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Renormalization of Gauge Field Theories with Spontaneous Symmetry Breaking in the Unitary Gauge

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Renormalization of Gauge Field Theories with Spontaneous Symmetry Breaking in the Unitary Gauge.

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Abstract: We derive renormalization prescriptions for Yang-Mills type theories with spontaneous symmetry breaking in the so called "unitary gauge" yielding a mass shell high energy behaviour as good as that of any manifestly renormalizable theory. Taking a smooth high energy behaviour of the S-matrix as sole criterion we show, that the Slavnov identities on the renormalization of the respective model in an equivalent manifestly renormalizable gauge are unnecessary. Our arguments apply only to models without massless particles.

I. Introduction

There exists up to now a variety of approaches towards the renormalization of gauge field theories with spontaneous symmetry breaking [1]. All investigations based on one or another manifestly renormalizable gauge version of the respective model aim at the verification of Slavnov identities [2] for the renormalized Green's functions. These identities establish relations between the renormalization constants. Moreover they are supposed to render a proof for unitarity.

For the sake of an obvious conceptual simplification it seems desirable to have renormalization rules at hand directly applicable to the unitary gauge. Such rules will simplify also to some extent concrete low order perturbation calculations. The main tool for setting up the renormalization in the unitary (U-) gauge will be a simple proof for the invariance of the S-matrix under point transformations of the field variables [3]. The details of this proof tell us how to construct from two Lagrangians related to each other through a point transformation of their fields the corresponding sets of Green's functions giving rise to the same S-matrix.
If one interpolates by a point transformation a renormalizable and a formally nonrenormalizable Lagrangian - that is in our case a gauge field model in the renormalizable (R-) and U-gauge respectively - one has a great amount of freedom in specifying the off mass shell behaviour of the formally nonrenormalizable theory. In other words, the construction prescriptions emerging from the S-matrix equivalence theorem leave a great deal of off-mass shell indetermenateness. This fact has already been observed by Steinmann [4] in the context of the renormalization of massive electrodynamics in the Proca gauge. Steinmann's method consists in a recursive solution of the Glaser, Lehmann, Zimmermann unitarity relations. Our arguments will be based on the renormalization construction of Epstein and Glaser [5].

As our procedure relies heavily on mass shell equivalence, we can treat rigorously only models without massless particles. We select as concrete

examples for our discussion the abelian Higgs-model [6] and a nonabelian simple SU(2)model [7]. The extension of our methods to more complicated gauge field models (that is more sophisticated gauge groups) including spinors is straightforward.
In section II of the present paper we recapitulate the rules taken over from [3], which have to be obeyed in the renormalization of a point transformed Lagrangian in order to guarantee S-matrix invariance. Section III is devoted to a discussion of the abelian Higgs-model. We demonstrate first the mass shell cancellations among the contributions from the point transformed gauge fixing term and the point transformed Faddeev-Popov (F.P.) Lagrangian.[12] Next we state sufficient conditions to be imposed on the renormalization in the R-gauge in order that after the point transformation, a manifestly unitary theory emerges. It turns out that the conditions on the Green's function in R-gauge are weaker than Slavnov identities: It is possible to define a unitary theory with a smooth[1.)] mass shell behaviour starting from gauge noninvariant renormalization prescriptions in R-gauge. Gauge invariance (i.e. Slavnov identities) is only needed to relate the renormalization constants, that is, to minimalize the number of independent parameters in the theory.
In section IV we describe the modifications necessary in comparison with the abelian Higgs-model for the discussion of the nonabelian SU(2) model.

1.) The adjective smooth denotes here and in the following a mass shell high energy behaviour as good as that of a renormalizable theory. This is a generalization of the notion of tree graph unitarity [8].

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II. The invariance of the S-matrix under point transformations

For simplicity we consider in this section a Lagrangian with exclusively scalar particles. The generalization of our procedure to vector particles is straightforward.

Let L and L' be two Lagrangians related through a point transformation

$$\mathcal{L} = \mathcal{L}(\varphi, \partial_\mu \varphi),$$
$$\mathcal{L}'(\varphi) = \mathcal{L}(\varphi + h, \partial_\mu(\varphi + h)), \tag{1}$$

$\varphi = \{\varphi_i, 1 \leqslant i \leqslant n\}$ denotes a collection of scalar particle fields with possibly different masses m_i and $h = \{h_i\}$ the corresponding collection of transformation functions, which are supposed to have a formal power series expansion in the fields φ_i

$$h_i(z)\big|_{z=0} = \frac{\partial h_i}{\partial z_j}\Big|_{z=0} = 0, \tag{2}$$
$$1 \leqslant i, j \leqslant n, \qquad z = (z_1, \dots, z_n).$$

It is convenient to consider an interpolating Lagrangian

$$\mathcal{L}_\lambda = \mathcal{L}(\varphi + \lambda h, \partial_\mu(\varphi + \lambda h)), \qquad 0 \leqslant \lambda \leqslant 1$$

and to construct the derivatives of the $\mathcal{L}_\lambda$- Green's functions with respect to the parameter λ. To start with we try to make the S-matrix equivalence of L and L' in tree graph approximation transparent. With

$$\int dx_1 \dots dx_n \{T(:q_1(x_1): \dots :q_n(x_n):)\}^t$$

we denote the set of all tree graphs (which are not vacuum to vacuum) corresponding to the time ordered product of the normal ordered Fock space operators: $:q_1: \dots :q_n:$.

The set of all tree graphs of the Lagrangian $\mathcal{L}_\lambda$ is

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$$T(\lambda) = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int dx_1 \dots dx_n \left\{ T(:\bar{\mathcal{L}}_\lambda(x_1): \dots :\bar{\mathcal{L}}_\lambda(x_n):) \right\}^t ,$$

$$\frac{dT(\lambda)}{d\lambda} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \sum_{j=1}^{n} \int dx_1 \dots dx_n$$

$$\left\{ T(:\bar{\mathcal{L}}_\lambda(x_1): \dots : \frac{d\bar{\mathcal{L}}}{d\lambda}(x_j) : \dots \quad :\bar{\mathcal{L}}_\lambda(x_n):) \right\}^t , \qquad (3)$$

$$\bar{\mathcal{L}}_\lambda = \mathcal{L}_\lambda - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{m^2}{2} \varphi^2 ,$$

$$m^2 = \{ m_i^2 \} , \qquad m^2 \varphi^2 = \sum_{i=1}^{n} m_i^2 \varphi_i^2 .$$

$\frac{d\bar{\mathcal{L}}}{d\lambda}$ can be manipulated as follows:

$$\frac{d\bar{\mathcal{L}}}{d\lambda} = \partial_\mu \varphi \partial^\mu h - \sum_i m_i^2 \varphi_i h_i + \frac{d\hat{\mathcal{L}}}{d\lambda} =$$

