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Synthesis of the Nucleon-Nucleon Interaction at "Low" and "High" Energies

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Synthesis of the Nucleon - Nucleon Interaction at "Low" and "High" Energies[+]

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There is no question that any understanding of the two-nucleon interaction at low and medium energies will rest on our knowledge of the nucleon - nucleon (antinucleon) potential communicating with the crossed channel singularities. If we would only be interested describing the two-nucleon system such a potential is not necessary. However, it is most useful in various other aspects such as the deuteron problem, nucleon - nucleon bremsstrahlung, nuclear matter calculations, nucleon - antinucleon bound states, etc. Moreover, solely the potential approach is sufficiently comprehensive as to accumulate and put to account the several pieces of information we have on the two-nucleon interaction.

In the last couple of years the phenomenological potentials such as the Hamada-Johnston[1] and Reid[2] potentials have been successfully replaced by physical one-boson-exchange (OBE) potentials[3-9]. However, the status of the medium range forces and the actual shape of the core is still confusing, and it seems to me that any improvement of this situation will only come from higher energies (probing shorter distances) and adopting concepts of high energy physics.

So far there is no synthesis of the nucleon - nucleon interaction at low and high energies. But there is a great deal of interesting physics to be done.

Before I proceed presenting some concepts how this confusion might be overcome or resolved let me first briefly review where we stand theoretically. I understand that the phenomenological aspect of the present status of the nucleon - nucleon potentials (in particular its merits) will be reviewed in Kermode's talk. So I will ignore that topic.

In Table I I have compiled some candidates of OBE models[9] waiting for their final approval. It is striking (and I consider this a great success) that the various models now tend to agree on the magnitude of the vector meson (i. e., the ω and ρ) coupling constants, and even the coupling constants of everybody's favourite scalar mesons seem to agree (apart from Nagels et al.) when expressed in terms of some effective coupling constant G_{eff} (for the definition see Table I). What is surprising, however, is that the coupling constants do not take notice of the fine-structure of the various models, i.e., relativistic (off-shell) effects, short distance behavior (cut-off), parameterization of the (scalar) two-pion exchange, etc. In this respect hardly two

Ref.		$(g^V_\omega)^2/4\pi$	$(g^V_\rho)^2/4\pi$	g^T_ρ/g^V_ρ	$g^2_\sigma/4\pi$	m_σ	$g^2_\epsilon/4\pi$	m_ϵ	$g^2_\delta/4\pi$	G^2_{eff} c)	cutt-off
3	Relativistic integral equation with generalized OBE potential	9.05	2.42	4.78	1.4	400	6.8	700	-	22.65	vector: $\exp\{(\alpha(t)-1)\ln s\}$ others: $(m^2-\Lambda)/(t-\Lambda)$
4	Schrödinger equation with regularized OBE potential	7.87	3.12	4.96	1.93	416	7.8	1070	1.83	18.05	$[\Lambda/(t-\Lambda)]^2$
5	Schrödinger equation with regularized OBE potential	10.00	2.33	5.18	-	-	13.9	783 b)	1.39	20.00	$[\Lambda/(t-\Lambda)]^2$
6	Blankenbecler - Sugar equation with regularized OBE potential	10.91	3.07	4.98	-	-	14.35	715 b)	2.41	23.08	$\Lambda/(t-\Lambda)$
7	Gross-equation	9.00	4.00	3.70 a)	2.14	364	-	-	-	16.20	no cut-off
8	Schrödinger equation with hard core	12.00	2.78	6.07	-	-	26.58	760 b)	0.78	35.00	individual hard cores for the different partial waves

Table I   The most significant features of some recent OBE models

a) Input to the fit.

b) Finite width corrections.

c) $G^2_{eff} = g^2_\sigma/4\pi\,(1/m^2_\sigma) + g^2_\epsilon/4\pi\,(1/m^2_\epsilon)$, or in case of finite width corrections $G^2_{eff} = g^2_\epsilon/4\pi\,(1/(m^2_\epsilon + 2\,m_\pi\Gamma_\epsilon))$; the dimension is $(GeV)^{-2}$.

of the present OBE models agree, and we may conclude that the low-energy nucleon - nucleon interaction ($\lesssim$ 400 MeV) is not sensitive to any of the details quoted above and, hence, cannot settle this confusion. In particular, this questions the use of any potential adjusted to the nucleon - nucleon phase shifts in other fields apart from improving its present status.

