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Do Recent e^+e^- Annihilation Data Imply Violations of Scaling in Deep Inelastic Electron Scattering?

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Abstract

We conjecture that there is breaking of scaling in the large ω region in deep inelastic electron scattering as a reflection of the approximate constancy of $\sigma(e^+e^- \rightarrow \text{hadrons})$ observed from about 3.5 to 5 GeV c.m. energy.

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A quantitative model for inelastic ep scattering has recently been developed[1] based on "Off-diagonal Generalized Vector Dominance", a version of Generalized Vector Dominance (GVD)[2], in which diagonal ($Vp \to V'p$ with masses $m_V = m_{V'}$) as well as off-diagonal ($m_V \neq m_{V'}$) contributions to the virtual photon forward Compton amplitude are taken into account. The model successfully describes the data on inelastic ep scattering in the large ω' ($\omega' \gtrsim 10$) diffraction region. Scaling for ep scattering is derived from scaling for the total e^+e^- annihilation cross section $\sigma(e^+e^- \to \text{hadrons}) \sim 1/s$ and reasonable hadron physics: the vector meson proton total cross section $\sigma_{V_N p}$ has been assumed to be independent of the mass of the vector meson V_N ($N = 0, 1, \ldots$), and the diffraction dissociation type terms $V_N p \to V_{N'} p$ ($N \neq N'$) have been introduced such that they are compatible with our knowledge on diffraction dissociation in hadron hadron interactions.

In the present paper we wish to draw additional consequences from the model of ref. 1 by taking into account recent e^+e^- annihilation results[3,4], which indicate a breakdown of the simple $1/s$ scaling law for $\sigma(e^+e^- \to \text{hadrons})$ as soon as the c.m. energy becomes larger than roughly 3.5 GeV in the c.m. system. The calculation to be presented is exploratory in the spirit of predicting what we should possibly expect[*1] in the spacelike region as a reflection of the approximate constancy of e^+e^- annihilation beyond 3.5 GeV.

Let us start by recalling the essential features of the model recently proposed, referring to ref. 1 for details. Working with a Veneziano type spectrum of vector states V_N with masses

$$m_N^2 = m_o^2(1 + \lambda N), \quad N = 0, 1, \ldots \tag{1}$$

($m_o = m_\rho$ being equal to the $\rho^o(\omega, \phi)$ mass, neglecting the difference between these masses subsequently), by taking into accoung diagonal ($V_N p \to V_N p$) as well as (effective) off-diagonal contributions ($V_N p \leftrightarrow V_{N+1} p$) to the imaginary part of the virtual Compton forward amplitude, the transverse virtual photoabsorption cross section $\sigma_T(W, q^2)$ is given by

$$\sigma_T(W,q^2) = \sum_{V=\rho^o,\omega,\phi} \alpha\pi\sigma_{Vp} \cdot \tag{2}$$

$$\sum_{N=o}^{\infty} \frac{m_N^2}{\gamma_{V,N}} \frac{1}{(q^2+m_N^2)} \left\{ \frac{1}{(q^2+m_N^2)} \frac{m_N^2}{\gamma_{V,N}} - 2C_N \frac{1}{(q^2+m_{N+1}^2)} \frac{m_{N+1}^2}{\gamma_{V,N+1}} \right\} .$$

$\frac{1}{\gamma_{V,N}}$ in this expression is (the absolute value of) the vector meson photon coupling. C_N denotes the ratio of off-diagonal to diagonal vector meson nucleon forward amplitude,

$$C_N \equiv T_{V_N p \leftrightarrow V_{N+1} p} / T_{V_N p \to V_N p} , \tag{3}$$

for which the Ansatz

$$C_N = \text{const.} \left(\frac{m_N}{m_{N+1}}\right)^{1+2\delta} \cong \text{const.} \frac{m_N}{m_{N+1}} \left(1 - \delta\lambda \frac{m_o^2}{m_N^2}\right) \tag{4}$$

has been adopted[1]. For definiteness $C_N > 0$ was assumed. The minus sign in (2) originates from $1/\gamma_N \sim (-1)^N$ and yields destructive interference between different ingoing vector mesons as an essential feature of the model.