*)
$$= \partial_\mu (\partial_\mu \varphi h) - h_i (\vec{\Box} + m_i^2) \varphi_i$$

$$+ \hat{\mathcal{L}}_{\lambda ; \varphi_i} A_{ik} h_k + \partial_\mu \left(\frac{\delta \hat{\mathcal{L}}}{\delta(\partial_\mu \varphi_i)} A_{ik} h_k \right), \quad (4)$$

$$\hat{\mathcal{L}}_{\lambda ; \varphi_i} = \frac{\delta \bar{\mathcal{L}}_\lambda}{\delta \varphi_i} - \partial_\mu \left(\frac{\delta \bar{\mathcal{L}}_{1\lambda}}{\delta(\partial_\mu \varphi_i)} \right) + \lambda \frac{\partial h_k}{\partial \varphi_i} (\Box + m_k^2) \varphi_k , \text{*)}$$

$$A_{ik} = \left(\delta_{ik} + \lambda \frac{\partial h_i}{\partial \varphi_k} \right)^{-1} .$$

We insert (4) into equ. (3). The terms with total derivatives in front drop out. Of special interest is the contribution of

$h_i (\vec{\Box} + m_i^2) \varphi_i$ to (3):

$$i^n \int dx_1 \dots dx_n \{ T(: \bar{\mathcal{L}}_\lambda (x_1) : \dots : (-h_i) (\vec{\Box} + m_i^2) \varphi_i (x_j) : \dots : \bar{\mathcal{L}}_\lambda (x_n) : \}^t =$$

$$= i^n \int dx_1 \dots dx_n (: \dots (\vec{\Box} + m^2) \varphi \dots : \dots) -$$
$$- \sum_{l=1}^{n-1} (i)^{n-1} \int dx_1 \dots dx_{n-1} \{ T(: \bar{\mathcal{L}}_\lambda (x_1) : \dots : \bar{\mathcal{L}}_{\lambda ; \varphi_i} h_i (x_l) : \dots : \bar{\mathcal{L}}_\lambda (x_{n-1}) :) \}^t , \quad (5)$$

$$\bar{\mathcal{L}}_{\lambda ; \varphi_i} = \frac{\delta \bar{\mathcal{L}}_\lambda}{\delta \varphi_i} - \partial_\mu \left(\frac{\delta \bar{\mathcal{L}}}{\delta \partial_\mu \varphi_i} \right)$$

The first term denotes the graphs not contributing to the mass shell, where $(\Box + m^2)$ acts on an external line. The second term represents all possibilities in which $(\Box + m^2)$ acts on an internal propagator line thereby contracting it to a δ-function.

*) Repeated indices are summed over.

Writing $\bar{\mathcal{L}}_{\lambda;l}\, h_l$

$$\bar{\mathcal{L}}_{\lambda;l}\, h_l = -\, h_l\, \lambda \frac{\partial h_k}{\partial \varphi_l} (\vec{\Box} + m_k^2) \varphi_k + \hat{\mathcal{L}}_{\lambda;l}\, h_l$$

we can perform further contractions:

$$i^{n-1}\int dx_1 \ldots dx_{n-1} \{ T(: \bar{\mathcal{L}}_\lambda(x_1) : \ldots : h_l \lambda \frac{\partial h_k}{\partial \varphi_l} (\vec{\Box} + m_k^2) \varphi_k(x_j) : \ldots : \bar{\mathcal{L}}_\lambda(x_n) :) \}^t =$$
$$= i^{n-1} \int dx_1 \ldots dx_{n-1} : \ldots (\Box + m_k^2) \varphi_k(x_j) \ldots : \ldots \quad +$$
$$+ i^{n-2} \sum_{p=1}^{n-2} \int dx_1 \ldots dx_{n-2} \{ T(: \bar{\mathcal{L}}_\lambda(x_1) : \ldots : h_l \lambda \frac{\partial h_k}{\partial \varphi_l} \bar{\mathcal{L}}_{\lambda;\varphi_k}(x_p) : \ldots$$
$$\ldots : \bar{\mathcal{L}}_\lambda(x_{n-2}) :) \}^t . \tag{6}$$

Equ. (5) and (6) are the first steps in a rearrangement of tree graphs. Continuing the procedure it is easy to see, that the sum of the contracted graphs, not trivially vanishing on the mass shell, add up to geometric series

$$\int dx_1 \ldots dx_n \{ T(: \bar{\mathcal{L}}_\lambda(x_1) : \ldots : \hat{\mathcal{L}}_{\lambda;\varphi_k} A_{ki} h_i(x_i) : \ldots : \bar{\mathcal{L}}_\lambda(x_n) : \}^t \tag{7}$$

and thereby cancel with the noncontracted graphs

$$\int dx_1 \ldots dx_n \{ T(: \bar{\mathcal{L}}_\lambda(x_1) : \ldots : \frac{d\hat{\mathcal{L}}(x_i)}{d\lambda} : \ldots : \bar{\mathcal{L}}_\lambda(x_n) :) \}^t ,$$
$$\frac{d\hat{\mathcal{L}}}{d\lambda} = \frac{d\bar{\mathcal{L}}}{d\lambda} + h_k (\vec{\Box} + m_k^2) \varphi_k .$$

By use of the Epstein-Glaser renormalization construction it was shown in [3] that the cancellation mechanism, as demonstrated above for the tree graph approximation, works analogously in all orders of perturbation theory. Namely, the S-matrix invariance is guaranteed if one uses the free field equations of motion to define the contributions of $\lambda h_k \Box \varphi_k$ to the Green's functions as being equal to those emerging from $-(m_k^2 \lambda h_k \varphi_k)$ up to the terms whose structure is given by the tree graph contractions (7) non vanishing on mass shell.
The important point to note is that S-matrix invariance does not impose restrictions on the construction of time ordered (T-) products involving no constituent $\lambda h_k \Box \varphi_k$. This fact is the reason for the great deal of off-mass shell ambiguity as was mentioned in the introduction. Apart from the axioms of renormalization [9] there are no conditions to be matched in the construction of T-products involving no constituents $\lambda h_k \Box \varphi_k$ but at least one other λ dependent term of $\mathcal{L}_\lambda$ as factor.

