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Relations Between the Electroexcitation of Nucleon Resonances and the Deep Inelastic Continuum for Proton and Deuteron Targets

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Relations Between the Electroexcitation of Nucleon Resonances and the Deep Inelastic Continuum for Proton and Deuteron Targets

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Abstract

New data of the electroexcitation cross section on protons and deuterons in the resonance region are compared with the results in the deep inelastic region for four-momentum transfers $q^2 \leq 1.5$ $(GeV/c)^2$. The Bloom-Gilman sum-rule is shown to hold for $q^2 \geq 1.2$ $(GeV/c)^2$, while the Rittenberg-Rubinstein sum rule is saturated for $q^2 \geq 0.4$ $(GeV/c)^2$. It is shown that the sum rules do not saturate in a local way.

I. Introduction

A systematic investigation of the electroexcitation cross section on protons and deuterons in the resonance region for four-momentum-transfers $q^2 \leq 1.5\ (GeV/c)^2$ has been performed by the Karlsruhe-DESY collaboration[1]. Figs. 1 and 2 show the results of these experiments in a compressed representation.

The results of these measurements have been used to derive properties of the dominant nucleon resonances[2]. Another important aspect of these data is their connection to the deep inelastic electron scattering.

The relation between the deep inelastic and the resonance cross section has been discussed by various authors[3-6] and has been checked for the electroexcitation on the proton[3,7,8]. In the present paper we want to investigate the validity of the sum rules in the region of small four-momentum transfers ($q^2 \leq 1.5\ (GeV/c)^2$) in the case of proton and deuteron targets.

II. Sum Rules

In the case of constant four-momentum transfer Bloom and Gilman[3] derived the sum rule

$$\int_0^{\nu_{max}} \{\nu W_2(q^2, \ ) - F_2(\omega')\}\, d\nu = 0 \qquad (1)$$

assuming that the structure function νW_2 shows scaling behaviour in the deep inelastic region and that the forward scattering amplitude for virtual photons obeys a superconvergence relation. $F_2(\omega')$ is the scaling function in the deep inelastic region as determined by the SLAC-MIT group[9,10]. W_2 is the structure function in the resonance region, which can be derived from the twofold differential cross section:

$$\frac{d^2\sigma}{d\Omega dE} = \sigma_M \{W_2(q^2,\nu) + 2\ \mathrm{tg}^2\ \Theta/2\ W_1(q^2,\nu)\} \qquad (2)$$

σ_M = Mott cross section

q^2 = four momentum transfer

W_1, W_2 = structure functions

Θ = electron scattering angle

ν = energy loss of the electron during the scattering process

The structure functions W_1, W_2 are related to the absorption cross section of virtual photons σ_t, σ_ℓ according to

$$\frac{W_1}{W_2} = \frac{1 + \frac{\nu^2}{q^2}}{1 + \frac{\sigma_\ell}{\sigma_t}} \qquad (3)$$

We assume, in accordance with existing data[11,12]

$$\frac{\sigma_\ell}{\sigma_t} = 0.18 \qquad (4)$$

and calculate W_2 from the measured cross section (2) with the help of (3) and (4).

Rittenberg and Rubinstein[4] generalised the sum rule (1) by postulating a vector sum rule which allows to give a connection between deep inelastic data and resonance cross sections along any line in the q^2-ν plane kinematically accessible

$$\int_{\nu_1}^{\nu_2} \{\nu W_2(q^2,\nu) - F_2(\omega')\}\, d\nu = 0 \qquad (5)$$

A further ingredient of all sum rules is the choice of the scaling variable. The Bjorken Variable

$$\omega = \frac{2M\nu}{q^2} \qquad (6)$$

is prefered by theoretical arguments in the limit

$$\nu \to \infty, \quad q^2 \to \infty, \quad \omega = \text{finite}$$

while in the kinematical region accessible to experiments scaling behaviour is observed in an extended interval if one plots the data as a function of[3]

$$\omega' = \frac{2M\nu + M^2}{q^2} \qquad (7)$$

or of[4,6]

$$\omega_W = \frac{2M\nu + a^2}{q^2 + b^2} \qquad (8)$$

where a^2 and b^2 have been determined from fits[4,5,6] as

$$a^2 = 1.43 \left(\frac{\text{GeV}}{c}\right)^2$$

$$b^2 = 0.42 \left(\frac{\text{GeV}}{c}\right)^2$$

From light cone arguments[13] the variable

$$\omega_L = \frac{M}{|\vec{q}| - \nu} \tag{9}$$

is derived.

In the present paper the scaling variables (6) - (9) have been used to investigate the region of validity of finite energy sum rules for the inelastic electron scattering on proton and deuteron targets. Since $W_2(q^2,\nu) = O(q^2)$ because of gauge invariance, the zero has been cancelled by investigating the structure function[4-6]

$$\frac{\omega}{\omega_W} \nu W_2 \tag{10}$$

The sum rule then reads:

$$\int_0^{\nu_{max}} \frac{\omega}{\omega_W} \nu W_2(q^2,\nu)\, d\nu = \int_0^{\nu_{max}} F_2(\omega_W)\, d\nu \tag{11}$$

III. Results and Discussion

We have investigated the validity of the sum rule for different constant values of the four momentum transfer. The structure function W_2 is derived from the measured cross section[1] with the help of formulas (2) - (4) and interpolated along lines of constant invariant mass W by a cubic interpolation formula. The error of this interpolation is estimated to be of the order of less than 4%, while the typical error of the experimental data is 3% to 5%.

