$

$$\rho_V^{\alpha'\beta'}(m,n) = \frac{1}{4} J_m^{\alpha'\beta'} \cdot \sum_i K_n^{ii} \quad , \tag{4.16b}$$

so that

$$\rho_{N',V}^{i'j',\alpha'\beta'}(m,n) = \rho_{N'}^{i'j'}(m,n) . \rho_V^{\alpha'\beta'}(m,n)/X$$

$$X = \mathrm{tr}\ \rho_{N';V}(m,n) = \mathrm{tr}\ \rho_{N'}(m,n) = \mathrm{tr}\ \rho_V(m,n) \tag{4.16c}$$

$$= \frac{1}{4} \sum_\alpha J_m^{\alpha\alpha} \sum_i K_n^{ii}$$

where, as in (4.1, 12),

$$\sum_i K_n^{ii} = \sum_\alpha J_m^{\alpha\alpha} = 0 \text{ for } n = 1 \text{ or } 3,\ m = 2 \text{ or } 3 \quad . \tag{4.16d}$$

The relation (4.16c) shows that, in the factorising case, $\rho_{N';V}$ contains no information beyond $\rho_{N'}$ and ρ_V.

5. "Practical Meaning" of Our Factorisation Tests

In order to see how feasible our tests are experimentally, we consider in subsec. 5.A the simplifications that result if s-channel conservation holds for a) the mesonic helicities, b) the nucleonic ones, and c) both the nucleon and the meson helicities; experimentally, there are indications for this conservation of mesonic helicities [14],[15] in $\gamma N \to \phi N$, and of nucleonic helicities [18] in πN elastic scattering, at least as a rough [17] approximation at the 20 % level. Factorisation tests which persist even in the case of helicity conservation would, in actual practice, be easier to perform experimentally because these tests would refer to the dominant $\gamma N \to \phi N$ amplitudes. The other tests which exist only when helicity conservation does not hold refer to small $\gamma N \to \phi N$ amplitudes; factorisation properties of these small amplitudes are better studied through the density matrix than through the spin-averaged differential cross-section where the large amplitudes mask them.

In subsec. 5.B we shall see which of our tests can help one to distinguish between a pure Regge pole type and some mixture of a pole and a cut-type Pomeron. For this purpose, we consider the case of the relative reality of all amplitudes, as should hold for a pure pole type Pomeron.

Since the present section is a "feasibility study", we shall consider factorisation tests for only the ϕ -decay density matrix for a generally polarised initial state, i.e., the tests of the types (A) - (D) of subsec. 4.A.

A. s-Channel Helicity Conservation

Since ρ_V (the quantity under study in this section) involves a summation over nucleon helicities, conservation of s-channel mesonic helicities will be seen to be much more powerful than that of nucleonic ones. Our results are presented in tables (I a,b,c,d) corresponding to the tests of the types (A), (B), (C), (D) respectively. The completeness of the tests can be easily checked in the case of helicity conservation also.

Conservation of mesonic s-channel helicity gives

$$\text{Im}\,\rho_v^{+-}(2,n) = -\,\text{Re}\rho_v^{+-}(1,n),\quad n = 0,\ 2 \tag{5.1}$$

$$\text{and}\quad \text{Re}\rho_v^{+-}(2,n) = \text{Im}\rho_v^{+-}(1,n),\quad n = 1,\ 3 \tag{5.2}$$

without any reference to factorisation or M-purity.

Similarly, nucleonic helicity conservation gives

$$\rho_v^{\alpha\beta}(m,n) = 0 \quad \text{for } n = 1,\ 2 \tag{5.3}$$

without invoking M-purity or factorisation. For nucleonic and mesonic helicity conservation, only the n = 0 part of (5.1) is non-vanishing. Eqs. (5.1 to 3) are, of course, only some of the consequences of helicity conservation.

The tables show that for mesonic (or mesonic and nucleonic) helicity conservation, the only tests are for M-purity which, of course, may hold even without factorisation. Because of the experimental [14],[15] indication for mesonic helicity-conservation in $\gamma N \to \phi N$, it would, therefore, be relatively difficult to perform tests of the types (C) and (D) which test factorisation more crucially than the types (A) and (B). On the other hand, testing factorisation for the small (helicity-nonconserving) amplitudes relevant to the types (C) and (D) would be feasible through $\rho_v(m,n)$ and almost impossible through the overall cross-section.