So we are led to ask how we can improve our understanding of the two-nucleon system and find a more convincing way of describing the nucleon - nucleon interaction at short distances and of handling the medium range (two-pion exchange) forces.

Let us first concentrate on the question how we can probe the core region and gain more insight in the nature of the short-range forces.

The short distance behavior of the nucleon - nucleon interaction is nicely reflected by the large-momentum-transfer behavior of the deuteron form factors[10] as, e. g., $G_c(q^2)$. If the deuteron is treated as a bound state of the Bethe-Salpeter equation or, what is equivalent here, of the Blankenbecler-Sugar equation[11] we obtain for large q^2 assuming the Bethe-Salpeter interaction being of the asymptotic form[12] $V((q-k)^2) \underset{(q-k)^2\to\infty}{\simeq} [(q-k)^2]^{-1-\delta} \quad (\delta > 0)$:

$$G_C(q^2) \simeq G_E(q^2)\,[q^2]^{-\frac{3}{2}-\delta} \tag{1}$$

for the scalar and γ_5 coupling and

$$G_C(q^2) \simeq G_E(q^2)\,[q^2]^{-1-\delta} \tag{2}$$

for the vector coupling where G_E is the electric nucleon form factor, and similarly for the other form factors. The asymptotic behavior of the nucleon form factor is fairly well known[13] being $G_E(q^2) \simeq [q^2]^{-2}$ so that equs. (1) and (2) establish a direct relationship between the large-momentum-transfer behavior of the deuteron form factor and the nucleon - nucleon interaction. Unfortunately, the present status of the deuteron form factor data[14] does not allow any conclusions on the power δ . But we have learned[15] that a more informative experiment (at larger q^2) is considered at Stanford which certainly will be a sensitive probe of the nucleon - nucleon interaction. In equs. (1) and (2) double-scattering corrections are not taken into account as have been calculated by Blankenbecler and Gunion[16] and Chemtob, Moniz and Rho[17]. These contributions are expected to fall-off exponentially so that they do not survive asymptotically (provided that the potential is power behaved).

A further means of studying the nucleon - nucleon interaction at short distances is provided by the large-q^2 electro-disintegration of the deuteron (q is the photon four-momentum) near threshold. If we define the spin-averaged structure functions[18] W_1 and W_2 ($t = 0$):

$$W_{\mu\nu} = \sum_{p,n} \langle d | j_\mu(0) | p,n \rangle \langle p,n | j_\nu(0) | d \rangle (2\pi)^4 \delta^4(q+d-p-n)$$
$$= \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1 + \left(d_\mu - \frac{d\cdot q}{q^2} q_\mu\right)\left(d_\nu - \frac{d\cdot q}{q^2} q_\nu\right) \frac{1}{m_d^2} W_2 \, , \tag{3}$$

we find[19] for large q^2 and a Bethe-Salpeter interaction of the (asymptotic) form as given before

$$W_1 \underset{\omega' \approx 1}{\simeq} G_E^2(q^2) \; (\omega' - 1)^{2+2\delta} \tag{4}$$

for the scalar and γ_5 coupling and

$$W_1 \underset{\omega' \approx 1}{\simeq} G_E^2(q^2) \; (\omega' - 1)^{1+2\delta} \tag{5}$$

for the vector coupling where $\omega' = 1 - (d+q)^2/q^2$. Here we have assumed $G_E \simeq G_M$ for simplicity. A similar relation can also be derived for the structure function W_2. For small but fixed c.m. energy $s = (d+q)^2$ equs. (4) and (5) behave like $G_E^2(q^2)\;(q^2)^{-2-2\delta}$ and $G_E^2(q^2)\;(q^2)^{-1-2\delta}$ respectively as q^2 becomes large. In contrast to the situation of the deuteron form factor we have already good information here[20] up to $q^2 \approx 0.4\ \mathrm{GeV}^2$ which is worth analysing it with respect to δ even though q^2 might not cover an energy range being wide enough as to allow a precise conclusion. Experiments at higher q^2 would be welcome, and I understand there are various possibilities to do these kind of experiments (including the deuteron form factor, etc.), e.g., at DESY.