In figs. 3 and 4 we have plotted the values of νW_2 for hydrogen as determined from our experiment and the structure functions $F_2(\omega')$, $F_2(\omega_L)$ and $(\omega_W/\omega)F_2(\omega_W)$ derived from the data in the deep inelastic region[9,10]. The results are given for the four momentum transfers $q^2 = 0.4$ $(GeV/c)^2$ and $q^2 = 1.4$ $(GeV/c)^2$. For the high values of the four momentum transfer the structure functions saturate the sum rules, while at $q^2 = 0.4$ $(GeV/c)^2$ only the sum rule (11) for the structure function (10) is satisfied. The respective results for the deuteron are given in figs. 5 and 6. They show a behaviour analogue to that of the proton data. The light cone variable (9) does not improve the saturation compared to the Bloom-Gilman variable (7).

To demonstrate the saturation of the sum rule quantitatively in fig. 7 the ratio

$$\frac{I_2}{I_1} = \frac{\int_0^{\nu_{max}} \nu W_2(q^2,\nu)\, d\nu}{\int_0^{\nu_{max}} F_2(\omega')\, d\nu} \tag{12}$$

is plotted as a function of the four momentum transfer q^2. Results are given for two upper limits W_{max} of the invariant mass which is connected to ν_{max} by

$$W_{max} = \sqrt{M^2 + 2M\nu_{max} + |q^2|}$$

Taking into account that the systematic errors of both experiments[1,9] used in the analysis are of the order of 3% to 4%, from fig. 7 follows that the sum rule saturates for $q^2 \geq 1.2$ $(GeV/c)^2$. The behaviour for proton and deuteron data is the same.

Minimizing the ratio $(I_2/I_1)-1$ in the case of the structure function (10) the parameters a^2 and b^2 have been determined to

$$a^2 = (1.3 \pm 0.07)\ (\mathrm{GeV}/c)^2$$
$$b^2 = (0.4 \pm 0.02)\ (\mathrm{GeV}/c)^2 \qquad (13)$$

Within the error limits the results for proton and deuteron targets coincide, they are in good agreement with the results of foregoing analyses[4,5,6].

In fig. 8 the ratio

$$\frac{I_2}{I_1} = \frac{\int_0^{\nu_{max}} \frac{\omega}{\omega_W}\, \nu W_2(q^2,\nu)\, d\nu}{\int_0^{\nu_{max}} F_2(\omega_W)\, d\nu} \qquad (14)$$

has been plotted as a function of q^2 for an upper integration limit W_{max} = 1.8 GeV. The parameters in (14) are used for the plot. In the whole q^2-region covered by the present experiment (fig. 1 and 2) the sum rule is in good agreement with the proton and deuteron data, while differences are observed for the smaller value of the integration constant W_{max} = 1.3 GeV. From this follows that the sum rule saturates only if the contribution of all resonances is taken into account. Hence local duality[3,4] describes only in an approximate way the data in the resonance region.

In conclusion we have shown that the Bloom-Gilman sum rule holds for $q^2 \geq 1.2\ (\mathrm{GeV}/c)^2$ while the Rittenberg-Rubinstein sum rule holds for all four-momenta investigated for proton and deuteron targets. Local duality describes the data only approximately, a result which was obtained[14] in electron ^{12}C scattering too.

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FIGURE CAPTIONS

Fig. 1: Lines of constant νW_2 in the ν-q^2 plane for protons. Data were taken along the straight lines of constant scattering angle.

Fig. 2: Same as fig. 1 for deuterons.

Fig. 3: νW_2 of hydrogen as a function of ν for $q^2 = 0.4$ $(GeV/c)^2$. For comparison the scaling function in the deep inelastic region is plotted for the variable ω', the light cone variable ω_L and the variable ω_W.

Fig. 4: Same as fig. 3 for $q^2 = 1.4$ $(GeV/c)^2$.

Fig. 5: νW_2 of deuterium as a function of ν for $q^2 = 0.4$ $(GeV/c)^2$. For comparison the scaling function in the deep inelastic region is plotted for the variable ω', the light cone variable ω_L and the variable ω_W.

Fig. 6: Same as fig. 5 for $q^2 = 1.4$ $(GeV/c)^2$.

Fig. 7: Ratio (12) as a function of q^2 for two different values of the upper integration limit. Values for protons and deuterons are plotted.

Fig. 8: Ratio (13) as a function of q^2 for two different values of the upper integration limit. Results for protons and deuterons are plotted.