For the case of full (mesonic and nucleonic) helicity-conservation, M-purity tests of the type (A) require longitudinally polarised targets, and have not yet been performed. The only (M-purity) test[15] of the type (B)

$$\mathrm{Re}\rho_v^{+-}(1,0)/\rho_v^{++}(0,0) = M \tag{5.4}$$

requires only unpolarised targets and linearly polarised photons. Data [14] indicate (see sec. 6) that M equals +1 if (5.4) holds, though the errors are large. In fact, this is the only remaining test for unpolarised targets, and the ones of type (A) are the only ones for polarised targets if full helicity-conservation holds.

B. Relative Reality of All Amplitudes

For a pole type Pomeron, the amplitudes would all have the phase of the signature-factor, and therefore be all relatively real. In distinguishing between a pole and a cut of various types, one should of course, note that, by definition, a pole is not only M-pure, but also factorising.

Out of the coefficients $\rho_v^{\alpha\beta}(m,n)$ and $\rho_v^{\alpha\beta}(m',n)$ occuring in the M-purity tests (4.2) of the type (A), relative reality leaves only the

[15] This test is valid also if only mesonic or only nucleonic helicity is conserved.

eight with m = 3, n = 1 or 3 nonvanishing. Since a pole is M-pure, vanishing or otherwise of any of the 24 tests of type (A) does not give any crucial information beyond M-purity; of course M-impurity would hold for only the non factorising cut-type component of the Pomeron.

The n = 2 coefficients $\rho_v^{\alpha\beta}(m,n)$ of the tests (4.3) of the type (B) vanish[16] because of relative reality, their vanishing being, therefore, an evidence for a pole-type[17] Pomeron. If these do not vanish, the tests (4.3) would distinguish an M-pure cut component from an M-impure cut component being present in the Pomeron. The n = 0 coefficients of (4.3) do not go beyond testing M purity.

The coefficients $\rho_v^{\alpha\beta}(0,2)$ of also the test (4.6) of the type (C) vanish[17] for relative reality, their vanishing, therefore, provides evidence for a pole-type Pomeron. The coefficients $\mathrm{Im}\rho_v^{+-}(2,2)$, $\mathrm{Im}\rho_v^{+o}(2,2)$, $\mathrm{Im}\rho_v^{+-}(3,0)$ and $\mathrm{Im}\rho_v^{+o}(3,0)$ in (4.6) also vanish for relative reality. Then, factorisation requires also $\mathrm{Im}\rho_v^{+-}(3,2)$ and $\mathrm{Im}\rho_v^{+o}(3,2)$ to vanish; this is also clear from the internal consistency of (4.6) and the vanishing of $\rho_v^{\alpha\beta}(0,2)$ in that equation. Again, therefore, the vanishing of $\mathrm{Im}\rho_v^{+-}(3,n)$, of $\mathrm{Im}\rho_v^{+o}(3,n)$, n = 0 or 2, and of $\mathrm{Im}\rho_v^{+-}(2,2)$ and $\mathrm{Im}\rho_v^{+o}(2,2)$ is evidence for

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16 One should remember that exact nucleonic helicity conservation also gives a similar result (5.3), but experimentally this conservation is only approximate.

17 We are excluding the somewhat accidental possibility that 'relative reality' would hold also for cuts.

a pole type Pomeron. If these coefficients (and similarly $\rho_v^{\alpha\beta}(0,2)$) in (4.6) do not vanish, the corresponding tests in (4.6) would distinguish a factorising from a nonfactorising Pomeron of the 'cut-pole mixture' type.