Another possibility of probing the core region and, perhaps, the most suggestive step towards a synthesis for the time being is to look at the nucleon - antinucleon system, a field which has been neglected theoretically but which is developing experimentally[21,22] in the last couple of years and occurs to me being very attractive vor various reasons.

It is well known that the negative G-parity exchange contributions such as, e. g., the π and ω exchange change sign when passing to the nucleon - antinucleon system. This means, e. g., that the ω exchange which is largely responsible for the repulsive core of the two-nucleon interaction will become attractive in case of the nucleon - antinucleon system. Hence, a combined analysis of the nucleon - nucleon and nucleon - antinucleon system would allow to single out the ω exchange contribution (i. e., the core), and I am sure this will have a drawback on various models.

We would conclude that the nucleon - antinucleon system is likely to have a couple of bound states lying considerably below the deuteron. Indeed, Gray et al.[21] found evidence for a $\bar{p}n$ bound state of 83.3 MeV binding energy corresponding to a meson of mass 1794.5 MeV. If one takes the one-boson-exchange model seriously there should even be many other bound states around the masses of the well established meson resonances.

It is tempting to pursue the alternative (to the quark model) that the mesons are nucleon - antinucleon (or, more general, baryon - antibaryon) bound states emerging in a bootstrap-like pattern. This, indeed, has proven to be a fruitful idea[23]. As a matter of fact the bootstrap principle puts some severe constraints on the dynamics of the two-nucleon system, in particular at short distances, as it turns out[23] that the baryon - antibaryon system has a self-consistent solution (where the baryon - antibaryon bound states are forced to coincide with the input mesons) incorporating the low-lying mesons (such as the π, ρ, ω, etc.) which correspond to binding energies of the order of 1 GeV. I believe this is a very good source of information on the two-nucleon system, especially on the nature of the short range forces.

With the existence of deeply bound nucleon - antinucleon states the hard-core models are ruled out. If one tries to fit the core parameters (Λ, α, etc., see Tab. I) to one of the nucleon - antinucleon bound states, one should be aware of the fact that there might be a whole series of bound states as suggested by the bootstrap model.

Everybody agrees that the one-boson-exchange diagrams have to be corrected at large-momentum transfer. There are many other methods how this region can be probed. But let us now ask for the dynamical origin of those short-range corrections.

It is time to face the truth: There are no elementary ρ, ω, etc. mesons, but all of them lie on Regge trajectories. The connection between one-boson and Regge-pole exchange is well established by means of the Khurie-Jones representation[24]. Using the "new" strip approximation[25], the Regge potentials result, in first approximation, in a "form factor" correction being of the form[26], e. g., for ω exchange

$$F(t) \approx \exp\left[(t-m_\omega^2)/m_\omega^2\right] . \tag{6}$$

In general, the Regge potential reads (neglecting the spin of the nucleons and assuming only one trajectory)

$$\begin{aligned} V(s,t) &= g^2 \frac{\pi}{\sin \pi\alpha(t)} \left(K^{\varphi}_{\alpha(t)}(-z_t) \pm K^{\varphi}_{\alpha(t)}(z_t) \right) \\ &= \sum_J (2J+1)\left(1 \pm \exp[-i\pi J]\right) \frac{g^2}{\alpha(t)-J} \exp\left[(\alpha(t)-J)\varphi\right] P_J(z_t) \end{aligned} \tag{7}$$

where K^{φ}_{α} are the so called Khurie functions[27], and the sum is over the (fixed J) resonance poles associated with the t-channel trajectory. The coupling constant g may, in general, be t-dependent. The parameter or function φ widely determines the analytic properties of the potential. Note that the Khurie functions $K^{\varphi}_{\alpha}(-z_t)$ have a cut at $-\infty < -z_t \leq \cosh\varphi$. There are various possibilities of determining φ. The only constraint is that the spurious singularities inherent in the Regge-exchange terms be removed in order to define a proper potential[27].

