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Time-Reversal-Invariance-Like Relations for Spin-Effects
in Elastic and Inelastic Reactions;
Vector-Meson Photoproduction versus Compton Scattering from Nucleons

by



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ABSTRACT

For elastic scattering, relations between spin-effects (for example, the well-known asymmetry-polarisation equality) follow from time-reversal invariance. We show that if certain amplitude-combinations vanish, there are strikingly similar relations between spin-effects for elastic and also inelastic reactions. This vanishing of amplitude-combinations (denoted M-purity) corresponds asymptotically to purely natural or purely unnatural parity in the crossed channel. The M-purity relations hold for spin-configurations much more general than do the corresponding time-reversal-invariance relations.

The experimental evidence for purely natural parity exchanges in high energy vector meson photoproduction from nucleons is shown to be good for all amplitudes involving nonzero meson helicity, but less conclusive for the zero helicity ones. Using time-reversal-invariance and a vector meson-dominance argument, this implies no unnatural parity contributions in high energy Compton scattering from nucleons.

Because of this empirical evidence for M-purity in these two processes, a detailed application to spin-effects in Compton scattering and in vector meson photoproduction is made. Some time-reversal-invariance relations in Compton scattering resemble the corresponding M-purity relations though the applicability of the two is different, and there are examples where only one of the two exists. Out of our illustrations, the only M-purity relations which change in form due to the extra amplitudes present in the inelastic reaction are the M-purity analogue and extensions of the asymmetry-polarisation equality (of Compton scattering) referring to the photon; the change is the appearance of the elements ρ^{00} of the vector meson density-matrix ρ . Our other examples of M-purity relations do not change in form in going over from the elastic reaction (Compton scattering) to the inelastic reaction (vector meson photoproduction).

1. INTRODUCTION

For elastic processes, time-reversal-invariance (denoted hereafter as T-invariance) leads to the equality [1] between the recoil polarisation with an unpolarised target and the cross-section asymmetry with a polarised target. In general, there is no such result for an inelastic reaction. We show that if certain combinations of amplitudes vanish, there are such relations for even an inelastic reaction. We denote the vanishing of these amplitude-combinations by M-purity which corresponds, asymptotically, to having pure normality (natural or unnatural parity, but not a mixture) in the crossed channel. Spin effects in Compton scattering ($\gamma N \rightarrow \gamma N$) and in vector-meson-photoproduction ($\gamma N \rightarrow V N$) from nucleons are considered in detail in order to bring out the comparison between the relations following from T-invariance and from M-purity and in order to illustrate the changes in going from an elastic to an inelastic process. Though our emphasis is on the application to these two processes, M-purity relations can be seen (e.g. sec. 2) to be quite general.

It is worth emphasizing that T-invariance is a general principle, while M-purity is a model that becomes empirically interesting in certain cases. We are pointing out the similarity of some M-purity relations to some T-invariance relations, but clearly, there are many nonoverlapping implications of M-purity and of T-invariance. Even when M-purity relations resemble the corresponding T-invariance relations, the regions of applicability of the two are not always the same, as will be discussed in detail.

(1.1) Definition of M-purity; Comparison with Restrictions on Helicity

Amplitudes following from T-Invariance.

For the s-channel helicity amplitudes f_{ba}^{dc} of the process $ab \rightarrow cd$ where a, b, c and d also denote helicities of the corresponding particles, parity conservation gives [2]

$$f_{-b-a}^{-d-c} = \eta (-)^{a-c-(b-d)} f_{ba}^{dc} \quad (I.1)$$

where η depends on spins and intrinsic parities of a, b, c and d. For elastic processes and for $\gamma N \rightarrow VN$, $\eta = +1$. We define

$$n_{ba}^{dc} = \frac{1}{2} (f_{ba}^{dc} + (-)^{d-b} f_{-ba}^{-dc}) \quad (I.2a)$$

$$u_{ba}^{dc} = \frac{1}{2} (f_{ba}^{dc} - (-)^{d-b} f_{-ba}^{-dc}) \quad (I.2b)$$

so that

$$n_{-ba}^{-dc}/n_{ba}^{dc} = - u_{-ba}^{-dc}/u_{ba}^{dc} = (-)^{b-d} \quad (I.2c)$$

For the amplitude f_{ba}^{dc} , M-purity is defined¹ as $M = +1$ or $M = -1$:

$$M = -1 \text{ means } n_{ba}^{dc} = 0, \quad (I.3a)$$

$$M = +1 \text{ means } u_{ba}^{dc} = 0. \quad (I.3b)$$

The n- and u- type amplitudes will sometimes be called $M = +1$ and $M = -1$ amplitudes respectively. Using the parity relation (I.1), n_{b-a}^{d-c} is related to n_{ba}^{dc} and u_{b-a}^{d-c} to u_{ba}^{dc} . The amplitudes of Eqs. (I.2) correspond², asymptotically [3], to pure normality in the crossed channel $a\bar{c} \rightarrow \bar{b}d$. However, we shall need only the definitions (I.2) and (I.3) which hold for general values of the kinematical variables.

For the elastic process $ab \rightarrow cd$, T-invariance gives the relation [2]

$$f_{dc}^{ba} = (-)^{a-c-(b-d)} f_{ba}^{dc} \quad (I.4)$$

where the particles a and c are the same and so are b and d. The T-invariance relation (I.4) and the corresponding parity relation (I.1) for elastic scattering overlap for only the helicity combinations $b = -d$,

$a = -c$. Similarly, the T-invariance relation (I.4) overlaps with the M-purity restrictions (I.3) only for $(a = c, b = -d)$ and similarly, for $(a = -c, b = d)$. In fact, (I.4) implies $M = +1$ for the amplitudes f_{-ba}^{ba} and f_{b-a}^{ba} :

$$u_{-ba}^{ba} = 0, \quad M = +1 \quad (I.5a)$$

$$u_{b-a}^{ba} = 0, \quad M = +1 \quad (I.5b)$$

The T-invariance relation (I.4) does not restrict any of the n_{ba}^{dc} amplitudes, and restricts only some of the u_{ba}^{dc} amplitudes. We shall call M-purity for the amplitudes f_{-ba}^{ba} and f_{b-a}^{ba} as the M-purity of type 1; T-invariance gives $M = +1$ for this type in elastic processes; M-purity for other helicity amplitudes will be called M-purity of type 2.