The calculations for inelastic ep scattering in ref. 1 were based on the scaling law $\sigma(e^+e^- \to h) \sim 1/s$. Consequently, the vector meson photon couplings in (2) have been assumed to be given by[5]

$$\frac{1}{\gamma_N^2} = \frac{1}{\gamma_o^2} \frac{m_o^2}{m_N^2} , \tag{5}$$

which Ansatz yields $\sigma(e^+e^- \to h) = \frac{4\pi^2}{\lambda\gamma_\rho^2} \sigma_{\mu^+\mu^-} \cong 2.5\ \sigma_{\mu^+\mu^-}$, if the level spacing λ is chosen to be $\lambda = 2$. This magnitude of $\sigma(e^+e^- \to h)$, and the energy dependence of the cross section are quite consistent with experiment as long as $\sqrt{s} \lesssim 3.5$ GeV (see fig. 1). With the couplings (5), convergence of the total cross section (2) and correct normalization to photoproduction $(q^2 = 0)$ determine[1] the constants in (4) to be const. = 1/2 and $\delta = 0.28$, and the q^2 dependence is predicted as

$$\sigma_T(W,q^2) = \frac{\overline{m}^2}{q^2 + \overline{m}^2} \sigma_{\gamma p}(W) \tag{6}$$

with

$$\overline{m}^2 = \frac{\frac{1}{2} + \delta}{1 + \delta} m_\rho^2 = 0.61\ m_\rho^2, \tag{7}$$

corresponding to precocious scaling of the transverse part of the structure function νW_2

$$\nu W_{2T} \cong \frac{1}{4\pi^2\alpha} q^2 \sigma_T , \quad (\omega' \geq 10, \quad q^2 \geq 0.5 \text{ GeV}^2) . \tag{8}$$

Let us explore now the consequences for the spacelike region of the approximate constancy of $\sigma(e^+e^- \to h)$ as observed at CEA and SPEAR beyond 3.5 GeV c.m. energy. We thus assume $\sigma(e^+e^- \to h) \sim$ const. for $s_1 \leqq s \leqq s_2$ and $\sigma(e^+e^- \to h) \sim 1/s$ for $s \geqq s_2$. The limit $s_2 \to \infty$, corresponding to an indefinitely rising $R \equiv (e^+e^- \to h)/\sigma_{\mu^+\mu^-}$, is not excluded a priori, but a discussion of its implications will be far beyond the scope of this work. As is easily verified, constancy for $s_1 \leqq s \leqq s_2$ and scaling behaviour of $\sigma(e^+e^- \to h)$ for $s \geqq s_2$ (see fig. 1) are realized within our scheme by simply requiring

$$\frac{1}{\gamma_N^2} = \text{const.} = \frac{1}{\gamma_o^2} \frac{m_o^2}{m_{N_1}^2} , \quad \text{for } N_1 \leqq N \leqq N_2 \tag{9}$$

and

$$\frac{1}{\gamma_N^2} = \frac{1}{\gamma_o^2} \frac{m_{N_2}^2}{m_{N_1}^2} \frac{m_o^2}{m_N^2} , \qquad \text{for } N \geqq N_2, \tag{10}$$

where $N_{1,2}$ is related to $s_{1,2}$ by $s_{1,2} \equiv m_o^2(1 + \lambda N_{1,2})$. With (5) for $N \leqq N_1$, and (9) and (10) for $N_2 \geqq N \geqq N_1$ and $N \geqq N_2$, respectively, the summation of expression (2) yields