In the strip approximation, i. e., confining the dynamics requiring a potential to the strip $-s_R \leq s \leq s_R$, and let the potential solely describe the T-matrix beyond that region[25], we would have[28] $\varphi = \ln\left(z_R + \sqrt{z_R^2 - 1}\right)$ with $z_R = -1 - 2s_R/(4m^2-t)$. In this case the potential becomes complex for $s > s_R$ (i.e., the Khurie function K^{φ}_{α} has a cut for $s > s_R$) which manifests itself in the partial-wave summation (7) becoming divergent for $s > s_R$. The disadvantage of this approach is that the potential has a (left-hand) cut in t for $t < -s_R + 4m^2$ arising out of $\varphi(t)$. This cut is supposed to be present in the t-channel partial-wave amplitude but not in the potential.

Another choice would be $z_R = -1 - 2s/(4m^2 - t_R)$ where $t_R = (2m_\pi)^2$ ($t_R = (3m_\pi)^2$) for, e.g., ρ (ω) exchange[3]. The Regge potential so obtained has the correct (t-channel) threshold behavior (i.e., a $(2\,m_\pi)^2$ and $(3\,m_\pi)^2$ branch point due to the 2π and 3π channels coupled to the ρ and ω mesons respectively) which should be of great importance at low and medium energies. Here $\varphi(s)$ gives rise to a left-hand cut in s which again should not be present in the potential. Such a cut is, however, to be expected in the partial wave projection of the potential (in the s-channel) so that this cut does not bother us as we are dealing with partial waves anyway.

In my opinion φ is most adequately chosen by imposing the proper double-spectral function boundaries on the potential[27]. If the double-spectral function boundary is chosen according to the diagram shown in Fig. 1, i.e.

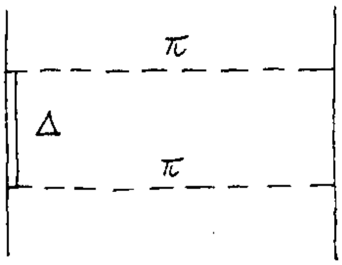

Fig. 1 Diagram contributing to the inelastic s-channel (elastic t-channel) double spectral function

$$s = b(t) = m^2 + m_\Delta^2 + \frac{2m^2 m_\Delta^2}{t - 4m_\pi^2} + \sqrt{\left(m^2 + m_\Delta^2 + \frac{2m^2 m_\Delta^2}{t - 4m_\pi^2}\right)^2 - m^4 - m_\Delta^4 + 2m^2 m_\Delta^2} \qquad (8)$$

which fairly well accounts for the elastic t-channel boundary, we would have $z_Z = -1 - 2\,b(t)/(4m^2 - t)$.

Under normal circumstances the series (7) converges very rapidly and in most cases it is sufficient to consider only the first term corresponding to the original single meson exchange contributions. Treating the ρ, ω, etc. mesons as Regge poles thus provides a dynamical cut-off depending (more or less) on the Regge trajectory $\alpha(t)$ which is known over a fairly wide range from the high energy data.

The last topic I like to discuss concerns the medium range forces being widely described by σ, ϵ, δ, etc. exchange. This subject has caused all kinds of misunderstandings apart from its intrinsic problems how to incorporate finite width corrections.

Everybody agrees that a description of the medium range forces in terms of a few pole terms is a very poor approximation as the two-pion s-wave phase shift[29] indicates a very broad resonance (or even more). Several authors[5,6,8] have faced this problem replacing the propagator of a stable meson by a Breit-Wigner ansatz. This approximation is, however, very crude as it does not take into account any left-hand singularities.