No essential change occurs in the type (D) tests for the case of relative reality; there are some simplifications because the coefficients $\mathrm{Im}\rho_v^{+-}(3,0)$ and $\mathrm{Im}\rho_v^{+o}(3,0)$ now vanish. The simplified tests are

$$\text{(D1)}': \quad [\rho_v^{++}(0,0)]^2 = [\rho_v^{++}(1,0)]^2 + [\mathrm{Im}\rho_v^{+-}(2,0)]^2 \quad ,$$

$$\text{(D2)}': \quad [\mathrm{Im}\rho_v^{+o}(2,0)]^2 = \frac{1}{2}\rho_v^{oo}(0,0)[\rho_v^{++}(0,0) + M\rho_v^{++}(1,0)] \quad , \qquad (5.5)$$

$$\text{(D3)}': \quad \frac{\mathrm{Re}\rho_v^{+o}(0,0)}{M\,\mathrm{Im}\rho_v^{+o}(2,0)} = \left\{\frac{\rho_v^{++}(0,0) - M\,\mathrm{Im}\rho_v^{+-}(2,0) - \rho_v^{++}(1,0)}{\rho_v^{++}(0,0) - M\,\mathrm{Im}\rho_v^{+-}(2,0) + \rho_v^{++}(1,0)}\right\}^M \quad ,$$

which should hold for a pure pole type Pomeron. If these tests are not satisfied, a 'cut-pole mixture' type Pomeron is indicated; factorisation for this mixture can be tested by the more general tests (D1), (D2) and (D3) given in table (Id).

The "feasibility" of the tests of this subsection can be studied exactly as in the preceding subsection 5.A.

6. Comparison with Data

The final state density matrix data are for only the ϕ-meson decay density matrix a) for unpolarised photons and targets [15] and b) for linearly polarised photons and unpolarised targets [14]. For the case a), one can show that there are no factorisation tests[18]. For b), tests of the type (B) are the only ones possible; one may also try to estimate the coefficients $\rho_V^{\alpha\beta}(3,0)$ using these data and assuming factorisation in the form of the (D) type tests.

The 9.3 GeV data, which is an average over the range $0.02 \leq |t| \leq 0.8$ $(\mathrm{GeV/c})^2$, serted in the (B) type tests (4.3) gives, for n = 0,

$$\frac{-0.08 \pm 0.12}{0.00 \pm 0.07} = \frac{-0.18 \pm 0.13}{-0.14 \pm 0.09} = \frac{0.44 \pm 0.15}{0.50 \pm 0.035} = \frac{0.20 \pm 0.11}{-0.01 \pm 0.06} = M \tag{6.1}$$

where we have used the relation[11]

$$\rho^m_{\alpha\beta}\Big|_{\text{Theirs}} = \rho_V^{\alpha\beta}(m,0)/\mathrm{tr}\rho_V(0,0)\Big|_{\text{Ours}} \tag{6.2}$$

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[18] This situation is not changed even if (separately) the recoil nucleon polarisation is also measured. In fact, the only nonvanishing coefficient $\tilde{\zeta}_2'(0,0)$ is already given, Eq. (4.11), by $\rho_V^{\alpha\beta}(m,n)$. Using [16] M-purity, (4.3), one gets $\tilde{\zeta}_2'(0,0) = M\,\mathrm{tr}\rho_V(0,2)$.

of our notation to that of refs. [14],[22]. Because of the rather large errors in (6.1) one cannot draw a firm conclusion, especially since the data are an average over a large t-range. However, the third[19] (and to a lesser extent, the second) ratio does indicate that if the Pomeron is M-pure (which is necessary for it to factorise), it has M = +1. Of course, all the four ratios in (6.1) can be regarded as consistent with M = +1.

The tests of the (D) type could be used to estimate the expected $\mathrm{Im}\rho_v^{+-}(3,0)$ and $\mathrm{Im}\rho_v^{+o}(3,0)$ using data [14] for linearly polarised photons, and assuming factorisation - for comparison with future data for circularly polarised photons. Unfortunately, the errors are large; for example, the value [14] of $\mathrm{Im}\rho^2_{1-1}$ violates the positivity bound in table 2 of ref. [22] on it, even if one allows a one standard-deviation error. From the relation (D1), therefore, one can only say that $\mathrm{Im}\rho_v^{+-}(3,0)$ is very small. Similarly, the relation (D3) is rendered ineffective. The relation (D2) does not involve $\mathrm{Im}\rho^2_{1-1}$ and gives

$$[\mathrm{Im}\rho_v^{+-}(3,0)]^2 \leqslant 0.012 \tag{6.3}$$

normalising to $\mathrm{tr}\rho_v(0,0) = 1$ and if the errors are interpreted literally to mean that a quantity quoted as x ± y lies between x + y and x - y. From

[19] This is confirmed also by the more accurate measurements [20] of the asymmetry Σ ; see ref. [14] for definition of Σ and for further references. See also footnote 20.