$$\sigma_T(W,q^2) = \sum_{V=\rho^o,\omega,\phi} \frac{\alpha\pi}{\gamma_V^2} \sigma_{Vp} \Big\{ \Big(\frac{1+\delta}{\lambda} + \frac{q^2}{\lambda^2 m_o^2}\Big)\Big(\frac{1}{x_o} - \frac{1}{x_1}\Big) + \frac{q^2}{\lambda^2 m_o^2}(\psi'(x_1)-\psi'(x_o))$$

$$+ \frac{m_o^2}{m_{N_1}^2} \Big[(\delta+ \frac{1}{2})(\psi(x_2)-\psi(x_1))+(1+ \frac{q^2}{\lambda m_o})(\frac{1}{2} +\delta+ \frac{q^2}{\lambda m_o})(\frac{1}{x_2} - \frac{1}{x_1})+ \frac{q^4}{\lambda^2 m_o^4}(\psi'(x_1)-\psi'(x_2))\Big]$$

$$+ \frac{m_{N_2}^2}{m_{N_1}^2} \Big[\Big(\frac{1+\delta}{\lambda} + \frac{q^2}{\lambda^2 m_o^2}\Big) \frac{1}{x_2} - \frac{q^2}{\lambda^2 m_o^2} \psi'(x_2)\Big]\Big\} , \tag{11}$$

where we have introduced the notation

$$x_o \equiv (q^2 + m_o^2)/\lambda m_o^2,$$

$$x_1 \equiv (q^2 + m_{N_1}^2)/\lambda m_o^2, \qquad (12)$$

$$x_2 \equiv (q^2 + m_{N_2}^2)/\lambda m_o^2,$$

and where $\psi(z)$ and $\psi'(z)$ denote the digamma function and its derivative respectively

$$\psi(z) \equiv \frac{d}{dz} \ln \Gamma(z) \sim \ln z - \frac{1}{2z} + O\left(\frac{1}{z^2}\right) ,$$

$$\psi'(z) \equiv \frac{d}{dz} \psi(z) \sim \frac{1}{z} + \frac{1}{2z^2} + O\left(\frac{1}{z^3}\right) . \qquad (13)$$

With the asymptotic expansions (13), the expression (10) for σ_T simplifies considerably. Evaluating σ_T at $q^2 = 0$ and equating to $\sigma_{\gamma p}$ determines δ, which is found to differ insignificantly for not too large values of s_2 from the result of ref. 1, $\delta = 0.28$. The final result following from (11) may then be written as*2

$$\sigma_T(W,q^2) = \left(\frac{\overline{m}^2}{q^2 + \overline{m}^2} + \frac{\overline{m}^2}{s_1} \ln \frac{q^2 + s_2}{q^2 + s_1}\right)\sigma_{\gamma p}/\left(1 + \frac{\overline{m}^2}{s_1} \ln \frac{s_2}{s_1}\right) , \qquad (14)$$

where $\overline{m}^2 = 0.61\, m_\rho^2$ as given in (7).