We end this chapter with a remark about the role of positive definiteness in the Epstein-Glaser renormalization method. The main achievement of this method is a rigorous proof of unitarity for Lagrangian field theories in manifestly positive definite Hilbert spaces. However the construction itself, that is, the recursive definition of distribution valued T-products as well as the proof for the existence of adiabatic limits (see [5]) does not refer to positive definiteness. The Epstein-Glaser construction recipes can be therefore equally well applied to a nonhermitean Lagrangian (which has of course no direct physical interpretation) with possibly wrong spin-statistic assignment in order to build up Green's functions fullfilling formal cutting rules and causality constraints. By imposing suitable normalization conditions on the two point functions one assures the existence of the adiabatic limit of the Green's functions and the existence of their "mass shell" restrictions. The quotation marks are put in order to indicate the formal meaning of a mass shell in this context.

Green's functions of Lagrangians - nonhermitean and/or with wrong spin - statistics assignment - are taken in the following as an intermediate mathematical remedy. They will be related to Green's functions of physically interpretable models with an underlying positive definite Hilbert space.

III. The abelian Higgs-model

a) Preliminaries

The Lagrangian of the abelian Higgs-model in t' Hooft's R-gauge after translation of the physical scalar field is

$$\begin{aligned}\mathcal{L}_R = & -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}\left|(\partial_\mu - ieA_\mu)(z + v + i\xi)\right|^2 \\ & -\frac{m^2}{2} z^2 - \frac{m^2}{2v} z(\xi^2 + z^2) - \frac{m^2}{8v^2}(\xi^2 + z^2)^2 \\ & -\frac{1}{2}(\partial_\mu A^\mu + M\xi)^2 + \partial_\mu \varphi \partial^\mu \varphi^* - M^2 |\varphi|^2 \\ & - M e \varphi^* z \varphi , \\ & F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \\ & M = ev. \end{aligned} \tag{1}$$

v is the translation parameter, φ and ξ denote the F.P and the longitudinal ghost resp..

To pass over from (1) to the U-gauge formulation of the model one has to apply on the scalar particle Z and the scalar longitudinal ghost ξ a point transformation

$$(i\xi + z + v) = e^{\frac{i\xi'}{v}} (z' + v) \tag{2}$$

Following the lines of chap. II we use also the interpolating point transformation

$$\begin{aligned} \xi &= \xi_\lambda + \lambda\left(\sin\frac{\xi_\lambda}{v}(z_\lambda + v) - \xi_\lambda\right), \\ z &= z_\lambda + \lambda\left(\cos\frac{\xi_\lambda}{v}(z_\lambda + v) - (z_\lambda + v)\right), \\ \xi_{\lambda=0} &= \xi, \quad z_{\lambda=0} = z, \quad \xi_{\lambda=1} = \xi', \quad z_{\lambda=1} = z'. \end{aligned} \tag{2a}$$

We set:

$$\mathcal{L}_\lambda = \mathcal{L}_R\,(A_\mu, \xi(\xi_\lambda, z_\lambda), z(\xi_\lambda, z_\lambda)) ,$$

$$\mathcal{L}_{\lambda=0} = \mathcal{L}_R ,$$

$$\mathcal{L}_{\lambda=1} = \mathcal{L}_U + \Delta\mathcal{L} \qquad (3)$$

$\mathcal{L}_U$ denotes the U-gauge Lagrangian with a Stückelberg-split [10] 2):

$$\mathcal{L}_U = -\frac{1}{4}\,(F_{\mu\nu})^2 + \frac{1}{2}\,\left|\partial_\mu z + ie\,(A_\mu - \frac{\partial_\mu \xi}{M})(z+v)\right|^2 -$$

$$-\frac{m^2}{2}z^2 - \frac{m^2}{2v}z^3 - \frac{m^2}{8v^2}z^4 - \frac{1}{2}\,(\partial_\mu A^\mu + M\xi)^2 . \qquad (3a)$$

ΔL comes out as the result of the point transformation (2) on the F.P. ghost part and the gauge fixing term of L_R (the latter without the term quadratic in the fields)

$$\Delta\mathcal{L} = -(\partial_\mu A^\mu + M\xi)M\left(\sin\frac{\xi}{v}(z+v) - \xi\right) -$$

$$-\frac{M^2}{2}\left(\sin\frac{\xi}{v}(z+v) - \xi\right)^2 + \partial_\mu\varphi^*\partial^\mu\varphi -$$

$$-M^2|\varphi|^2 - Me\varphi^*\left(\cos\frac{\xi}{v}(z+v) - v\right)\varphi \qquad (3b)$$

To prepare the ground for the subsequent discussion we make the following remark:

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2) For the U-gauge fields $z_{\lambda=1}$, $\xi_{\lambda=1}$ we omit the index λ.

The Lagrangians (1) and (3) are first of all synonymous with the corresponding sets of tree graphs. Using the B.P.H.Z. renormalization framework we would have to specify a set of counter terms with coefficients worked out as power series in the Planck constant h. However, as we rely on the Epstein-Glaser construction, we start from the Lagrangians (1) and (3) as they stand. That is, m and M are to be considered as the physical masses of the particles[3] and ν and e are taken as fixed parameters marking the starting point of the inductive Epstein-Glaser construction[4]. It will turn out that the choice of normalization of the R-gauge Green's functions in higher orders of perturbation theory is to a large extent arbitrary (within a minimal subtraction scheme), if we only require for the U-gauge theory interpolated from the R-gauge version a smooth mass shell high energy behaviour (besides causality and unitarity). We will state below the normalization conditions to be observed in order to keep contact with the results of other authors [1].

III. b Mass shell cancellation of ΔL contributions

The interpolation by the point transformation (2a) from (1) to (3) does not lead directly to the desired result because of the disturbing term ΔL. We have to show that the Green's functions involving vertices of Δ L cancel out on the mass shell.
We anticipate here one condition to be imposed on T - products

$$T(:\bar{\mathcal{L}}_u(x_1): \quad \ldots \quad : \bar{\mathcal{L}}_u(x_n):) \tag{4}$$

$$\bar{\mathcal{L}}_u = \mathcal{L}_u + \frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{2}(\partial_\mu \xi)^2 - \frac{1}{2}(\partial_\mu z)^2 + \frac{m^2}{2} z^2 + \frac{M^2}{2}\xi^2 + \frac{(\partial_\mu A^\mu)^2}{2} - M^2 A_\mu^2 .$$

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3) In the case of the R-gauge Lagrangian II.1 we don't insist on a rigorous particle interpretation.

4) In B.P.H.Z. terminology our procedure is a hard quantization.