(6.3) also, keeping in mind the large errors, one can only say that $|Im\rho_v^{+o}(3,0)|$ is expected to be small.

In summary therefore, present data[20] indicate that the Pomeron is M-pure with M = +1. Better and more data are needed to make more useful statements about Pomeron factorisation.

7. Summary and Discussion

Because of the suitability (see the Introduction) of $\gamma N \to \phi N$ for studying the Pomeron, we have considered testing Pomeron factorisation in this reaction, assuming that the Pomeron is the only driving force for $\gamma N \to \phi N$. This assumption is very natural in the conventional quark model, and is supported by data. Even conservatively speaking, the energy at which Pomeron dominance is expected in $\gamma N \to \phi N$ should be much lower than that in other reactions. Hence one need not go to extremely high energies for testing Pomeron factorisation in $\gamma N \to \phi N$. We have listed in detail (subsec. 4.A.1 and 4.A.2) the possible tests for the more feasible experiments easuring the ϕ-decay density matrix, and also shown (subsec. 4.B) how a measurement of the recoil nucleon polarisation adequately complements the ϕ-density matrix information in the factorising case.

A simple, but quite important, consequence of factorisation of helicity

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[20] The asymmetries P_σ and Σ mainly depend on the ratio $Re\rho_v^{+-}(1,0)/\rho_v^{++}(0,0)$ and do not provide really independent [16] tests of M-purity, because of (4.3).

amplitudes is M-purity as defined in Eqs. (3.6). This holds for any type of a driving mechanism (including arbitrary mixtures of cuts); M-purity corresponds to purely natural or purely unnatural parity exchanges in the crossed channel, to leading order in s. M-purity is a necessary (but not sufficient) consequence of factorisation, but of course, it could hold without any reference to factorisation.

Factorisation tests, therefore, are of two categories: M-purity tests and secondly, those testing the separability, Eq. (3.1), more crucially. To the first category belong the types (A) and (B), while the second category includes the types (C) and (D) for the ϕ-density matrix, see subsec. 4.A . These tests were shown in subsec. 4.A.3 to be exhaustive if only the ϕ-density matrix is measured. As summarised in tables (Ia,b,c,d), various types of target and photon polarisations are needed for the different tests.

The question, "which of the above tests are easy to perform experimentally?" was considered in subsec. 5.A by using the experimentally indicated [14],[15],[17],[18] criterion that amplitudes which do not conserve s-channel helicity are comparatively small. Some of the M-purity tests (of the types (A) and (B)) persist even for helicity conservation, and therefore, refer to dominant amplitudes. These tests should be easy to perform in contrast to the ones of the types (C) and (D). If mesonic helicity is exactly conserved, there are no tests of the types (C) and (D) left. These latter types which test factorisation more crucially than the other two types, therefore, refer to nondominant amplitudes. This is not purely discouraging, however. Factorisation for such small amplitudes would be almost impossible to test in the overall cross-section wherein the large (s-channel helicity-conserving) amplitues would dominate them. The reaction $\gamma N \rightarrow \phi N$, therefore, provides a chance to study Pomeron factorisation even for these small amplitudes through the ϕ-density matrix.

The question, "Can these tests reveal further information related to Pomeron factorisation?" was considered in subsec. 5.B by considering the special case of relative reality of all amplitudes - as is relevant to a pure pole-type Pomeron. Since the density matrix ρ_V simplifies for this relative reality, it was found that useful distinction between a pole Pomeron, an M-pure 'cut-pole mixture' Pomeron and an M-impure (and hence nonfactorising) 'cut-pole mixture' Pomeron could be made by considering some tests of the type (B) requiring a target polarised normal to the production plane. Similarly, distinction between a pole Pomeron, a factorising 'cut-pole mixture' Pomeron and a nonfactorising 'cut-pole mixture' Pomeron could be made by considering some tests of the type (C) requiring unpolarised targets and targets polarised normal to the production plane; this remark holds also for the (D) type tests which require only unpolarised targets.