Let us discuss the physical implications of our result (14). Quite trivially, for the limiting case $s_1 = s_2$ we have $\sigma(e^+e^- \to h) \sim 1/s$ for arbitrary s, and consequently σ_T in (14) reduces to the scaling expression (6). If we assume $s_2 \neq s_1$, corresponding to a constant e^+e^- annihilation cross section for $s_1 \leq s \leq s_2$, from (14) and (8), scaling will be violated approximately linearly in q^2 for $q^2 \leq s_2$, a scaling limit being reached for $q^2 \gg s_2$ only, when the log term becomes negligible. For $s_1 \simeq 12$ GeV^2, as indicated by the data[3,4], $\overline{m}^2/s_1 = 0.03$, and thus the slope of the scale breaking term is quite small as long as s_2 is only moderately large. Thus from the empirical fact that the onset of approximate constancy of $\sigma(e^+e^- \to \text{hadrons})$ is at about 3.5 GeV only, we expect scaling violations in deep inelastic ep scattering to be moderately large for not too large spacelike q^2 (keeping s_2 finite). As a numerical example, in fig. 2 (curve (b)), we first of all show the result obtained for νW_{2T} from constancy of $\sigma(e^+e^- \to \text{hadrons})$ in the CEA SPEAR range of 12 $\text{GeV}^2 \leq s \leq 25\ \text{GeV}^2$. Scaling violations become more dramatic as soon as s_2 is raised to e. g. $s_2 = 50\ \text{GeV}^2$ (corresponding to $R \equiv \sigma(e^+e^- \to h)/$ $/\sigma_{\mu^+\mu^-} \cong 10$ at $\sqrt{s_2} \cong 7$ GeV). In fact, the corresponding curve (c) on fig. 2 may be considered to be at variance with the plotted SLAC-MIT data as available in the low q^2 region. Thus, if our results are taken literally on the quantitative level, and not as an indication of qualitative trends only, one may even be bold enough to infer that $\sigma(e^+e^- \to h)$ should after all start to go down as $1/s$ not too far beyond the presently explored region. As long as s_2 is finite, νW_2 will eventually scale, the scaling limit being enhanced approximately by a factor s_2/s_1 compared with the result obtained from $\sigma(e^+e^- \to h) \sim 1/s$:

$$\nu W_{2T} \sim \frac{1}{4\pi^2\alpha} \bar{m}^2 \sigma_{\gamma p}(W \to \infty) \frac{s_2}{s_1} \frac{1}{(1 + \frac{\bar{m}^2}{s_1} \ln \frac{s_2}{s_1})} . \qquad (15)$$

For $s_2 \cong 25\ \text{GeV}^2$ and $s_1 \cong 12\ \text{GeV}^2$ (SPEAR range) the enhancement factor is about 2, the scaling limit being reached at $q^2 \cong 100\ \text{GeV}^2$ only, however. As mentioned, a discussion of the hypothesis that $\sigma(e^+e^- \to h)$ stays constant, i. e. $s_2 \to \infty$, corresponding to an indefinitely rising ratio $\sigma(e^+e^- \to \text{hadrons})/$ $/\sigma_{\mu^+\mu^-}$, is certainly far beyond the scope of this work.

Experimentally, tests of scaling for large ω' require lepton beam energies in the 100 GeV range presently available at FNAL. It is of great interest in connection with our conjecture that an indication for a positive violation of scaling of roughly 20 % for $\omega' > 9$ and $5\ \text{GeV}^2 \leq q^2 \leq 10\ \text{GeV}^2$ has actually been reported quite recently as a first result from the FNAL muon beam experiment[7].

Let us add a few additional remarks concerning our result. If instead of $\sigma(e^+e^- \to \text{hadrons}) \sim$ const. we assume a fall-off somewhat weaker than $1/s$, e. g. $1/\sqrt{s}$, which may also be compatible with available data, the corresponding scaling violations are also present, but are somewhat smaller, depending, of course, on the values of s_1 and s_2. As regards the longitudinal photoabsorption cross section σ_S, not considered so far, we have estimated that even with $\sigma(e^+e^- \to \text{hadrons})$ being constant for $s_1 \leq s \leq s_2$, the prediction[2] for the ratio $\sigma_S/\sigma_T \sim \ln q^2/m_\rho^2$ remains essentially unchanged. We thus expect an additional small logarithmic violation of scaling when instead of the transverse part the whole structure function νW_2 is taken into consideration.