For $\gamma N \rightarrow \gamma N$, Eqs. (I.5) are the only restrictions that T-invariance puts on helicity amplitudes, if one takes (as we shall do) parity-conservation (I.1) for granted. The fact that M-purity and T-invariance restrictions are related is contained in (I.5). While (I.4) is for elastic processes like $\gamma N \rightarrow \gamma N$, the M-purity restrictions can hold for any general reaction like $\gamma N \rightarrow VN$. It is now easy to imagine, for any general reaction, M-purity relations which resemble the T-invariance relations between spin-effects in elastic scattering. This is further illustrated in sec. 2 where the asymmetry-polarisation equality following from T-invariance is compared with its M-purity analogues.

While M-purity implies $M = +1$ or $M = -1$, T-invariance makes only some $M = -1$ amplitudes vanish. Of course, M-purity may also hold for only certain particular amplitudes.

(1.2) Interest in M-purity Relations; Why the Processes $\gamma N \leftarrow \gamma N$ and $\gamma N \rightarrow VN$?

The only established evidence against T-invariance comes from neutral kaon decays; there, too, the relative strength of the T-non-

invariant amplitude is very small $\sim 10^{-3}$. So, T-invariance is quite general, but M-purity is a model which is interesting in certain situations.³ A good point about the M-purity relations is their applicability to any general reaction even if the corresponding time-reversed process be very hard to achieve experimentally, e.g.,
 $\text{projectile} + \text{nucleon} \rightarrow \text{nucleon} + \text{anything}$. Secondly, very high energy diffractive processes may have a dominant natural parity contribution in the crossed channel; there are many experimentally interesting reactions in this category. In fact, present data on $\gamma N \rightarrow \phi N$ support this hypothesis for the normality of the Pomeron.⁴ Thirdly, combinations of contributions which are expected to be M-pure can be formed in different processes, and M-purity relations of the type we consider can be used as tests of the M-purity of these combinations. In this sense, M-purity relations are relevant also to the dynamical interpretation of amplitude-analyses.

Our detailed application of the comparison between T-invariance relations and M-purity relations to $\gamma N \rightarrow \gamma N$ and $\gamma N \rightarrow V N$ is motivated by several reasons. These reactions are easily accessible experimentally and offer enough spin-complications so as to make this comparison feasible; some spin-effects are relatively easy to study experimentally because the vector meson density-matrix, polarised photons and polarised targets are already available. In contrast, in πN elastic scattering, for example, T-invariance and also the vanishing of unnatural parity contributions already follow from the parity relation (I.1) because of the spinlessness of the pion - thus making the desired comparison impossible. One wants to consider an example in which there are some nonoverlapping restrictions on helicity amplitudes due to T-invariance and to M-purity, and in which T-invariance and M-purity do imply restrictions that go beyond those already implied by parity-conservation. One such experimentally interesting case is $\bar{A} N \rightarrow \bar{A} N$.

where \bar{A} is a spin- $\frac{1}{2}$ baryon, but because both \bar{A} and γ have only two helicities, this example is not essentially different from $\gamma N \rightarrow \gamma N$ which we are considering in detail. From $\bar{A}N \rightarrow \bar{A}N$ data, there is no positive experimental evidence of M-purity, while our example has the advantage that $\gamma N \rightarrow VN$ data (and to a large⁵ extent, also $\gamma N \rightarrow \gamma N$) give evidence of being dominated by natural parity ($M = +1$) contributions. Next, the M-purity relations in $\gamma N \rightarrow VN$ (and $\gamma N \rightarrow \gamma N$) predict some observables, and are therefore testable by future measurements. Looked at the other way, since the value of these predicted observables follows from only M-purity (which is experimentally supported), their measurement will not reveal⁶ further dynamical information for this process; this statement is already of some interest.

Though $\gamma N \rightarrow VN$ looks very similar to $\gamma N \rightarrow \gamma N$, the helicity zero component of the vector meson does give this process some essential features of an inelastic reaction so that a comparison between the M-purity relations in $\gamma N \rightarrow VN$ and in $\gamma N \rightarrow \gamma N$ is capable of showing some possible modifications due to inelasticity. Comparison between the M-purity relations and the T-invariance relations in $\gamma N \rightarrow \gamma N$ shows how the M-purity ones go beyond the T-invariance ones.

We restrict our illustrations of the various comparisons to the following spin effects for an initial state polarised in generality: cross-section asymmetries, recoil nucleon and final photon polarisations, and the vector meson decay density-matrix. We shall not consider the correlations between the polarisations of the final nucleon and the final photon/vector meson; these correlations are difficult to measure experimentally. Our purpose is to mention some simple illustrations of the resemblance between M-purity relations and T-invariance relations among observable spin-effects, rather than to consider the complete set of these relations.

The plan of the paper: Section 2 gives the standard asymmetry-polarisation equality and its M-purity analogues, and compares the two; the generality of the polarisation configuration under which this analogue holds has been pointed out. Section 3 is devoted to some further notation used for $\gamma N \rightarrow \gamma N$ and $\gamma N \rightarrow VN$ in the subsequent parts of the paper. In sec. 4, we consider, for $\gamma N \rightarrow \gamma N$, some T-invariance relations and compare them with their M-purity analogues; the main points are summarised in table 1. The experimental evidence for M-purity in $\gamma N \rightarrow VN$, its comparison with and implications (based on a vector dominance argument) for $\gamma N \rightarrow \gamma N$ are considered in sec. 5. The subsequent section is devoted to the M-purity relations for $\gamma N \rightarrow VN$, closely compared with those (of sec. 4) for $\gamma N \rightarrow \gamma N$; some comments on the modifications due to the inelastic feature of $\gamma N \rightarrow VN$ are included. In sec. 7, an overall summary of the paper and a short discussion are given. The Appendix gives a general treatment of cross-section asymmetries in $\gamma N \rightarrow \gamma N$ and $\gamma N \rightarrow VN$, in particular, the information obtainable from these asymmetry-measurements; only parity-conservation has been assumed.