In the normal product expansion of expressions like (4) $\partial_\mu \xi$ and A_μ have to occur in the Wick ordered operator parts only in the combination $B_\mu = \left(A_\mu - \frac{\partial_\mu \xi}{M} \right)$

$$T\left(:\mathcal{L}_u(x_1): \ldots : \mathcal{L}_u(x_n):\right) =$$

$$= \sum_{\{\alpha,\beta\}} :\left(B^{\mu_1}(x_1)\right)^{\alpha_1} \chi^{\beta_1}(x_1) \ldots \left(B^{\mu_n}(x_n)\right)^{\alpha_n} \chi^{\beta_n}(x_n): T^{\alpha,\beta}(x_1 \ldots x_n) \quad (5)$$

$\{\alpha, \beta\}$ = all Wick contractions

$T^{\alpha,\beta}$ denote the vacuum expectation values of the T-products of the respective submonomials of $\mathcal{L}_u$.

The factorization property (5) is an obvious necessity in view of the fact that it is the field B_μ, which corresponds to the physical vector particle. The Stückelberg split of B^μ into A^μ and $\partial_\mu \xi$ is an auxiliary remedy.

Assume that all T-products of type (4) fullfilling the requirement (5) have already been constructed. We go on to define T-products involving one term

$$B(x) = (\partial_\mu A^\mu + M\xi)(-M)\left(\sin\frac{\xi}{v}(\chi + v) - \xi\right) \quad \text{of } \Delta\mathcal{L}$$

as factor

$$T^{n+1} = T\left(:\mathcal{L}_u(x_1):\ldots:\mathcal{L}_u(x_n)::B(x_{n+1}):\right) \qquad (6)$$

The construction of the expressions (6) proceeds by induction in n. Assume that T-products of type (6) with less than (n+1) points have already been defined. In order to construct the (n+1)-point product (6) along Epstein and Glaser's lines we consider first

$$D^{n+1}(x_1 \ldots x_n ; x_{n+1}) = \sum \left[T(J', x_{n+1}), \overline{T}(J)\right] (-1)^{|J|} \qquad (7)$$

$$J \cup J' = \{x_1 \ldots x_n\}$$
$$J \cap J' = \emptyset$$
$$J \neq \emptyset$$

$$T(J', x_{n+1}) = T(:\mathcal{L}_u(x_{i_1}):\ldots:\mathcal{L}_u(x_{i_\ell})::B(x_{n+1}):),$$

$$\bar{T}(J) = \bar{T}(:\mathcal{L}_u(x_{j_1}):\ldots:\mathcal{L}_u(x_{j_k}):),$$

$$(x_{i_1}\ldots x_{i_\ell}) = J', \quad (x_{j_1}\ldots x_{j_k}) = J$$

$\bar{T}$ denotes antichronological ordering.

A proper definition of the (n+1) point product is obtained by 'splitting' (see [5]) D^{n+1}, that is, one looks for a retarded product $R_{n+1}(x_1\ldots x_n;\, x_{n+1})$ whose support is contained in

$$V^- = \{x = \{x_1\ldots x_{n+1}\} \in R^{4(n+1)};\, (x_i - x_{n+1})^2 \geq 0,\, (x_i^0 - x_{n+1}^0) \geq 0,\, i < n+1\}$$

such that the support of $(D^{n+1} + R^{n+1})$ is in $V^+ = -V^-$. (The support of D^{n+1} is contained in $V^+ \cup V^-$ if up to this point we followed the Epstein-Glaser recipes).

The T-product (6) is then given by

$$T^{n+1} = R(x_1\ldots x_n;\, x_{n+1}) - \sum_{\substack{J \cup J' = \{x_1\ldots x_n\} \\ J \cap J' = \phi \\ J \neq \phi}} \bar{T}(J)\,T(J', x_{n+1})(-1)^{|J|} \qquad (8)$$

Note that the second term on the right hand side of (8) as well as D^{n+1} is already defined under the assumptions made above.

Taking into account the factorization property (5) and the identity

$$\langle_0 T((\partial_\mu A^\mu + M\xi)(x)\,(A_\lambda - \frac{\partial_\lambda \xi}{M})\,y\,)_0\rangle^f =$$

$$= \langle_0(\partial_\mu A^\mu + M\xi)(x)\,(A_\lambda - \frac{\partial_\lambda\xi}{M})\,(y)_0\rangle^f = 0 \qquad 5.)$$

5.) The free field propagators resp. Wightman functions to be used can be read off from the quadratic terms in (1) or (3):

$$\langle_0 T(A^\mu(x)A^\nu(y))_0\rangle^f = -\frac{ig^{\mu\nu}}{(2\pi)^4}\,\frac{\int d^4p\, e^{-ip(x-y)}}{p^2 - M^2 + i\epsilon}$$

.... etc.

it is easy to see that the following induction hypothesis for $T^{(n)}$ is consistent and that it can be reproduced for T^{n+1}: The factor $(\partial_\mu A^\mu + M\xi)$ appears in the normal product expansion of the operator expressions (6), (7), (8) always in the Wick ordered operator parts. All terms in which $(\partial_\mu A^\mu + M\xi)$ is contracted into an internal propagator or Wightman function line vanish identically.

Next we inspect T-products involving two factors

$$B = (\partial_\mu A^\mu + M\xi)\, M\left(\sin\frac{\xi}{v}(z+v) - \xi\right)$$

Exploiting the permissible ambiguities in the construction of the retarded functions we achieve the following net result: In the normal product expansion of

$$T(:\bar{\mathcal{L}}_u(x_1):\ldots:\bar{\mathcal{L}}_u(x_n)::B(x_{n+1})::B(x_{n+2}):) \qquad (9)$$

appear besides terms:

$$:(\partial_\mu A^\mu + M\xi)\ldots:\ldots$$

expressions of the form

$$(-i)\delta(x_{n+1}-x_{n+2})\, T(:\bar{\mathcal{L}}_u(x_1):\ldots:\bar{\mathcal{L}}_u(x_n)::M^2\left(\sin\frac{\xi}{v}(z+v)-\xi\right):(x_{n+1})) \qquad (10)$$

and terms "with ghost loop structure" (to be explained below).

The contributions (10) to (9) arise, naively speaking, from the contraction of two factors $(\partial_\mu A^\mu + M\xi)$ into the same propagator line

$$\langle_0 T((\partial_\mu A^\mu + M\xi)(x_{n+1})(\partial_\mu A^\mu + M\xi)(x_{n+2}))_0\rangle^+ \sim \delta(x_{n+1}-x_{n+2}) \qquad (11)$$

with no other internal line connecting the vertices x_{n+1} and x_{n+2}. Terms in which a second line connects the vertices x_{n+1} and x_{n+2} in (11) are striked out. This way of proceeding is justified by invoking the permissible ambiguities of renormalization: terms with factors

$\langle_0 T((\partial_\mu A^\mu + M\xi)(x)(\partial_\mu A^\mu + M\xi)(y))_0\rangle^+$ appear

in the inductive construction first at places where they only contribute to the totally coinciding point of the respective T-product distribution.