Thus summarizing, within the framework of GVD, we are led to conjecture that positive violations of scaling, approximately linear in q^2, are to be expected in the large ω' diffraction region*[3] of ep scattering as a consequence of the approximate constancy of e^+e^- annihilation in the CEA SPEAR energy range. Indications for such an effect may have been found in the FNAL muon beam experiment[7]. Should scaling violations of roughly the magnitude we are predicting not be confirmed in future experiments at large ω', such a situation would seem to be difficult to understand within the framework of GVD. Even though the general concept that the q^2 dependence in deep inelastic scattering is due to the propagation of vector states, i. e.

$$\sigma_T(W,q^2) = \int \frac{m^2 \rho(m^2,m'^2,W) \; m'^2 \; dm^2 \; dm'^2}{(q^2 + m^2)(q^2 + m'^2)} , \tag{16}$$

would clearly not be affected, a rather artificial cancellation between hadronic vector state nucleon amplitudes would have to be invoked to cancel the large coupling of the photon to higher mass states reflected in the constancy of the CEA SPEAR e^+e^- annihilation cross section.

Acknowledgement

It is a pleasure to thank Tom Walsh for helpful comments.

Footnotes

*1 The results to be presented quantitatively in this paper have been qualitatively indicated in ref. 1. While in the precess of writing up the present communication, we obtained a preprint be H.T. Nieh (ITP-SB-79-22, Stony Brook) in which the conjecture of breaking of scaling is put forward in a way closely related to our reasoning.

*2 One may convince oneself that the q^2 dependence in (14) (as well as in (6)) is derivable within the diagonal formulation[2] of GVD by introducing a $\sigma_{V_N p} \sim 1/m_N^2$ law[5]. Such a decreasing cross section may thus appear to be equivalent to introducing destructively interfering off-diagonal terms. Assuming a $\sigma_{V_N p} \sim 1/m_N^2$ behaviour seems at variance[1], however, with the diagonal approximation. Moreover, the mentioned equivalence is lost, as soon as exclusive processes are considered[6], as e. g. ρ^o electroproduction.

*3 Well known qualilative lifetime arguments based on the uncertainty principle suggest that values of $\omega' \gg 50$ may actually be necessary to fully see the scaling violation predicted in Fig.2 (see e.g. Nieh, loc.cit[+1]).

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Figure Captions

Fig. 1 The e^+e^- annihilation cross section as a function of the c.m. energy. The data are from ref. 8. The theoretical curves show the GVD prediction and its modification to fit the approximate constancy from $\sqrt{s_1}$ to $\sqrt{s_2}$ in order to estimate its influence on deep inelastic ep scattering.

Fig. 2 Predictions for the transverse part of νW_2 as a function of q^2 at fixed[*3] ω' for different assumptions on e^+e^- annihilation. The data points have been computed[9] from the measured 6^o, 10^o and 18^o data[10] with a model[2] for σ_S/σ_T, which is consistent with the separation data available.