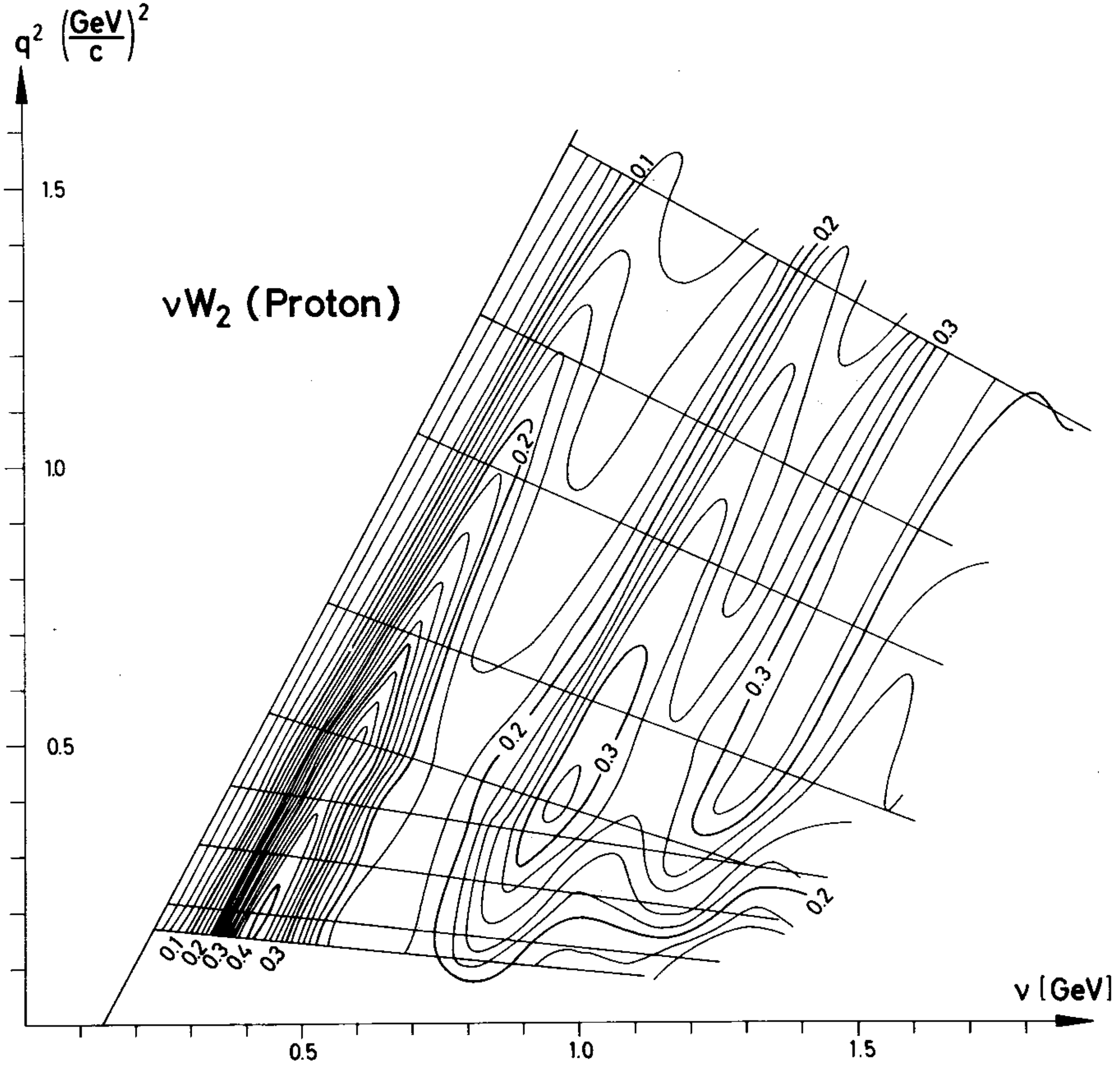
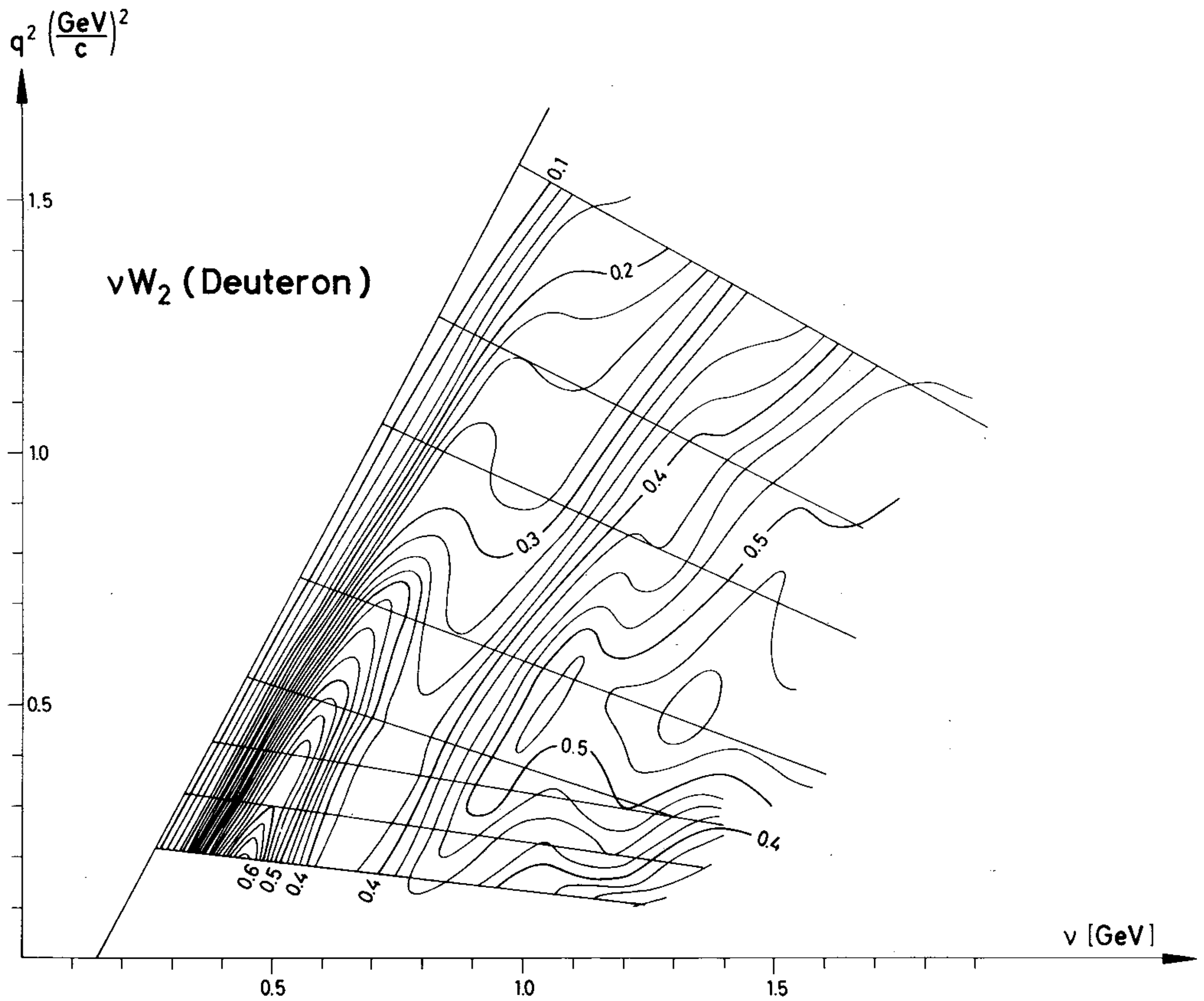