Expressions of the form (10) can be defined such that they vanish together with the T-products involving the M^2-terms of $\Delta\mathcal{L}$:

$$T\left(:\bar{\mathcal{L}}_u(x_1):\dots:\bar{\mathcal{L}}_u(x_n)::(-M^2)\left(\sin\frac{\xi}{v}(z+v)-\xi\right)^2(x_{n+1}):\right)$$

Finally we have to exemplify what we mean by ghost loop structure: Consider the normal product expansion of the two point functions

$$:B(x_1)::B(x_2):\;=\;:eM\left(\cos\frac{\xi}{v}(z+v)-v\right)(x)\,eM\left(\cos\frac{\xi}{v}(z+v)-v\right)(y): \left(\langle_0\,\xi(x)\,\xi(y)_0\rangle^f\right)^2+\dots \tag{12}$$

$$:F.P(x)::F.P.(y)=(-1):eM\left(\cos\frac{\xi}{v}(z+v)-v\right)(x)\,eM\left(\cos\frac{\xi}{v}(z+v)-v\right)(y): \langle_0\,\varphi(x)\varphi^*(y)_0\rangle^f\langle_0\varphi^*(x)\,\varphi(y)_0\rangle^f+\dots \tag{13}$$

$$F.P=\varphi^*Me\left(\cos\frac{\xi}{v}(z+v)-v\right)\varphi$$

The relative minus sign between the terms written out in (12) and (13) comes from the Fermi quantization prescription for the F.P. ghosts.

It can be easily verified with the help of the Epstein-Glaser method that contributions to T-products with equal graphical structure - the simplest example noted in (12) and (13) - can be defined to be equal. The additional signature factor for the F.P. ghosts render the cancellation among the terms with ghost loop structure (see equ. 12) and the original F.P. ghost loop contributions.

By now it should be clear what strategy one has to follow in the inductive construction of T-products with $\Delta\mathcal{L}$ factors, in order to achieve their mass shell cancellation. We confine ourselves to spell out the basic ingredients giving the following (exhaustive) enumeration of situations one encounters in the normal product expansion of T-products involving $\Delta\mathcal{L}$ terms[6.)]:

α.) At least one factor $(\partial_\mu A^\mu + M\xi)$ appears in the Wick ordered operator part. These terms drop out on the mass shell because of transversality of the physical vector particle.

β.) Terms in which two factors $(\partial_\mu A^\mu + M\xi)$ are contracted into the same propagator line cancel all contributions derived directly from

$$-\frac{M^2}{2}\left(\sin\frac{\xi}{v}(z+v) - \xi\right)^2$$

γ.) Expressions with contractions of $(\partial_\mu A^\mu + M\xi)$ with fields of $\mathcal{L}_u$ vanish identically. The necessary and sufficient condition to achieve this result is the factorization property (5).

δ.) All parts of T-products not listed under $\alpha) - \gamma)$ with $(\partial_\mu A^\mu + M\xi)(-M)(\sin\frac{\xi}{v}(z+v) - \xi)$ factors have ghost loop structure and vanish together with the corresponding original F.P. ghost loop contributions. The crucial points leading to this conclusion are first the Fermi quantization prescription for the F.P. ghosts and second the simple relation between the point transformed gauge fixing term (i.e. that part which is linear in λ $(\lambda = 1)$) and the point transformed F.P. ghost Lagrangian

$$\text{F.P.} = -\varphi^* Me\left(\cos\frac{\xi}{v}(z+v) - v\right)\varphi\,,$$

$$\lambda(\partial_\mu A^\mu + M\xi)(-M)\left(\sin\frac{\xi}{v}(z+v) - \xi\right)\Big|_{\lambda=1}$$ are connected by

$$M\frac{\partial}{\partial\xi}\left(-M\left(\sin\frac{\xi}{v}(z+v) - \xi\right)\right) = -Me\left(\cos\frac{\xi}{v}(z+v) - v\right). \qquad (14)$$

6.) We use for this purpose a somewhat abbreviated and therefore 'naive' language. It is easy to attain a standard satisfying all pretensions of rigour by penetrating through a lengthy and straightforward discussion using at every step the Epstein-Glaser method.

III. c Renormalization prescriptions in the unitary gauge

It is important for the subsequent discussion to note which parts of $\mathcal{L}_U$ come from λ dependent and λ independent terms of $\mathcal{L}_\lambda$ respectively.

$$
\begin{aligned}
\mathcal{L}_\lambda = & -\tfrac{1}{4}\left(F_{\mu\nu}\right)^2 + \tfrac{1}{2}\Big|\left(\partial_\mu - ieA_\mu\right)\Big((z+v)(1-\lambda) + \\
& + \lambda \cos\tfrac{\xi}{v}(z+v) + i\xi(1-\lambda) + i\lambda \sin\tfrac{\xi}{v}(z+v)\Big)\Big|^2 - \\
& - \tfrac{m^2}{2}\left(z + \lambda\left(\cos\tfrac{\xi}{v}(z+v) - (z+v)\right)\right)^2 - \\
& - \tfrac{m^2}{2v}\left(z + \lambda\left(\cos\tfrac{\xi}{v}(z+v) - (z+v)\right)\right)\Big(\left[\xi(1-\lambda) + \lambda \sin\tfrac{\xi}{v}(z+v)\right]^2 + \\
& + \left[z + \lambda\left(\cos\tfrac{\xi}{v}(z+v) - (z+v)\right)\right]^2\Big) - \\
& - \tfrac{m^2}{8v^2}\left(\left[\xi(1-\lambda) + \lambda \sin\tfrac{\xi}{v}(z+v)\right]^2 + \left[z + \lambda\left(\cos\tfrac{\xi}{v}(z+v) - (z+v)\right)\right]^2\right)^2 \\
& - \tfrac{1}{2}\left(\partial_\mu A^\mu + M\left(\xi + \lambda\left(\sin\tfrac{\xi}{v}(z+v) - \xi\right)\right)\right)^2 \\
& + \partial_\mu \varphi\, \partial^\mu \varphi^* - M^2|\varphi|^2 - Me\,\varphi^* z \varphi,
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
\mathcal{L}_U = & -\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}\left(\partial_\mu \xi\right)^2 + \tfrac{1}{2}\left(\partial_\mu z\right)^2 - \\
& - \tfrac{m^2}{2}z^2 - \tfrac{m^2}{2v}z^3 - \tfrac{m^2}{8v^2}z^4 - \tfrac{1}{2}\left(\partial_\mu A^\mu\right)^2 - \tfrac{M^2}{2}\xi^2 + \\
& + \tfrac{e^2}{2}\left(A^\mu\right)^2(z+v)^2 + 2e\,A_\mu \partial^\mu \xi\, z + e\,A_\mu \partial^\mu z\,(1-\lambda)\,\xi + \\
& + \tfrac{\lambda^2}{2}\left(\left(\partial_\mu \xi\right)^2\left(z^2 + 2zv\right)\right) + \lambda^2 e^2 A_\mu \partial^\mu \xi\, z^2 + \\
& + O\left(\lambda(1-\lambda)\right)\Big|_{\lambda=1}.
\end{aligned}
\tag{16}
$$

We are interested in (15) only in so far as we extract from the interpolating $\mathcal{L}_\lambda$ theory the rules to be respected in the renormalization of the much simpler Lagrangian $\mathcal{L}_U = \mathcal{L}_{\lambda=1} - \Delta\mathcal{L}$, in order to obtain a smooth mass shell behaviour.