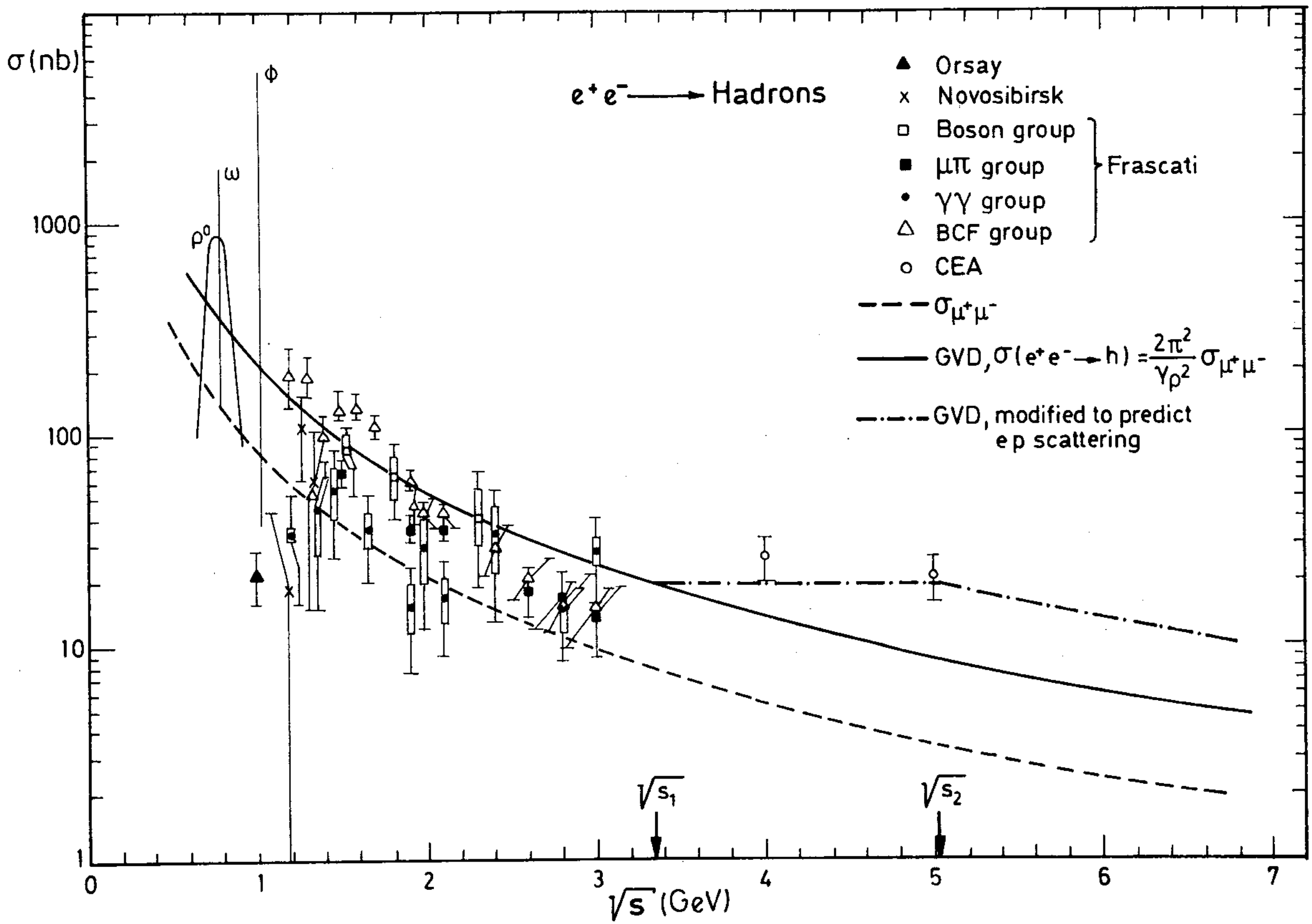
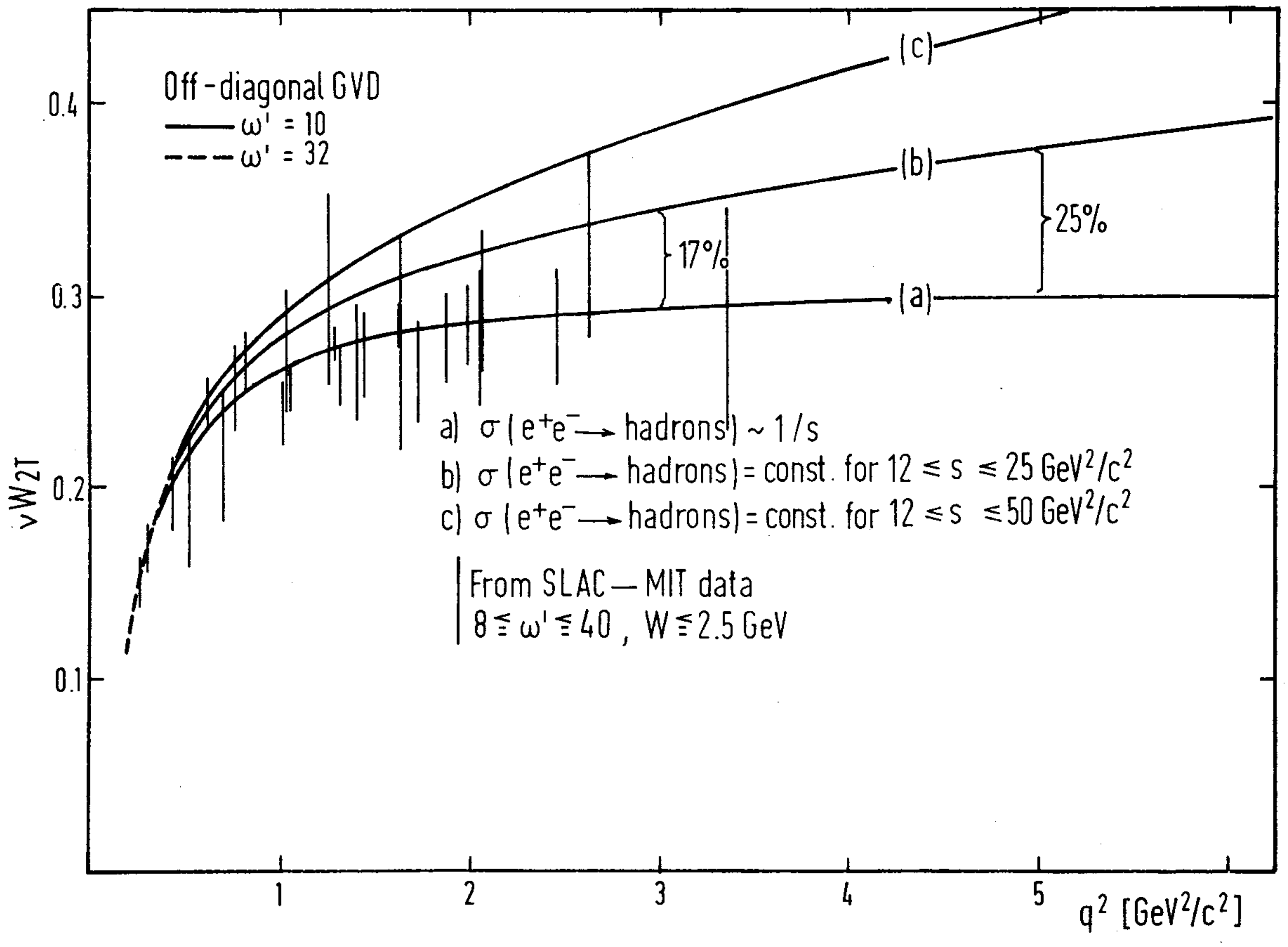

σ(nb)
1000
100
10
1
0
1
2
3
4
5
6
7
$\sqrt{s}$ (GeV)
$e^+e^- \longrightarrow$ Hadrons
Orsay
Novosibirsk
Boson group
μπ group
γγ group
BCF group
CEA
Frascati
$\sigma_{\mu^+\mu^-}$
GVD, $\sigma(e^+e^- \to h) = \frac{2\pi^2}{\gamma_\rho^2}\sigma_{\mu^+\mu^-}$
GVD, modified to predict
e p scattering
ρ°
ω
φ
$\sqrt{s_1}$
$\sqrt{s_2}$

Fig. 1



Off-diagonal GVD
ω' = 10
ω' = 32
0.4
0.3
0.2
0.1
νW2T
(c)
(b)
(a)
25%
17%
a) σ (e+e− → hadrons) ~ 1/s
b) σ (e+e− → hadrons) = const. for 12 ≤ s ≤ 25 GeV²/c²
c) σ (e+e− → hadrons) = const. for 12 ≤ s ≤ 50 GeV²/c²
From SLAC — MIT data
8 ≤ ω' ≤ 40 , W ≤ 2.5 GeV
1
2
3
4
5
q² [GeV²/c²]

Fig. 2