Present data give some indication for the M-purity (with M = + 1) of the Pomeron - its having a purely natural-parity character - especially [20] at small momentum transfers $-t \sim 0.2$ GeV^2, see also sec. 6. While M-purity is required by factorisation, it does not prove factorisation. More and better data are needed to confirm this M-purity, as embodied in tests of the types (A) and (B), and also to confront with experiment the types (C) and (D) which test factorisation more crucially. One could get a more complete picture of Pomeron factorisation by measuring also the recoil nucleon polarisation, but only the ϕ-density matrix can already give a lot of information.

In conclusion, we repeat that because of the rather clean nature of the reaction $\gamma N \to \phi N$, and because of its being a good laboratory for investigating the Pomeron, it is only natural to appeal to this reaction before one hopes to get a complete picture of Pomeron factorisation. Some of the relevant factorisation tests are relatively easy; others are not so easy. Those of the latter variety refer to nondominant amplitudes which may be "uniquely" studied in $\gamma N \to \phi N$, as noted above. The importance of the question of Pomeron factorisation justifies an experimental investigation of the tests presented in this paper.

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Table Captions

Table Ia: Influence of conservation of mesonic, of nucleonic, and of both (called "full") mesonic and nucleonic helicity conservation on the factorisation tests of type (A) for the ϕ -meson density-matrix expansion coefficients of Eq. (2.6)

Table Ib: Same as for Table Ia, but for tests of type (B)

Table Ic: Same as for Table Ia, but for tests of type (C)

Table Id: Same as for Table Ia, but for tests of type (D)

Table Ia

Type of test; Eq.	Tests in the general case	Tests in the case of helicity - conservation			Remarks
		Mesonic	Nucleonic	Full	
(A); (4.2)	$\rho_v^{oo}(m,n)=\rho_v^{++}(m,n)$ $= Re\rho_v^{+-}(m,n)$ $= Re\rho_v^{+o}(m,n)$ $= Im\rho_v^{+-}(m',n)$ $= Im\rho_v^{+o}(m',n) = 0$, $m = 2$ or 3, $m' = 0$ or 1, $n = 1$ or 3 ++++++	$\rho_v^{++}(3,n)$ $= Re\rho_v^{+-}(2,n)$ $= Im\rho_v^{+-}(1,n) = 0$, $n = 1$ or 3 ++++++ The only tests for a target polarised in the production plane ++++++	Same as in the general case with n = only 3. ++++++	Same as in the 'mesonic' case with n = only 3 ++++++ The only tests for a polarised target ++++++	The other $\rho_v^{\alpha\beta}(m,n)$ of the general case vanish as a result of the appropriate helicity conservation. ++++++ Need targets polarised in the production plane. ++++++ Only M-purity tested
	Total no. of tests = 24	Total no. of tests = 6	Total no. of tests = 12	Total no. of tests = 3	

Table Ib

Type of test; Eq.	Tests in the general case	Tests in the case of helicity - conservation			Remarks
		Mesonic	Nucleonic	Full	
(B); (4.3)	$-\frac{\rho_v^{oo}(1,n)}{\rho_v^{oo}(0,n)} = \frac{\rho_v^{++}(1,n)}{Re\rho_v^{+-}(0,n)} = \frac{Re\rho_v^{+-}(1,n)}{\rho_v^{++}(0,n)} = \frac{-Re\rho_v^{+o}(1,n)}{Re\rho_v^{+o}(0,n)} = M = 1/M$ $n = 0$ or 2	$\frac{Re\rho_v^{+-}(1,n)}{\rho_v^{++}(0,n)} = M$, $n = 0$ or 2 ++++++ The only tests for an unpolarised target or one polarised normal to the production plane ++++++	Same as in the general case[a] with n = only 0 ++++++	Same as in the 'mesonic' case with n = only 0 ++++++ The only tests for an unpolarised target ++++++	The other $\rho_v^{\alpha\beta}(m,n)$ of the general case vanish as a result of the appropriate helicity conservation ++++++ Need unpolarised targets or targets polarised normal to the production plane ++++++ Only M-purity tested
	Total no. of tests = 8	Total no. of tests = 2	Total no. of tests = 4	Total no. of tests = 1	

a) The vanishing of the coefficients $\rho_v^{\alpha\beta}(m,n)$ for n = 2 would follow also if the Pomeron were a pure pole, see subsec. 5.B.