FIG.1



$q^2 \left(\frac{GeV}{c}\right)^2$
1.5
1.0
0.5
νW_2 (Deuteron)
0.1
0.2
0.3
0.4
0.5
0.5
0.4
0.6
0.5
0.4
0.4
ν [GeV]
0.5
1.0
1.5

FIG.2

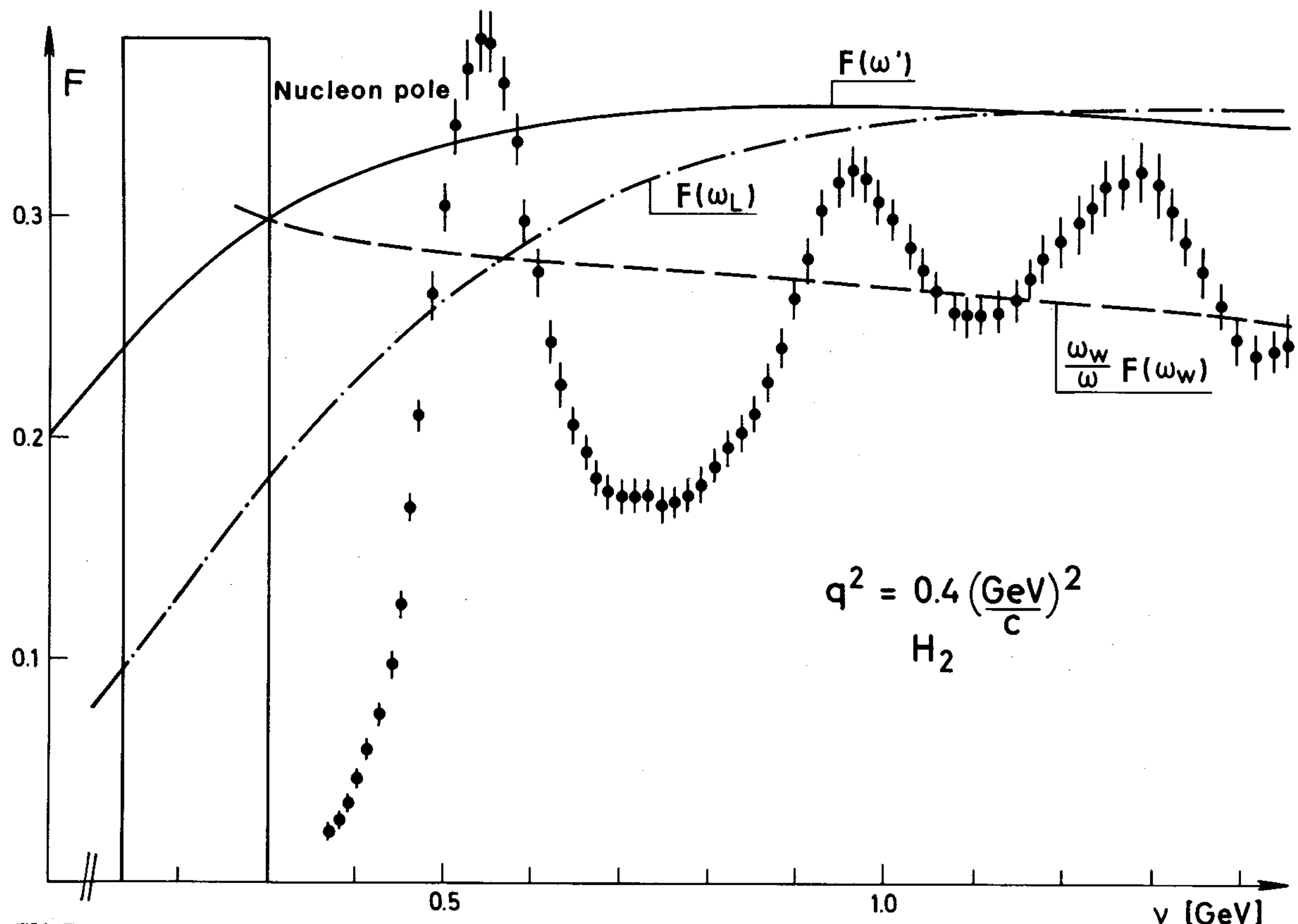
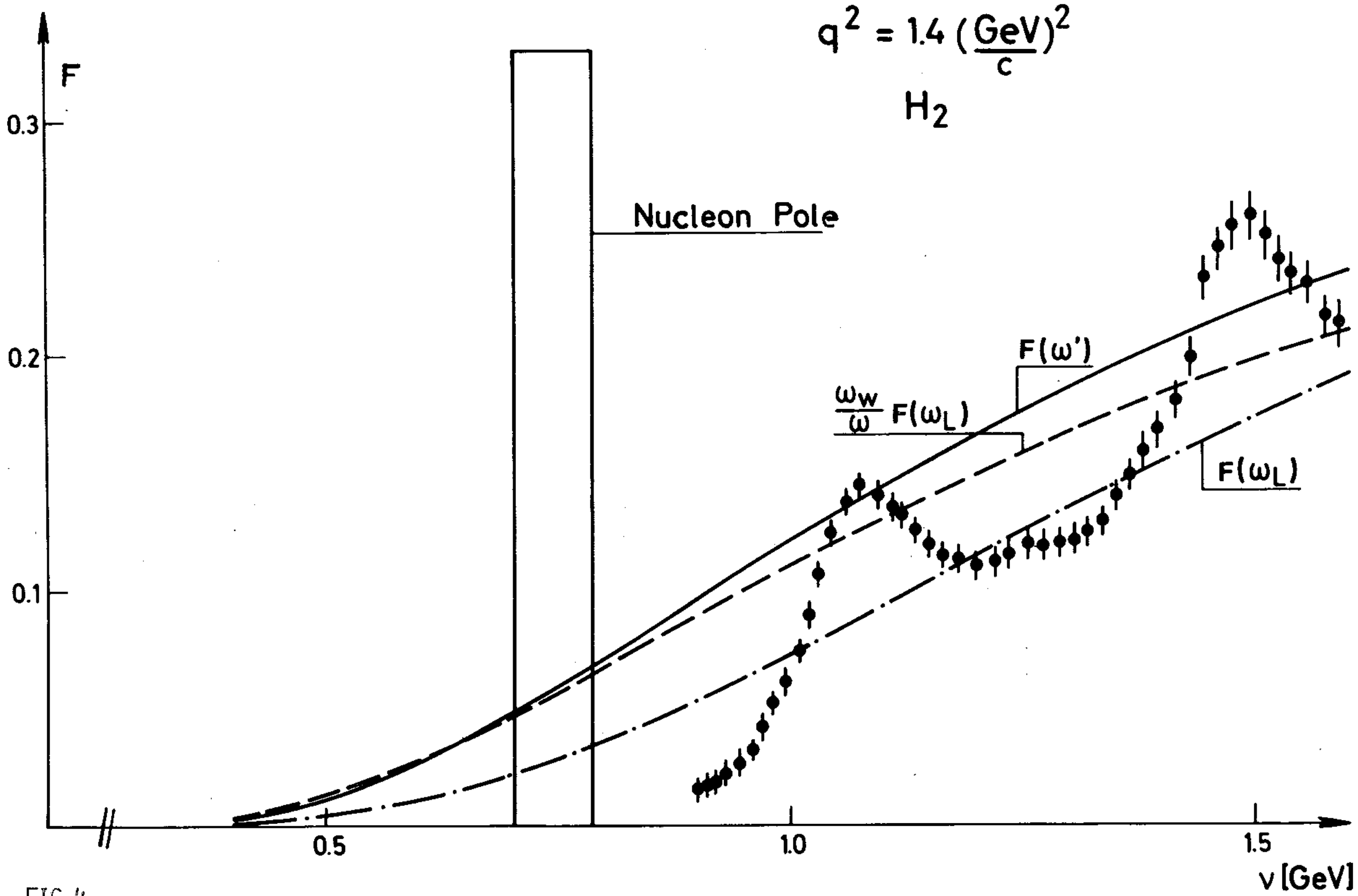
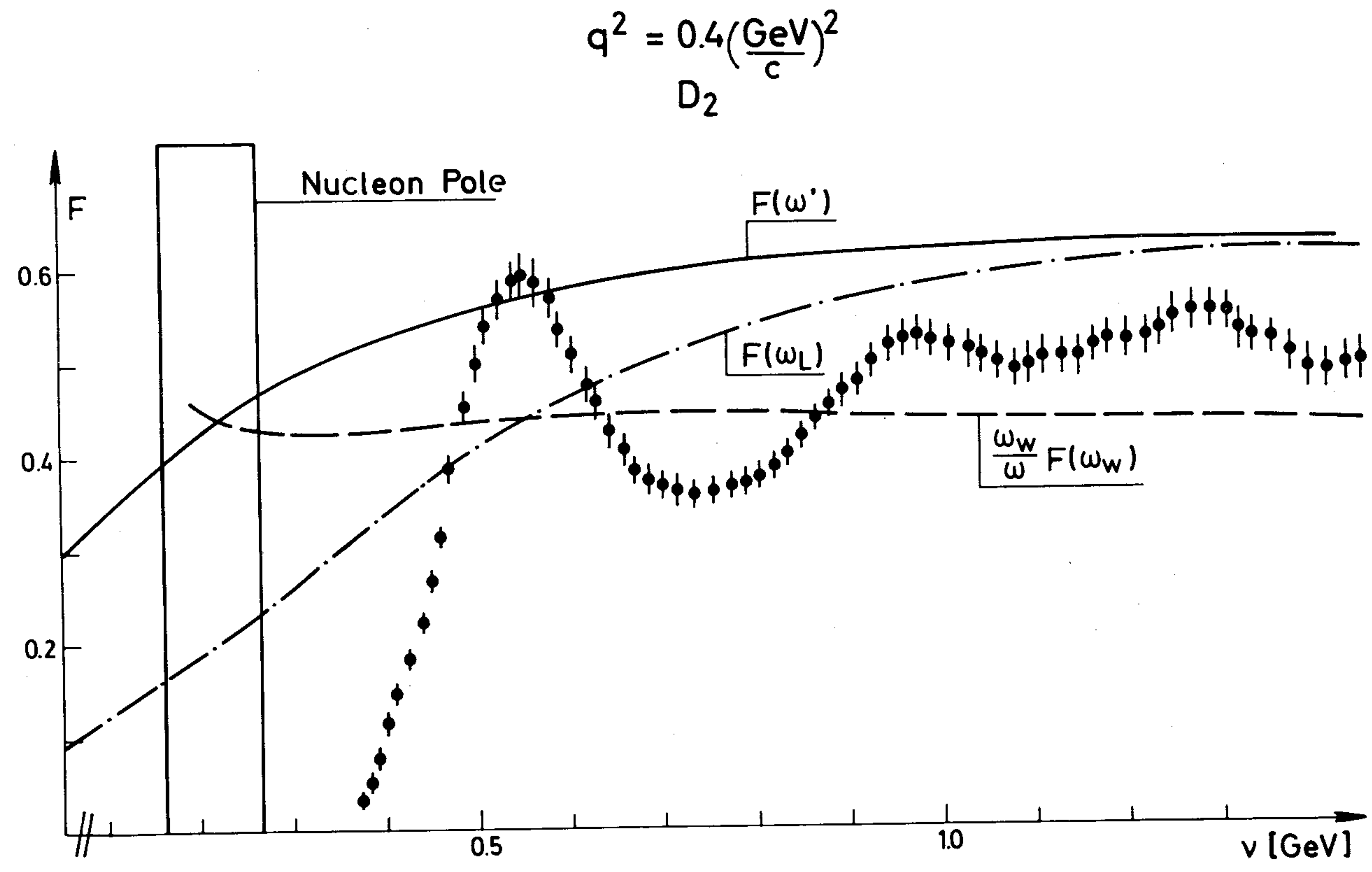