For a first reading of this paper, secs. 4, 5, 6 and the Appendix may be omitted. For the reader who is not interested in details, the tables are a convenient summary of subsecs. (4.1) and (4.2) and sec. 6.

2. THE ASYMMETRY - POLARISATION EQUALITY FOR ELASTIC PROCESSES, AND ITS M-PURITY ANALOGUES.

Here, we consider the cross-section asymmetry A with the spin- $\frac{1}{2}$ particle b polarised perpendicular to the plane of the reaction

$$a + b \rightarrow X + b' \quad (\text{II.1})$$

and compare it with the polarisation \bar{P} (perpendicular to the plane of the reaction) of the spin- $\frac{1}{2}$ particle b' when b is unpolarised. The polarisation state of the system X is not observed and the system a

may be arbitrarily polarised.

The polarisation \bar{P} is

$$\bar{P} = -2 \operatorname{Im} \left(\sum_{j, \alpha, \beta, \theta} f_{j\beta}^{+\alpha} \rho_a^{\beta\theta} f_{j\theta}^{-\alpha*} \right) / D, \quad (\text{II.2})$$

$$D = \sum_{\ell, j, \alpha, \beta, \theta} f_{j\beta}^{\ell\alpha} \rho_a^{\beta\theta} f_{j\theta}^{\ell\alpha*} \quad (\text{II.3})$$

where ρ_a is the density-matrix for the particle a; the helicities β, j, α and ℓ in the amplitude $f_{j\beta}^{\ell\alpha}$ for the process (II.1) refer to the particles a, b, X and b' respectively; the helicities $\pm \frac{1}{2}$ are denoted by \pm . The asymmetry A is

$$A = p \left[\sum_{\ell, j, k, \alpha, \beta, \theta} f_{j\beta}^{\ell\alpha} (\sigma_2)^{jk} \rho_a^{\beta\theta} f_{k\theta}^{\ell\alpha*} \right] / D, \quad (\text{II.4})$$

where σ_2 is the Pauli matrix and p is the polarisation of b.

Using the hermiticity of ρ_a , one can show that

$$A = M p \bar{P} \quad (\text{II.5})$$

$$\text{if either } f_{-\theta}^{-\alpha} = M f_{+\theta}^{+\alpha} \quad (\text{II.6})$$

$$\text{or/and } f_{-\theta}^{+\alpha} = -M f_{+\theta}^{-\alpha} \quad (\text{II.7})$$

where $M = +1$ or -1 . The conditions (II.6) and (II.7) obviously mean M-purity for these amplitudes; α and θ are arbitrary. One may note that for the corresponding elastic process, T-invariance gives $M = +1$ in only (II.7) for only $\alpha = \theta$; T-invariance does not refer to (II.6).

For the case when the particle a is unpolarised, the matrix ρ_a is essentially the unit matrix and using the T-invariance relation (I.4), one gets the standard asymmetry-polarisation equality [1] for elastic scattering

$$A = p \bar{P}. \quad (\text{II.8})$$

When the particle a is polarised, T-invariance relates A not to \bar{P} , but to some elements of the final-state joint density-matrix which

measure correlations between the two final particles.

In the simple case when a and X are spinless, just the parity relation (I.1) gives $M = +1$ (-1) when the product of the intrinsic parities of a and b is the same as (opposite to) the corresponding product for X and b' ; the relation (II.5) then gives (II.8) for $\pi N \rightarrow \pi N$, $\pi N \rightarrow K\Lambda$, $\epsilon N \rightarrow \epsilon N$, etc.; and

$$A = - p \bar{P} \quad (II.9)$$

for $\pi N \rightarrow \epsilon N$ where ϵ is a $J^P = 0^+$ particle; no reference to T -invariance need be made. For elastic scatterings $\pi N \rightarrow \pi N$ and $\epsilon N \rightarrow \epsilon N$, T -invariance gives $M = +1$ for only the amplitudes (II.7) with helicities $\alpha = \theta = 0$, leading to (II.8), or (II.5) with $M = +1$. On the other hand, the M -purity relation (II.5) would hold even if (II.7) be not valid, but only (II.6) is true; this is an example of the standard equality (II.8) being satisfied even if T -invariance did not hold [6].

So, the M -purity relation (II.5) holds in a much more general situation than the T -invariance relation (II.8), the differences being in the requirements on 1) the nature of a and X , and of b and b' , and 2) the spin-states of a and X . The T -invariance relation (II.8) requires the particles b and b' (and similarly, a and X) to be the same, and $a = \text{unpolarised}$ ⁷. In getting (II.5), however, there is no such restriction on the identity of the particles b and b' (or of a and X), nor on the polarisation state of a ; a and X could have different spins which need not even be known; b and b' could have all quantum numbers (except spin) different; the relation (II.5) would hold for the rather general process

$$\text{Anything} + b \rightarrow b' + \text{Anything} \quad (II.10)$$

where b and b' have $\text{spin}=\frac{1}{2}$.

We shall not discuss extensions of these considerations to other spin values for b and/or b' ; the case of unequal spins for b and b' offers no interesting comparison to the T-invariance relation (II.8). In $\gamma N \rightarrow \gamma N$ and $\gamma N \rightarrow VN$, we shall encounter M-purity relations similar to (II.5), but involving spin-1 effects, others involving spin- $\frac{1}{2}$ effects, and also those involving both spin-1 effects and spin- $\frac{1}{2}$ effects.

The example of this section illustrates the point that M-purity relations for spin-effects can hold for more general configurations than do the corresponding T-invariance ones. In some cases, there may be no corresponding T-invariance relation, this is the case for relations due to M-purity of type 2. It is interesting that any statement resembling the asymmetry polarisation theorem is at all possible for a general inelastic reaction; in fact, the generality of the configuration for which (II.5) holds is remarkable as compared to that for (II.8).