The main result of chap. II was, that an appropriate handling of the point transformed part of the kinetic energy, which is linear in λ, is uniquely responsible for S-matrix invariance.
In particular S-matrix invariance does not impose restrictions on the definition of T-products not involving this term. We used the indeterminateness left over after the requirement of S-matrix invariance already tacitly in the preceding section, when we constructed the T-products with $\Delta\mathcal{L}$ vertices. We exploit it further by constructing the contributions to the Green's functions of terms $\mathcal{O}(1-\lambda)$ in (16) such that they cancel out for $\lambda = 1$.

We are left over with the Lagrangian:

$$\begin{aligned}\mathcal{L}_U = & -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\left((\partial_\mu\xi)^2 + (\partial_\mu z)\right)^2 + \frac{e^2}{2}A_\mu^2(z+v)^2 - \\ & - \frac{m^2}{2}z^2 - \frac{M^2}{2}\xi^2 - \frac{(\partial_\mu A^\mu)^2}{2} - \frac{m^2}{2v}z^3 - \frac{m^2}{8v^2}z^4 + \\ & + 2eA_\mu\partial^\mu\xi z + \lambda^2\Big[\frac{(\partial_\mu\xi)^2}{2}(z^2+2zv) + \\ & + e^2A_\mu\partial^\mu\xi z^2\Big]\Big|_{\lambda=1}\,. \end{aligned} \qquad (17)$$

Note that the remaining λ^2 terms are not subject to restrictions imposed by S-matrix invariance.

We are now in a position to read off from (17) the renormalization prescriptions in the U-gauge: Construct the T-products involving only λ independent terms with a minimal number of subtractions. Add the T-products with λ dependent terms of dimension 5 and 6 such that a unitary and causal theory emerges (especially, the factorization property III. b.(5) has to be fullfilled). The infinitely many subtractions necessary for these T-products are, apart from the axiomatic constraints, to be chosen arbitrarily.

Remarks: 1.) A minimal subtraction scheme for the T-products of the λ-independent terms provides in view of the S-matrix equivalence of R- and U-gauge a smooth mass shell behaviour for the whole theory.

2.) The unitarity and causality constraints are fullfilled if one proceeds along the following lines: The T-products of the λ independent terms are constructed so that the formal cutting rules (with respect to the λ independent part of the interaction) and the causality relations are respected. Using the Epstein-Glaser construction this is automatically built in. All c-number two point kernels of the normal product expansion of λ independent T-products have to be assigned the correct mass and wave function renormalization (that is, a pole at the point of the physical mass with unit residue). The prescriptions, given so far, are quite general and not at all restricted to gauge field models. The only specific rule to be observed in order that the λ independent Green's functions can be completed by the λ dependent ones to a manifestly unitary and causal theory concerns the factorization property III. b(5): (Amputated) graphs differing from each other by one attached external line, which is either A_μ or $\partial_\mu \xi$ line[7.)], are defined equal.

[7.)] That is, terms in the normal product expansion of the T-products, which go over into one another by substituting one field A_μ in the Wick ordered operator part by field $\frac{\partial_\mu \xi}{M}$.

Obeying the same rule also in the construction à la Epstein-Glaser of the λ dependent T-products we fullfill the factorization requirement III. b. (5) which together with the cutting rules (now with respect to the whole Lagrangian $\bar{\mathcal{L}}_U$), the causality relations and the correct mass and wave function renormalizations render the axiomatic respectability of the U-gauge theory in the Stückelberg-split formulation.

3.) Using the normalization conventions for graphs with vertices emerging from the λ independent terms of (17), as they are specified for the same graphs in the R-gauge approaches of references [1] , we obtain with our recipes the same S-matrix results as by calculations based on R-gauge prescriptions of [1] . However these normalization prescriptions are not a necessity in order that we can pass over via the λ - interpolation from R- to U-gauge. We have for this purpose to follow the (weaker) rules given under 2.).
In particular, no Slavnov identities for the R-gauge Green's functions are needed.

4.) Different choices of the infinitely many subtractions, mentioned under 2.), lead to a finite number of different S-matrices corresponding to the different possibly non gauge invariant normalization prescriptions in the R-gauge.

IV. The nonabelian SU (2) model

In t'Hooft's R-gauge the model under consideration is given by [7]

$$
\begin{aligned}
\mathcal{L}_R = & -\frac{1}{4}(\vec{B}_{\mu\nu})^2 + \frac{1}{4}\,\mathrm{tr}\,(D_\mu C\,(D^\mu C)^*) - \\
& - \frac{m^2}{2}\,z^2 - \frac{m^2}{2\sqrt{2}F}\,z\,(z^2 + \vec{K}^2) - \frac{m^2}{16F^2}(z^2 + \vec{K}^2)^2 - \\
& - \frac{1}{2}(\partial_\mu \vec{g}^\mu + M\vec{K})^2 + \partial_\mu \vec{\varphi}^*\,\partial^\mu \vec{\varphi} - M^2 \vec{\varphi}^*\vec{\varphi} + \\
& + g\,\partial_\mu \vec{\varphi}^*(\vec{g}^\mu \otimes \vec{\varphi}) - \frac{Mg}{2}\,\vec{\varphi}^*(z + \vec{K}\otimes)\vec{\varphi}\,, \qquad (1)
\end{aligned}
$$

$$\vec{B}_{\mu\nu} = \partial_\mu \vec{g}_\nu - \partial_\nu \vec{g}_\mu + g\,\vec{g}_\mu \otimes \vec{g}_\nu\,,$$

$$M = g\cdot\frac{F}{\sqrt{2}}\,,$$

$$
\begin{aligned}
D_\mu C = & \;\partial_\mu (z\,\mathbb{1} + i\,\vec{K}\cdot\underline{\tau}) + \\
& + (\partial_\mu - i\,\frac{g}{2}\,\vec{g}_\mu\cdot\underline{\tau})((z + \sqrt{2}F)\mathbb{1} + i\,\vec{K}\cdot\tau)\,,
\end{aligned}
$$

$$\vec{K}\cdot\underline{\tau} = i\sum K_i\cdot\tau_i\,,$$

τ = Pauli matrices, $\mathbb{1}$ = unit matrix,

F is the translation parameter of the spontaneous symmetry breaking,

z denotes a physical scalar particle. $\vec{K}$ and $\vec{\varphi}$ stand for the longitudinal and F.P. ghosts respectively.