FIG.3



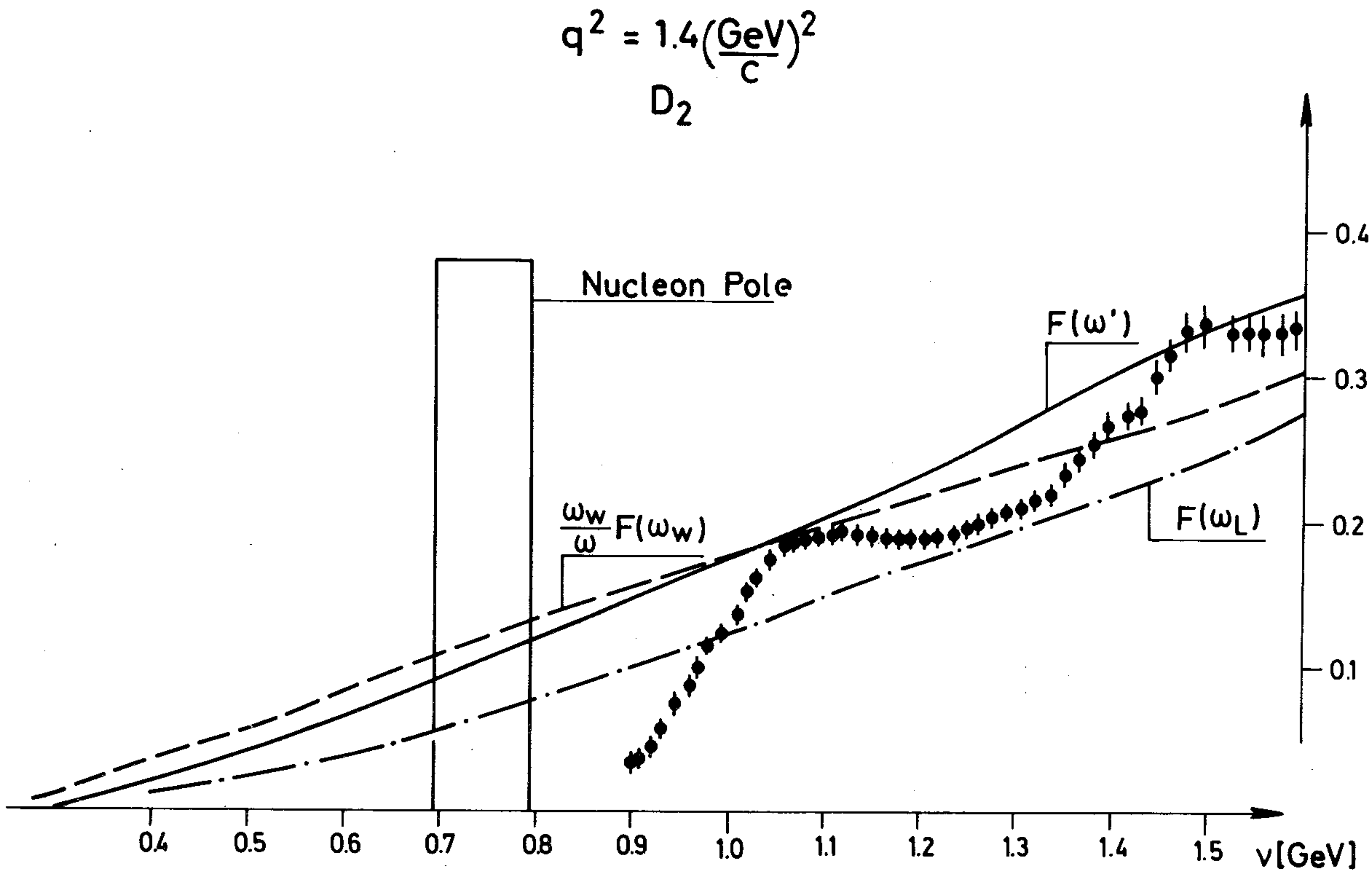
$q^2 = 1.4\,(\frac{GeV}{c})^2$
H_2
F
0.3
0.2
0.1
Nucleon Pole
$F(\omega')$
$\frac{\omega_W}{\omega}\,F(\omega_L)$
$F(\omega_L)$
0.5
1.0
1.5
ν [GeV]