The importance of the theorem (II.8) can hardly be overestimated: a measurement of A avoids the need of a subsidiary experiment to measure \bar{P} . Similar remarks would hold for M-purity relations of the type of (II.5) for situations in which there is evidence for M-purity. Looked at the other way, a measurement of both sides of (II.8) would test the underlying symmetry; similar is the case of testing M-purity by using relations like (II.5).

3. SOME FURTHER NOTATION FOR $\gamma N \rightarrow VN$ AND $\gamma N \rightarrow \gamma N$

We use s-channel helicity amplitudes $f_{i\alpha}^{i'\alpha'}(s,t)$; four-momenta and helicities are defined as follows: (p_μ, i) for the target nucleon N ; (p'_μ, i') for the recoil nucleon N' ; (k_μ, α) for the initial photon γ ; and (k'_μ, α') for the final photon γ' or vector meson V . The invariants are $s = -(p + k)^2$ and $t = -(p' - p)^2$.

As in refs. [7] and [8], the polarisation of the photon beam and of the target nucleon are described by the conventional density-matrices

$$\rho_Y = \frac{1}{2}(1 + \vec{P} \cdot \vec{\sigma}) \equiv \frac{1}{2} \sum_{\mu=0}^3 P_{\mu} \sigma_{\mu} \quad (\text{III.1})$$

and

$$\rho_N = \frac{1}{2}(1 + \vec{\zeta} \cdot \vec{\sigma}) \equiv \frac{1}{2} \sum_{\mu=0}^3 \zeta_{\mu} \sigma_{\mu} \quad (\text{III.2})$$

where the "four-vector" notation implies $P_0 = \zeta_0 = 1$, $\sigma_0 =$ the unit matrix, and $\vec{\sigma}$ represents the three Pauli matrices. The vector

$$\vec{P} = |\vec{P}| (-\cos 2\phi, -\sin 2\phi, 0)$$

describes linearly polarised photons with an angle ϕ between the reaction plane (taken as the XZ plane) and the polarisation vector

$$\vec{e} = (\cos \phi, \sin \phi, 0)$$

of the photons; P_3 corresponds to circular polarisation. For the target nucleon, ζ_1 and ζ_2 are the transverse polarisations respectively in and normal to the reaction plane, and ζ_3 is similarly the degree of longitudinal polarisation. The parameters P_j and ζ_j are restricted

$$|\zeta_j| \leq 1 \text{ and } |P_j| \leq 1, \quad (j = 1, 2, 3).$$

The (unnormalised) joint density-matrix of the vector meson-nucleon final state is

$$\rho_{N',V}^{i'j',\alpha'\beta'} = \sum_{i,j,\alpha,\beta} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_Y^{\alpha\beta} f_{j\beta}^{j'\beta'*} \quad (\text{III.3a})$$

where the vector-meson V can also be the final photon (γ'). The (unnormalised) density-matrices $\rho^{\alpha'\beta'}$ of the vector-meson and $\rho_{N'}^{i'j'}$ of the final nucleon are respectively

$$\begin{aligned} \rho^{\alpha'\beta'} &= \sum_{i'} \rho_{N',V}^{i'i',\alpha'\beta'} \\ &= \sum_{i',i,j,\alpha,\beta} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_Y^{\alpha\beta} f_{j\beta}^{i'\beta'*} \end{aligned} \quad (\text{III.3b})$$

and

$$\begin{aligned} \rho_{N'}^{i'j'} &= \sum_{\alpha'} \rho_{N',V}^{i'j',\alpha'\alpha'} \\ &= \sum_{\alpha',i,j,\alpha,\beta} f_{i\alpha}^{i'\alpha'} \rho_N^{ij} \rho_Y^{\alpha\beta} f_{j\beta}^{j'\alpha'*} \end{aligned} \quad (\text{III.3c})$$

The (unnormalised) density-matrix ρ_Y , of the final photons in $\gamma N \rightarrow \gamma N$ is defined in the same way as ρ for the vector meson in $\gamma N \rightarrow VN$. The normalisation of the helicity amplitudes is provided by the differential cross-section σ as

$$C\sigma = \sum_{i', \alpha'} \rho_{N', V}^{i' i', \alpha' \alpha'} = \sum_{i'} \rho_{N'}^{i' i'} = \sum_{\alpha'} \rho^{\alpha' \alpha'} = \text{tr } \rho, \quad (\text{III.4})$$

$$C = (2\pi/E^*)^{-2},$$

where E^* is the initial photon energy in the C.M. system. The polarisation \vec{P}' of the final photon and the polarisation $\vec{\zeta}'$ of the final nucleon are described analogously to the corresponding initial polarisations in (III.1, 2):

$$\begin{aligned} \zeta_1' &= 2 \text{Re } \rho_{N'}^{+-} / \text{tr } \rho_{N'}, \\ \zeta_2' &= -2 \text{Im } \rho_{N'}^{+-} / \text{tr } \rho_{N'}, \\ \zeta_3' &= (\rho_{N'}^{++} - \rho_{N'}^{--}) / \text{tr } \rho_{N'}; \\ P_1' &= 2 \text{Re } \rho_Y^{+-} / \text{tr } \rho_Y, \\ P_2' &= -2 \text{Im } \rho_Y^{+-} / \text{tr } \rho_Y, \\ P_3' &= (\rho_Y^{++} - \rho_Y^{--}) / \text{tr } \rho_Y, \end{aligned} \quad (\text{III.5})$$

where + and - stand for the nucleon helicities $+\frac{1}{2}$ and $-\frac{1}{2}$, and for the photon⁸ helicities +1 and -1 respectively.