To pass over to the U-gauge we perform the following substitutions in (1):

$$\left((z+\sqrt{2}F)\mathbb{1}+i\vec{K}\cdot\underline{\tau}\right)=e^{\frac{i\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}}(z'+\sqrt{2}F)\,\mathbb{1}, \tag{2}$$

$$\frac{i\vec{q}_\mu\cdot\underline{\tau}}{2}=e^{\frac{i\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}}\left(\frac{i}{2}\right)\underline{\tau}\left(\vec{q}_\mu^{\,\prime}-\frac{\partial_\mu\vec{K}'}{M}\right)e^{\frac{-i\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}}+$$

$$+\frac{1}{g}\left(\partial_\mu e^{\frac{i\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}}\right)e^{\frac{-i\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}}, \tag{3}$$

$$\mathcal{L}_R=\mathcal{L}_R(\vec{q}_\mu,\vec{K},z),$$

$$\mathcal{L}_R\left(\vec{q}_\mu(\vec{q}_\mu^{\,\prime},\vec{K}',z'),\vec{K}(\vec{K}',z'),z(\vec{K}',z')\right)=\mathcal{L}_U+\Delta\mathcal{L}, \tag{4}$$

$$\mathcal{L}_U=-\frac{1}{4}(B'_{\mu\nu})^2+\frac{1}{2}\left|\partial_\mu z'+\frac{ig}{2}\left(\vec{q}_\mu^{\,\prime}-\frac{\partial_\mu\vec{K}'}{M}\right)(z'+\sqrt{2}F)\right|^2-$$

$$-\frac{m^2}{2}z^2-\frac{m^2}{2\sqrt{2}F}z^3-\frac{m^2}{16F^2}z^4$$

$$-\frac{1}{2}\left(\partial_\mu\vec{q}^{\,\prime\mu}+M\vec{K}'\right)^2. \tag{4a}$$

The point transformations (2) and (3) have to be executed one after the other because otherwise the recipes of chapter II are not applicable[8.)].

The renormalization prescriptions for $\mathcal{L}_U$ which give a smooth mass shell behaviour are analogous to those for the abelian model (and are justified by the same arguments): Identify the part of $\mathcal{L}_U$ coming from L_R and the part coming in through the interpolation (in the language of the preceding chapter the λ dependent and λ independent terms)

$$
\begin{aligned}
\mathcal{L}_U = & -\frac{1}{4}\left(\vec{B}'_{\mu\nu}\right)^2 + \frac{1}{2}\left(\partial_\mu z'\right)^2 - \frac{m^2}{2} z'^2 - \frac{m^2}{2\sqrt{2}F} z'^3 - \\
& - \frac{m^2}{16F^2} z'^4 - \frac{1}{2}\left(\frac{g}{2}\vec{g}'^\mu (z+\sqrt{2}F)\right)^2 - \\
& - g\,\vec{g}'_\mu \frac{\partial^\mu \vec{K}'}{M}\sqrt{2}F z' - \frac{1}{2}\left(\left(\partial_\mu \vec{g}'^\mu\right)^2 + \left(M\vec{K}'\right)^2\right) + \\
& + \frac{\lambda^2}{2}\left[\left(\frac{g}{2}\frac{\partial_\mu \vec{K}'}{M}\right)^2 \left(z^2 + 2\sqrt{2}\, zF\right) + \frac{g^2}{2}\frac{\partial_\mu \vec{K}'}{M}\vec{g}'^\mu z^2\right]\Bigg|_{\lambda=1} \qquad (5)
\end{aligned}
$$

$$
\vec{B}'_{\mu\nu} = \partial_\mu \vec{g}'_\nu - \partial_\nu \vec{g}'_\mu + g(\lambda\partial_\mu \vec{K}' + \vec{g}'_\mu) \otimes (\lambda\partial_\nu \vec{K}' + \vec{g}'_\nu)\Big|_{\lambda=1} .
$$

Define the T-products of λ idependent factors with a minimal number of subtractions. Add the T-products with λ dependent terms so that correct

---

8.) The interpolation method described in chapter II applies to Lagrangians with interactions involving no higher than first derivatives. The point transformed term $-\frac{1}{2}(\partial_\mu \vec{g}^\mu + M\vec{K})^2$ contains a second derivative of $\vec{K}$. If we perform the point transformations (2), (3) one after the other a $\Box\vec{K}$ appears only in the second step as part of the transformation function of $\vec{g}^\mu$, where it does not matter.

mass and wave function renormalizations, cutting rules with respect to $\mathcal{L}_U$, causality relations and the transscribed factorization property III.b.5 are satisfied. The remarks at the end of the section III.c apply without exception also to the nonabelian model.

The only point requiring some further discussion in comparison with the abelian case concerns the mass shell cancellations of $\Delta\mathcal{L}$ terms. The points α.) to γ.) of III.b can be immediately taken over to the nonabelian model. All that remains to be done is to find a similiarly simple connection as (14) between the λ dependent part of the gauge fixing term and the point transformed F.P. ghost Lagrangian. The latter can be written in the form

$$\begin{aligned}
F.P. &= \partial_\mu \vec{\varphi}^* \partial^\mu \vec{\varphi} - M^2 \vec{\varphi}^* \vec{\varphi} + g\partial_\mu \vec{\varphi}^* (\vec{\underset{\sim}{g}}^\mu \otimes \vec{\varphi}) - \\
&\quad - \frac{Mg}{2} \vec{\varphi}^* (\underset{\sim}{z} + \underset{\sim}{\vec{K}} \otimes) \vec{\varphi}, \\
\frac{i\vec{\underset{\sim}{g}}_\mu \cdot \underline{\tau}}{2} &= e^{i\frac{\vec{K}'\underline{\tau}}{\sqrt{2}F}} \frac{i}{2} \underline{\tau} \left(\vec{g}'_\mu - \frac{\partial_\mu \vec{K}'}{M}\right) e^{-i\frac{\vec{K}'\underline{\tau}}{\sqrt{2}F}} + \frac{1}{g}\left(\partial_\mu e^{i\frac{\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}}\right) e^{-i\frac{\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}}, \\
\underset{\sim}{\vec{K}} &= \frac{\vec{K}'}{\sqrt{\vec{K}'^2}} \sin \frac{\sqrt{\vec{K}'^2}}{\sqrt{2}F}, \qquad \underset{\sim}{z} = z' \cos \frac{\sqrt{\vec{K}'^2}}{\sqrt{2}F}.
\end{aligned} \tag{6}$$