Table Ic

Type of test; Eq.	Tests in the general case	Tests in the case of helicity-conservation
(C); (4.6)	$\frac{\rho_v^{++}(m,2)}{\rho_v^{++}(m,0)} = \frac{\rho_v^{oo}(m,2)}{\rho_v^{oo}(m,0)} = \frac{Re\rho_v^{+-}(m,2)}{Re\rho_v^{+-}(m,0)}$ $= \frac{Re\rho_v^{+o}(m,2)}{Re\rho_v^{+o}(m,0)} = \frac{Im\rho_v^{+-}(m',2)}{Im\rho_v^{+-}(m',0)}$ $= \frac{Im\rho_v^{+o}(m',2)}{Im\rho_v^{+o}(m',0)}$, $m = 0$[a] , $m' = 2$ or 3 ++++++ Need unpolarised targets and those polarised normal to the production plane. ++++++ Total no. of (independent) tests = 7	Nucleonic: Coefficients with n = 2 vanish leaving no test of the type (C). The same, of course, holds for '<u>full</u>' helicity-conservation. ++++++ Mesonic: Combined with the two tests for the 'mesonic' case of table Ib, Eq. (5.1) leads to $\rho_v^{++}(0,2)/\rho_v^{++}(0,0) = Im\rho_v^{+-}(2,2)/Im\rho_v^{+-}(2,0)$ which is the only surviving relation out of those in the previous column. No independent test of the type (C), therefore, remains. ++++++ Total no. of (independent) tests = 0

[a]See subsec. 4.A.2 for m = 1 which is not independent of m = 0.

Table Id

Type of test	Tests in the general case	Tests in the case of helicity-conservation
(D)	<u>(D1)</u>: $[\rho_v^{++}(0,0)]^2 = [\rho_v^{++}(1,0)]^2 + [\mathrm{Im}\rho_v^{+-}(2,0)]^2 + [\mathrm{Im}\rho_v^{+-}(3,0)]^2$ <u>(D2)</u>: $[\mathrm{Im}\rho_v^{+o}(2,0)]^2 + [\mathrm{Im}\rho_v^{+o}(3,0)]^2 = \frac{1}{2}\rho_v^{oo}(0,0)[\rho_v^{++}(0,0) + M\rho_v^{++}(1,0)]$ <u>(D3)</u>: $\mathrm{Re}\rho_v^{+o}(0,0)\{[\rho_v^{++}(0,0) - M\,\mathrm{Im}\rho_v^{+-}(2,0) + M\rho_v^{++}(1,0)]^2 + [\mathrm{Im}\rho_v^{+-}(3,0)]^2\}$ $= M\,\mathrm{Im}\rho_v^{+o}(2,0)\{[\rho_v^{++}(0,0) - M\,\mathrm{Im}\rho_v^{+-}(2,0)]^2 - [\rho_v^{++}(1,0)]^2 - [\mathrm{Im}\rho_v^{+-}(3,0)]^2 -$ $- 2\mathrm{Im}\rho_v^{+o}(3,0)\,\mathrm{Im}\rho_v^{+-}(3,0)[\rho_v^{++}(0,0) - M\,\mathrm{Im}\rho_v^{+-}(2,0)]\}$ ++++++ Total no. of tests = 3	<u>Mesonic</u>: (D1) becomes $\lvert\rho_v^{++}(0,0)\rvert = \lvert\mathrm{Im}\rho_v^{+-}(2,0)\rvert$ which follows from the $n^v = 0$ examples of the mesonic case of table Ib and of Eq. (5.1), leaving no independent tests of the type (D) because (D2) and (D3) become null identities without any reference to factorisation. The same is, of course, true also for the case of '<u>full</u>' helicity-conservation. ++++++ <u>Nucleonic</u>: No change in the form of the type (D) relations which refer to an unpolarised target, and involve a complete sum over all nucleonic helicities. ++++++ Total no. of independent tests = 3 for the nucleonic case, 0 for the other two cases.