For a given initial polarisation configuration (P_m, ζ_n) , $(m, n) = \text{fixed}$, but $\neq 0$, we expand the cross-sections σ and the density-matrices

$\rho_{N', V}$, ρ , $\rho_{N'}$ and ρ_Y , as for example

$$\rho(P_m, \zeta_n) = \rho(0, 0) + P_m \rho(m, 0) + \zeta_n \rho(0, n) + P_m \zeta_n \rho(m, n) \quad (\text{III.6})$$

where the expansion coefficients $\rho(m, n)$ are very convenient bilinears of amplitudes, and $\rho(P_0, \zeta_0) = \rho(0, 0)$ for the unpolarised case; in (III.6) the initial density-matrices are

$$\rho_Y = \frac{1}{2}(1 + P_m \sigma_m), \quad \rho_N = \frac{1}{2}(1 + \zeta_n \sigma_n), \quad (m, n) = \text{fixed}.$$

For the expansion coefficients, one gets, for example,

$$\rho^{\alpha'\beta'}(m,n) = \frac{1}{4} \sum_{i',\alpha,\beta,i,j} f_{i\alpha}^{i'\alpha'} (\sigma_m)^{\alpha\beta} (\sigma_n)^{ij} f_{j\beta}^{i'\beta'*} \quad (\text{III.7})$$

The coefficients $\rho(m,n)$, $\rho_N(m,n)$, $\rho_{N,V}(m,n)$, contain all the information; no loss of generality is incurred by not considering mixtures of these polarisations in the initial state. One can expand only the unnormalised density matrices as in (III.6), but not the normalised ones

$$\hat{\rho}(P_m, \zeta_n) = \rho(P_m, \zeta_n) / \text{tr } \rho(P_m, \zeta_n) \quad (\text{III.8})$$

which are not simple polynomials in P_m and ζ_n ; similarly, ζ'_j and P'_j of (III.5) cannot be so expanded. Our unnormalised "actual" polarisations \hat{P}'_j and $\hat{\zeta}'_j$ defined as⁹

$$\hat{P}'_j(P_m, \zeta_n) = P'_j(P_m, \zeta_n) \text{tr } \rho_Y(P_m, \zeta_n) \quad (\text{III.9a})$$

$$\text{and } \hat{\zeta}'_j(P_m, \zeta_n) = \zeta'_j(P_m, \zeta_n) \text{tr } \rho_N(P_m, \zeta_n) \quad (\text{III.9b})$$

can indeed be expanded like (III.6), as for example

$$\hat{\zeta}'_j(P_m, \zeta_n) = \hat{\zeta}'_j(0,0) + P_m \hat{\zeta}'_j(m,0) + \zeta_n \hat{\zeta}'_j(0,n) + P_m \zeta_n \hat{\zeta}'_j(m,n) \quad (\text{III.10})$$

Similarly, the "universally normalised actual" polarisations

$$\hat{P}_j(P_m, \zeta_n) = \hat{P}'_j(P_m, \zeta_n) / \text{tr } \rho_Y(0,0) \quad (\text{III.11a})$$

$$\text{and } \hat{\zeta}_j(P_m, \zeta_n) = \hat{\zeta}'_j(P_m, \zeta_n) / \text{tr } \rho_N(0,0) \quad (\text{III.11b})$$

can be expanded in terms of their coefficients, as in (III.6). The definitions (III.11) obviously imply

$$\hat{P}_j(P_0, \zeta_0) = P_j(P_0, \zeta_0) \quad (\text{III.12a})$$

$$\text{and } \hat{\zeta}_j(P_0, \zeta_0) = \zeta_j(P_0, \zeta_0) \quad (\text{III.12b})$$

A universal normalisation independent of initial polarisation has been used by Schilling et al [8]:

$$\rho_{\lambda\lambda'}(P_m, \zeta_n) \Big|_{\text{Theirs}} = \rho^{\lambda\lambda'}(P_m, \zeta_n) / \text{tr } \rho(0,0) \Big|_{\text{Ours}} \quad (\text{III.13a})$$

so that

$$\left. \frac{\rho_{\lambda\lambda'}(P_m, \zeta_n)}{\rho_{\Lambda\Lambda'}(P_M, \zeta_N)} \right|_{\text{Theirs}} = \left. \frac{\rho^{\lambda\lambda'}(P_m, \zeta_n)}{\rho^{\Lambda\Lambda'}(P_M, \zeta_N)} \right|_{\text{Ours}} \quad (\text{III.13b})$$

This comparison (III.13) is important for our use of $\gamma N \rightarrow \gamma N$ data [5] (sec. 5) because their [8] notation has been used in ref. [5].

It is worth noting that for the following combinations of (m, n) , parity-conservation makes $\text{tr } \rho (= \text{tr } \rho_N)$ vanish [7,8]:

$$\rho_N^{++}(m,n) = - \rho_N^{--}(m,n) \quad \text{and} \quad \rho_Y^{++}(m,n) = - \rho_Y^{--}(m,n) \quad (\text{III.14})$$

for $m = (0 \text{ or } 1), n = (1 \text{ or } 3):$ and $m = (2 \text{ or } 3), n = (0 \text{ or } 2).$.
(III.15)

The notation for cross-section asymmetries is given in Eqs. (A.10 to 12) of the Appendix.

The notational phrase "nth equality of (V.1 or 2)" are defined when Eqs. (V.1 and 2) first appear (sec. 5).