Introducing the transformation matrix A_{ik} of the left handed nonlinear chiral transformation

$$e^{\frac{i\vec{K}'\underline{\tau}}{\sqrt{2}F}} \rightarrow e^{i\frac{\vec{\alpha}\cdot\underline{\tau}}{2}} e^{i\frac{\vec{K}'\cdot\underline{\tau}}{\sqrt{2}F}} = e^{i\frac{K''\underline{\tau}}{\sqrt{2}F}}$$

$$A_{ik} = \left.\frac{\partial K_i''}{\partial \alpha_k}\right|_{\vec{\alpha}=0}$$

we can write (6) as follows:

$$F.P. = \partial_\mu \varphi^{i*} \left(\left(\frac{\delta}{\delta(\partial_\mu K_j')} - \frac{\delta}{\delta g'_{\mu,j}}\right) \underset{\sim}{g}^{\mu,i}\right) A_{jk} \partial_\mu \varphi^k +$$

$$+ M\partial_\mu \varphi^i \frac{\delta A^{\mu i}}{\delta K'_j} A_{jk} \varphi_k - M^2 \varphi^{i*} \frac{\delta K^i}{\delta K'_j} A_{jk} \varphi^k \tag{7}$$

Equ. (7) suggests a point transformation of the F.P. ghosts:

$$\varphi^{i*} = \tilde{\varphi}^{i*}, \qquad \tilde{\varphi}^i = A_{ik} \varphi_k \tag{8}$$

which translates (7) into

$$\begin{aligned} F.P. = {} & \partial_\mu \tilde{\varphi}^{i*} \Big(\Big(\frac{\delta}{\delta(\partial_\mu K'_j)} - \frac{\delta}{\delta(g'_{\mu,j})} \Big) \underset{\sim}{g}^{\mu i} \partial_\mu \tilde{\varphi}^j - \\ & - M^2 \tilde{\varphi}^{i*} \frac{\delta \tilde{K}_i}{\delta K'_j} \tilde{\varphi}^j + M \partial_\mu \tilde{\varphi}^{i*} \frac{\delta \underset{\sim}{g}^{\mu i}}{\delta K'_j} \tilde{\varphi}_j , \end{aligned} \tag{9}$$

and $\Delta\mathcal{L}$ into

$$\begin{aligned} \Delta\mathcal{L} = {} & F.P.(\tilde{\varphi}^*, \tilde{\varphi}, K', z') - \frac{1}{2} (\partial_\mu \underset{\sim}{\vec{g}}^\mu + M \underset{\sim}{\vec{K}})^2 \\ & + \frac{1}{2} (\partial_\mu \vec{g}'^\mu + M \vec{K}')^2 . \end{aligned} \tag{10}$$

Equations (9) and (10) represent the analogon to the relations III.b.14 for the abelian Higgs model. They guarantee, that those parts of the T-products with factors

$$(\partial_\mu \vec{g}'^\mu + M\vec{K}')(\partial_\mu \underset{\sim}{\vec{g}}^\mu + M \underset{\sim}{\vec{K}} - (\partial_\mu \vec{g}'^\mu + M\vec{K}'))$$

not listed under III.b. α) - γ) have ghost loop structure and can therefore be manipulated to vanish together with the $\tilde{\varphi}^*, \tilde{\varphi}$ ghost loop contributions. One has to note in this context, that also the point transformed ghosts $\tilde{\varphi}^*, \tilde{\varphi}$ have to obey Fermi statistics.

V Conclusion

The main purpose of this paper was to single out the essential ingredients necessary for the renormalization of unitary gauge field theories with smooth mass shell behaviour. Slavnov identities do not belong to these essentials.(They may be rather regarded as a convenient technical remedy, in proving unitarity in R-gauge formulations (see [11])). It is an open question what kind of anormalies we obtain starting from a non gauge invariant renormalization in the R-gauge (only fulfilling the weaker conditions of chapters III and IV) and interpolating to the manifestly unitary gauge. Our conjecture is that the exclusion of anormalies in the equations of motion and in the Ward identities for the spontaneously broken reflection symmetry of the scalar particle, together with the requirement of mass shell smoothness, fix the S-matrix uniquely.

References

[1] Lectures by G. t'Hooft and B.W. Lee in "Renormalization of Yang Mills Fields and Applications to Particle Physics, CNRS, Marseille, June 1972, where the original work by G. t'Hooft, B.W. Lee, M. Veltman, J. Zinn-Justin is reported; O. Piguet, Construction of a Strictly Renormalizable Effective Lagrangian For The Massive Abelian Higgs Model, to appear in Comm. Math. Phys.; J.H. Lowenstein, A. Rouet, R. Stora, W. Zimmermann, Renormalizable Models With Broken Symmetries, lectures given at the International Summer School on "Renormalization and Invariance in Quantum Field Theory, Capri, July 1973; C. Becchi, A. Rouet, R. Stora, lecture given at an informal meeting on 'Recent Progress in Lagrangian Field Theory and Applications',Marseille, June 1974.

[2] A.A. Slavnov, Theor. Math. Phys. 10, No. 2, 153 (1972)

[3] R. Flume, The Invariance of the S-Matrix under Point Transformations in Renormalized Perturbation Theory, to appear in Comm. Math. Phys.

[4] O. Steinmann, Axiomatic Field Theory and Quantum Electrodynamics: The Massive Case, Universität Bielefeld preprint, Bi - 74/13, June 1974.

[5] H. Epstein and V. Glaser, CERN preprint TH 1344 (1971) and CERN preprint TH 1400 (1971).

[6] P. Higgs, Phys. Rev. 145, 1156 (1966)

[7] See for example G. t'Hooft, Nucl. Phys. B35, 167 (1971).

[8] C.H. Llewellyn Smith, Phys. Letts. B46, 233 (1973); J.M. Cornwall, D.N. Levin and G. Tiktopoulos, Phys. Rev. Letts. 30, 1268 (1973)

[9] K. Hepp, Les Houches Lectures (1970)

[10] E.C.G. Stückelberg, Helv. Phys. Acta 30, 209 (1956)

[11] C. Becchi, A. Rouet, R. Stora, The Abelian Higgs Kibble Model. Unitarity Of The S-Operator, CNRS Marseille preprint 74/P. 625, June 1974.

[12] L.D. Faddeev and U.N. Popov, Phys. Letters 25B, 29 (1967).