The Regge-potentials discussed above are best suited as to incorporate finite width corrections. The problem then is to cook up any idea about the σ, etc. trajectories. In principle, the trajectories can be separated from the phase shifts by means of

$$-\operatorname{Im}\alpha(t)\,\xi_{\pi\pi} + \frac{1}{2}\ln\frac{\alpha(t)-J}{\alpha^*(t)-J} \underset{t\geq 4m^2}{=} \delta_J(t) \qquad (9)$$

being a consequence of ($\pi\pi$-)elastic unitarity and assuming that the coupling constant g (cf. equ. (7)) has no further cuts in t. Here $\xi_{\pi\pi}$ means the analogue of ξ for pion dynamics. This can be continued below threshold using

$$\alpha(t) = \alpha(\infty) + \frac{1}{\pi}\int dt'\,\frac{\operatorname{Im}\alpha(t')}{t'-t}\,. \qquad (10)$$

In practice, equ. (10) might, however, need some extra information on the large-t behavior of $\operatorname{Im}\alpha(t)$.

The main problem of the medium range forces lies, however, in the fact that the σ, etc. exchange is superimposed by the pomeron (having vacuum quantum numbers). To my knowledge the pomeron has not been taken into account explicitly though it is implicitly contained in the dispersion-theoretical two-pion exchange potentials to be discussed later on.

In the last couple of years the question of the dynamical origin of the pomeron has been largely settled. Drechsler[30] has shown that the pomeron exchange can be generated by multiple exchange of reggeons (or, e.g., pions) considering a sequence of excited intermediate states between the reggeons (pions)

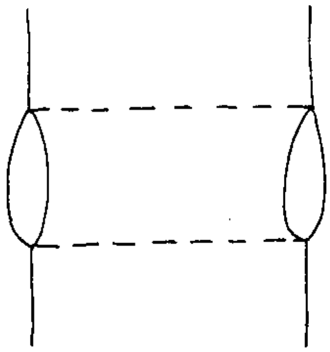

Fig. 2 Double-Regge (pion) exchange generating the pomeron. The dotted lines are reggeons (or, e. g., ordinary pions). The bubbles represent any excited intermediate state such as $\Delta(1236)$, etc.

as shown in Fig. 2. The lowest excited intermediate state one here can think of would be the Δ (1236).

Such inelastic contributions have been analysed with respect to the low-energy nucleon - nucleon interaction by several authors. It has been argued[31-33] that these contributions have the same effect as exchange of a scalar boson at least for not too short distances. This is, however, not to be confused with the "true" σ , etc. exchange contribution but has to be ascribed to the pomeron. Note that the pomeron has a $J = 0$ component (cf. equ. (7)) which has the same quantum numbers as the σ, etc.

We conclude that both the σ, etc. exchange and the pomeron contribution have to be included in the potential. The Regge potential (7) provides a canonical scheme of parameterizing the pomeron contribution as the pomeron coupling constant and the pomeron trajectory are well known from the high energy data (for details see Ref. 28).

Another (though much more elaborate) way of treating the medium range forces has been pursued by Cottingham and Vinh Mau and others[34-38] starting from a Mandelstam representation and the empirical πN- and $\pi\pi$-phase shifts as input. This approach avoids disentangling the various contributions to the two-pion exchange forces as it implicitly contains the σ , etc. and pomeron exchange. True, here are no free parameters involved. But if the once iterated pion-exchange is taken out the rest is not reliable at all as it depends very much on the actual πN- and $\pi\pi$-phase shifts. So, there will also be little hope that this method can be used to learn something about the shape of the pomeron. Nevertheless, it is a nice enterprise.

There are many other interesting features how the theory of low- and medium-energy nucleon - nucleon interaction could benefit from concepts of high energy physics and higher energy data than I could discuss in this talk. Even so not all the points I mentioned can be taken up immediately since we need better data at higher energies (say up to 1200 MeV) and higher q^2.

I like to thank the members of the session group for lively discussions.

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