4. SOME T-INVARIANCE AND M-PURITY RELATIONS IN $\gamma N \rightarrow \gamma N$

Assuming (as throughout) parity-conservation, there are only eight independent helicity amplitudes for $\gamma N \rightarrow \gamma N$:

$$f_{++}^{++}, f_{-+}^{++}, f_{--}^{++}, f_{+-}^{++}, f_{-+}^{+-}, f_{++}^{+-}, f_{+-}^{+-} \text{ and } f_{++}^{+-} . \quad (\text{IV.1})$$

Out of these, the first two are not restricted by the T-invariance relation (I.4), while the last four are:

$$f_{-+}^{++} = - f_{++}^{++} \quad (\text{IV.2a})$$

$$f_{+-}^{++} = f_{++}^{+-} \quad (\text{IV.2b})$$

giving

$$u_{-+}^{++} = 0, u_{++}^{-+} = 0, \quad (\text{IV.3a})$$

$$u_{+-}^{++} = 0, u_{++}^{+-} = 0, \quad (\text{IV.3b})$$

as examples of Eqs. (I.5). The relations

$$f_{--}^{++}/f_{++}^{--} = f_{-+}^{+-}/f_{+-}^{-+} = -1 \quad (\text{IV.4})$$

involving the remaining two amplitudes of (IV.1) are given by both parity-conservation (I.1) and T-invariance (I.4); (IV.4) does not restrict any n- or u- type amplitudes. We distinguish the consequences of (IV.3) which is characteristic of T-invariance from those of (IV.4). Because of (IV.3), the only independent nonvanishing u-amplitudes are

$$u_{++}^{++} \quad \text{and} \quad u_{--}^{++}, \quad (\text{IV.5})$$

while the n-amplitudes

$$n_{++}^{++}, n_{--}^{++}, n_{-+}^{++} \quad \text{and} \quad n_{+-}^{++} \quad (\text{IV.6})$$

allowed by parity-conservation are also allowed by T-invariance.

(4.1) T-Invariance Relations

Using only parity-conservation which includes the T-invariance constraint (IV.4), one gets [7]

$$\tilde{\zeta}_2^i(0,0) = \tilde{P}_1^i(1,2) \quad (\text{IV.7})$$

$$\tilde{\zeta}_2^i(1,0) = \tilde{P}_1^i(0,2) \quad (\text{IV.8})$$

$$\tilde{\zeta}_2^i(0,2) = \tilde{P}_1^i(1,0) \quad (\text{IV.9})$$

$$\text{and} \quad \tilde{\zeta}_2^i(1,2) = \tilde{P}_1^i(0,0) \quad (\text{IV.10})$$

These can be expressed in terms of the actual polarisations $\tilde{P}_1^i(P_i, \zeta_j)$ and $\tilde{\zeta}_2^i(P_k, \zeta_k)$, and also in terms of the cross-section asymmetries defined in the Appendix. The four relations (IV.7 to 10) combine to give

$$\tilde{P}_1^i(P_1, \zeta_2) = P_1 \zeta_2 \tilde{\zeta}_2^i\left(\frac{1}{P_1}, \frac{1}{\zeta_2}\right) \quad (\text{IV.11})$$

where the argument on the right hand side expresses the relevant values of P_1 and ζ_2 respectively. Because $|P_1| \leq 1$ and $|1/P_1| \leq 1$, this relation is meaningful for only $P_1 = \pm 1$ and similarly, for $\zeta_2 = \pm 1$, giving

$$\tilde{P}'_1(P_1, \zeta_2) = P_1 \zeta_2 \tilde{\zeta}'_2(P_1, \zeta_2) \quad \text{for } P_1 = \pm 1, \zeta_2 = P_1 \text{ or } -P_1. \quad (\text{IV.12})$$

Further use of the T-invariance restriction (IV.3) in (IV.7 and 10) gives

$$\tilde{\zeta}'_2(0,0) = C \sigma(0,2) \quad (\text{IV.13})$$

$$\text{and} \quad \tilde{P}'_1(0,0) = C \sigma(1,0) \quad (\text{IV.14})$$

which provide examples of the standard asymmetry-polarisation theorem for elastic scattering. In fact, using the asymmetries $A(P_0, \pm \zeta_2)$ and $A(\pm P_1, \zeta_0)$ of Eqs. (A.15 and 16) of the Appendix, the above two relations read, in terms of the normalised polarisations ζ'_2 and P'_1 as

$$A(P_0, \pm \zeta_2) = \zeta_2 \cdot \zeta'_2(P_0, \zeta_0) \quad (\text{IV.15})$$

$$\text{and} \quad A(\pm P_1, \zeta_0) = P_1 \cdot P'_1(P_0, \zeta_0) \quad (\text{IV.16})$$

which are the equality (II.8) corresponding to the nucleon and the photon respectively.

Another consequence of T-invariance (IV.3) is to relate [7] different final photon polarisations

$$\tilde{P}'_2(2,2) = \tilde{P}'_3(3,2) \quad (\text{IV.17})$$

$$\text{and} \quad \tilde{P}'_2(3,0) = -\tilde{P}'_3(2,0) \quad (\text{IV.18})$$

which can be expressed in terms of the actual polarisations as, for example

$$P_2 [\tilde{P}'_3(P_3, \zeta_2) - \tilde{P}'_3(P_3, \zeta_0)] = P_3 [\tilde{P}'_2(P_2, \zeta_2) - \tilde{P}'_2(P_2, \zeta_0)] \quad (\text{IV.19})$$

$$\text{and} \quad P_2 \tilde{P}'_2(P_3, \zeta_0) = -P_3 \tilde{P}'_3(P_2, \zeta_0). \quad (\text{IV.20})$$

T-invariance (IV.3) also gives relations [7] between the final nucleon polarisations in the reaction plane and final photon polarisations:

$$\tilde{P}_2'(0,1) = \tilde{\zeta}_1'(2,0), \quad (\text{IV.21})$$

$$\tilde{P}_2'(0,3) = -\tilde{\zeta}_3'(2,0), \quad (\text{IV.22})$$

$$\tilde{P}_3'(0,3) = \tilde{\zeta}_3'(3,0), \quad (\text{IV.23})$$

and $\tilde{P}_3'(0,1) = -\tilde{\zeta}_1'(3,0) \quad (\text{IV.24})$

which may be rewritten in terms of the actual polarisations as, for example

$$P_2 \tilde{P}_2'(P_0, \zeta_1) = \zeta_1 \tilde{\zeta}_1'(P_2, \zeta_0), \quad (\text{IV.25})$$

$$P_2 \tilde{P}_2'(P_0, \zeta_3) = -\zeta_3 \tilde{\zeta}_3'(P_2, \zeta_0), \quad (\text{IV.26})$$

$$P_3 \tilde{P}_3'(P_0, \zeta_3) = \zeta_3 \tilde{\zeta}_3'(P_3, \zeta_0), \quad (\text{IV.27})$$

and $P_3 \tilde{P}_3'(P_0, \zeta_1) = -\zeta_1 \tilde{\zeta}_1'(P_3, \zeta_0). \quad (\text{IV.28})$

(4.2) M-Purity Relations

In subsec. 4.1 were considered some relations as consequences of the M-purity ($M = +1$) of type 1, as embodied in (IV.3). If one has full M-purity (of type 2 as well), simplifications in these relations occur, and some new relations hold; we consider the two separately.