FIG.4



$q^2 = 0.4\left(\frac{GeV}{c}\right)^2$
D_2
Nucleon Pole
F
0.6
0.4
0.2
$F(\omega')$
$F(\omega_L)$
$\frac{\omega_W}{\omega} F(\omega_W)$
0.5
1.0
ν [GeV]

FIG.5



$q^2 = 1.4\left(\frac{GeV}{c}\right)^2$
D_2
Nucleon Pole
$F(\omega')$
$\frac{\omega_w}{\omega}F(\omega_w)$
$F(\omega_L)$
0.4
0.3
0.2
0.1
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
ν[GeV]

FIG.6

BLOOM - GILMAN
SUM - RULE

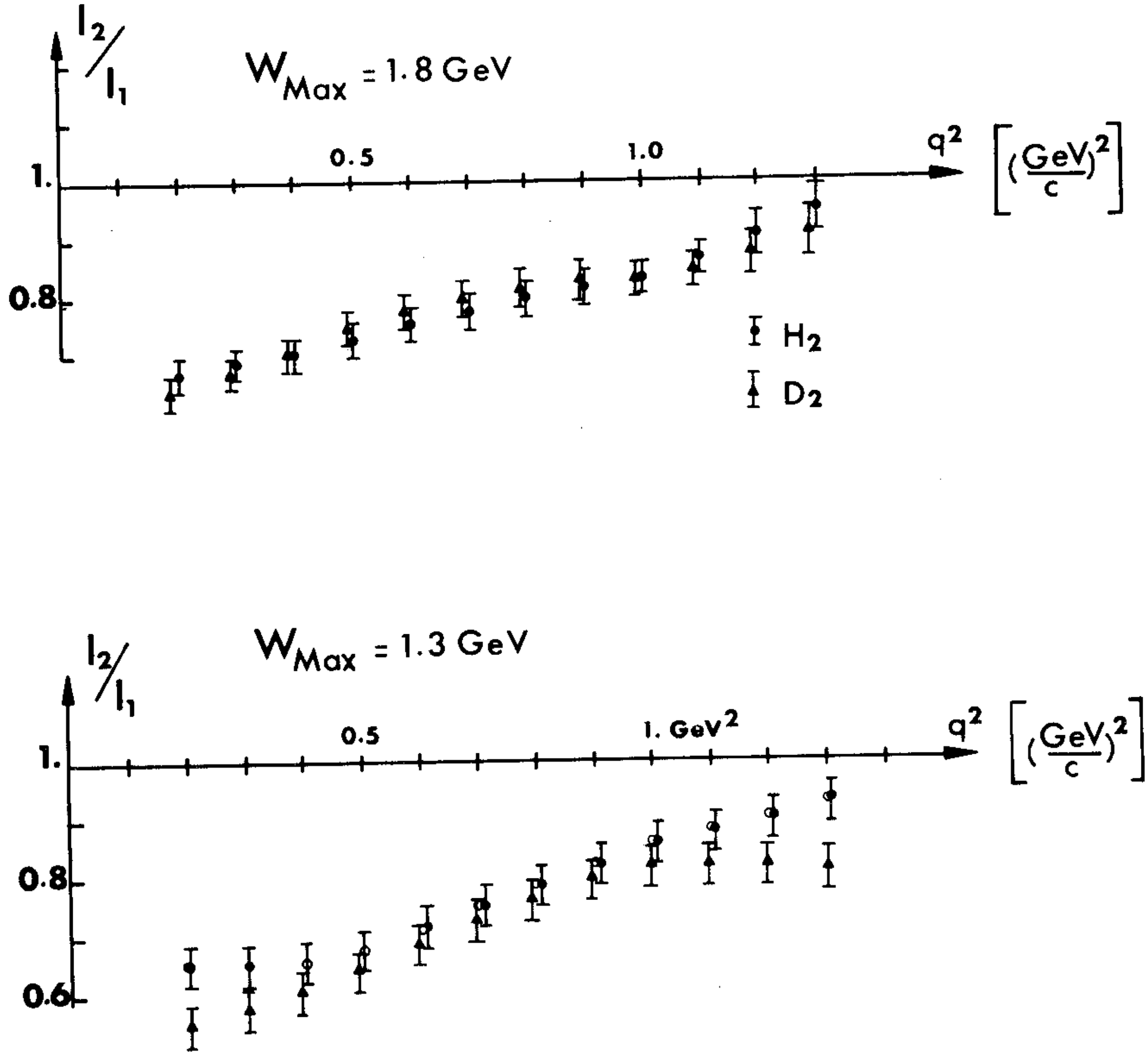


FIG.7

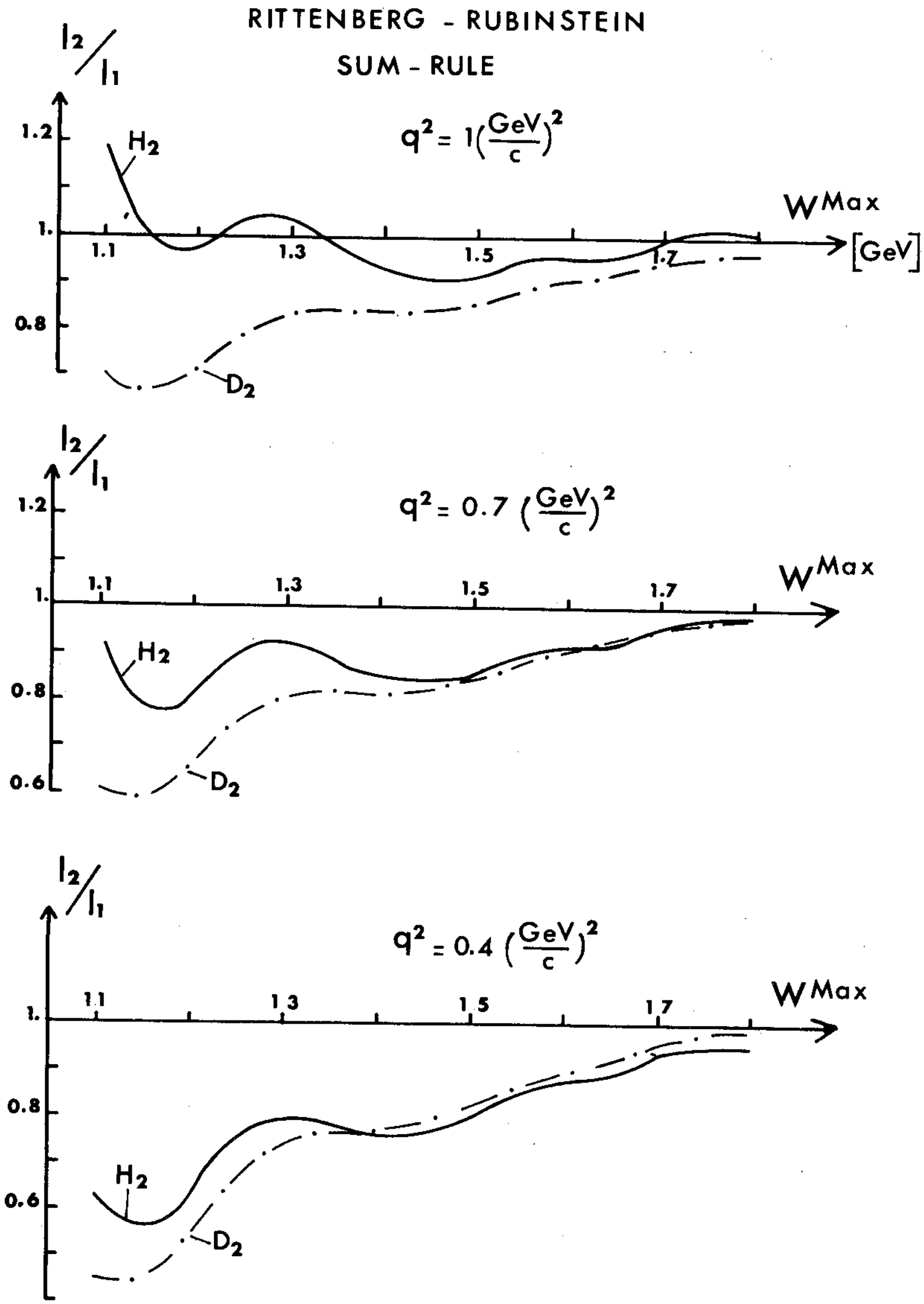

RITTENBERG - RUBINSTEIN
SUM - RULE
I_2/I_1
$q^2 = 1 \left(\frac{GeV}{c}\right)^2$
1.2
1.
0.8
1.1
1.3
1.5
1.7
W^{Max}
[GeV]
H_2
D_2
I_2/I_1
$q^2 = 0.7 \left(\frac{GeV}{c}\right)^2$
1.2
1.
0.8
0.6
1.1
1.3
1.5
1.7
W^{Max}
H_2
D_2
I_2/I_1
$q^2 = 0.4 \left(\frac{GeV}{c}\right)^2$
1.
0.8
0.6
1.1
1.3
1.5
1.7
W^{Max}
H_2
D_2

FIG.8