(4.2a) M-Purity Modifications in the T-Invariance Relations of subsec. (4.1)

The relations (IV.7 to 10) involving recoil nucleon polarisations normal to the reaction plane become, under M-purity,

$$\tilde{\zeta}_2'(0,0) = \tilde{P}_1'(1,2) = M C \sigma(0,2) \quad (\text{IV.29})$$

$$\tilde{\zeta}_2'(1,0) = \tilde{P}_1'(0,2) = M C \sigma(1,2) \quad (\text{IV.30})$$

$$\tilde{\zeta}_2'(0,2) = \tilde{P}_1'(1,0) = M C \sigma(0,0) \quad (\text{IV.31})$$

$$\tilde{\zeta}_2'(1,2) = \tilde{P}_1'(0,0) = M C \sigma(1,0) \quad (\text{IV.32})$$

Out of these only (IV.29 and 32) have T-invariance analogues in (IV.13 and 14). In fact, the four cross-section coefficients in (IV.29 to 32) are the only nonvanishing ones for $\gamma N \rightarrow \gamma N$ and $\gamma N \rightarrow \gamma N$ under M-purity; all the nonvanishing cross-section asymmetries are then related to recoil nucleon polarisations perpendicular to the reaction plane (or, equivalently, to the corresponding final photon polarisations \hat{P}_1'). Combining the four relations (IV.29 to 32), full M-purity gives

$$\zeta_2'(P_1, \zeta_2) = M C \zeta_2 \sigma(P_1, 1/\zeta_2) \quad (\text{IV.33})$$

$$\text{and} \quad \hat{P}_1'(P_1, \zeta_2) = M C P_1 \sigma(1/P_1, \zeta_2) \quad (\text{IV.34})$$

where the arguments on the right hand side express the relevant values of P_1 and ζ_2 respectively. Because of the restrictions $|P_1| \leq 1$, $|1/P_1| \leq 1$, (IV.34) is meaningful for only $P_1 = \pm 1$, and similarly, (IV.33) for $\zeta_2 = \pm 1$ giving

$$\zeta_2'(P_1, \zeta_2 = \pm 1) = \pm M C \sigma(P_1, \zeta_2 = \pm 1) \quad (\text{IV.35})$$

$$\text{and} \quad \hat{P}_1'(P_1 = \pm 1, \zeta_2) = \pm M C \sigma(P_1 = \pm 1, \zeta_2) \quad (\text{IV.36})$$

which read as

$$\zeta_2'(P_1, \zeta_2 = \pm 1) = \pm M \quad (\text{IV.37})$$

$$\text{and} \quad \hat{P}_1'(P_1 = \pm 1, \zeta_2) = \pm M \quad (\text{IV.38})$$

in terms of the normalised polarisations. The M-purity relations (IV.37 and 38) have the interesting feature that there is no dependence¹⁰ on the polarisation of the "other" particle: if the initial nucleon (photon) is fully polarised, so is the final nucleon (photon), independent of the value of the initial photon (nucleon) polarisation $P_1(\zeta_2)$, the relative sign between the initial and final nucleon (photon) polarisations being given by M.

The relation of (IV.29 and 32) to cross-section asymmetries is, apart from the factor M, the same as in (IV.15 and 16). The relation of (IV.30) to various cross-section asymmetries may be written in terms of the double asymmetry $A(\pm P_1, \pm \zeta_2)$ and the single asymmetries $A(P_0, \pm \zeta_2)$ and $A(\pm P_1, \zeta_0)$ as

$$\zeta_2'(1,0) = \frac{MC\sigma(0,0)}{P_1\zeta_2} \left\{ -1 + \frac{A(\pm P_1, \zeta_0) + A(P_0, \pm \zeta_2)}{A(\pm P_1, \pm \zeta_2)} \right\} \quad (IV.39)$$

or equivalently in terms of these single asymmetries and $A(P_1, \pm \zeta_2)$ as

$$\zeta_2'(1,0) = \frac{MC\sigma(0,0)}{P_1\zeta_2} \left\{ A(P_1, \pm \zeta_2) (1 + A(\pm P_1, \zeta_0)) - A(P_0, \pm \zeta_2) \right\} \quad (IV.40)$$

where, of course, the left hand side may be expressed in terms of the actual polarisations as for example $[\zeta_2'(P_1, \zeta_0) - \zeta_2'(P_0, \zeta_0)]/P_1$ or replaced by the corresponding expressions for $\tilde{P}_1'(0,2)$. The relations (IV.39 and 40) expressing the recoil nucleon polarisations in terms of cross-section asymmetries have no T-invariance analogue because the "other" particle (the photon in $\zeta_2'(1,0)$ or the nucleon in $\tilde{P}_1'(0,2)$) is initially polarised.

The relation (IV.31) shows that under M-purity, the final state polarisation coefficients $\zeta_2'(0,2)$ and $\tilde{P}_1'(1,0)$ when the corresponding initial particle is polarised, but the "other" particle initially unpolarised, are determined by the unpolarised cross-section $\sigma(0,0)$.

The relations (IV.29 to 32) also provide interesting M-purity analogues of the depolarisation relation of πN scattering¹¹

$$\text{Final Polarisation} = D \cdot \text{Initial Polarisation} + P \quad (IV.41)$$

where all the polarisations are perpendicular to the reaction plane;
P is the recoil polarisation obtained with an unpolarised target;
D is the depolarisation coefficient; for $\pi N \rightarrow \pi N$, $D = +1$ and for

$\pi N \rightarrow \pi N$, $D = -1$. The relation closest to (IV.41), arising from (IV.31), reads in terms of the 'universally normalised actual' polarisations as

$$\hat{\zeta}_2'(P_0, \zeta_2) = M\zeta_2 + \hat{\zeta}_2'(P_0, \zeta_0) \quad (\text{IV.42})$$

or equivalently,

$$\hat{P}_1'(P_1, \zeta_0) = MP_1 + \hat{P}_1'(P_0, \zeta_0), \quad (\text{IV.43})$$

the role of the depolarisation coefficient being taken by M now. The corresponding relations arising from (IV.29, 30, 32) look less simple, but are straightforward to write.

Coming to the M -purity analogues of (IV.17, 18), one gets

$$\hat{P}_2'(2, 2) = M \hat{P}_3'(3, 2) \quad (\text{IV.44})$$

$$\hat{P}_2'(3, 0) = -M \hat{P}_3'(2, 0). \quad (\text{IV.45})$$

Eqs. (IV.17, 18) are the $M = +1$ examples of these relations.

The T -invariance relations (IV.21 to 24) between final nucleon polarisations in the reaction plane and final photon polarisations involve interference between amplitudes of the n - and the u - type so that for full M -purity, they become identities of the $0 = 0$ type.

(4.2b) New M -Purity Relations

Here we mention only two relations resembling (IV.44 and 45) between final photon polarisations:

$$\hat{P}_2'(2, 0) = M \hat{P}_3'(3, 0) \quad (\text{IV.46})$$

$$\hat{P}_2'(3, 2) = -M \hat{P}_3'(2, 2) \quad (\text{IV.47})$$

Taken together, the M -purity relations (IV.44 to 47) cover the case of both unpolarised targets and targets polarised normal to the reaction plane. Written in terms of the actual polarisations, (IV.46 and 47)

read respectively as

$$P_3 \hat{P}'_2(P_2, \zeta_0) = M P_2 \hat{P}'_3(P_3, \zeta_0) \quad (\text{IV.48})$$

$$\text{and } P_3 \hat{P}'_3(P_2, \zeta_2) = -P_2 \hat{P}'_2(P_3, M\zeta_2) \quad (\text{IV.49})$$

where the T-invariance result (IV.18) has been used in (IV.49), and $M\zeta_2$ in the argument on the right hand side of (IV.49) gives the relevant value of ζ_2 .

(4.3) Remarks on the Comparison between the T-Invariance and the M-Purity Relations in $\gamma N \rightarrow \gamma N$.

A convenient summary of subsecs. (4.1) and (4.2) is given in table 1. In some cases, there are only T-invariance relations and no corresponding useful M-purity ones (no: 5); in other cases, there are only M-purity relations but no corresponding T-invariance relations (nos; 6b,7,8,10); in still other cases one has both the T-invariance relations and their corresponding M-purity analogues (nos: 2, 3,6a;4,9). The fact that M-purity relations like the asymmetry polarisation equality can hold even when the "other" particle is polarised is illustrated by item no: 7 which is based on (IV.30). Another example where an M-purity relation goes beyond T-invariance is the depolarisation-like relations (IV.42, 43) based on (IV.31).

There is some vector meson-dominance argument (subsec. (5.2)) for full M-purity (with $M = +1$) in $\gamma N \rightarrow \gamma N$; the argument is not model-independent. To decide whether data favour only T-invariance (M-purity of type 1), or full M-purity, one can consider situations where either only T-invariance relations or only M-purity relations exist. For example, are the T-invariance relations (IV. 21 to 28) non-trivial, or only null identities as full M-purity would give? Examples of the other type are to test the consequences (IV.39, 40; 42, 43; 46, 47)

of full M-purity; these consequences do not follow if one had only T-invariance. The above tests require measurements of final state polarisations.

5. EXPERIMENTAL EVIDENCE FOR M-PURITY IN $\gamma N \rightarrow V N$; IMPLICATIONS FOR

$$\gamma N \rightarrow \gamma N$$

(5.1) $\gamma N \rightarrow V N$

Data on the vector meson density matrix are available [5] for linearly polarised photons, and unpolarised target nucleons. In order to examine the M-purity implications of these data, we recall [4] the M-purity restrictions on elements of the vector meson density matrix for various initial polarisations. Stated in terms of the density-matrix coefficients $\rho^{ij}(m,k)$, these relations read as

$$-\frac{\rho^{00}(1,k)}{\rho^{00}(0,k)} = \frac{\text{Re } \rho^{+-}(0,k)}{\rho^{++}(1,k)} = \frac{\text{Re } \rho^{+-}(1,k)}{\rho^{++}(0,k)} = -\frac{\text{Re } \rho^{+0}(1,k)}{\text{Re } \rho^{+0}(0,k)} = M = 1/M \quad (\text{V.1})$$

and

$$-\frac{\text{Im } \rho^{+-}(2,k)}{\rho^{++}(3,k)} = \frac{\text{Im } \rho^{+-}(3,k)}{\rho^{++}(2,k)} = M = 1/M \quad (\text{V.2})$$

where k is 0 or 2. The equality of the first, second, third and fourth expressions in (V.1) to M would be called the first, second, third and fourth equalities of (V.1); and similarly for (V.2). The denominators in (V.2) do not appear in the decay distribution of the vector meson. If one had M-purity of only type 1, one would obtain the second equality of (V.1) for $k = 0$, the third equality of (V.1) for $k = 2$, the first equality of (V.2) for $k = 2$ and the second equality of (V.2) for $k = 0$. The first and the last equalities of (V.1) obviously involve amplitudes with zero V-helicity.

For our purposes, the available data [5] refer to only (V.1) for $k = 0$ (unpolarised targets). The highest energy (9.3 GeV photon energy)

data on $\gamma N \rightarrow \rho N$ show that all the density-matrix elements in (V.1) except those in its third equality are rather small, and consistent with being zero, especially for small momentum transfers. The elements in the third equality are both equal, giving

$$\text{Re } \rho^{+-}(1,0)/\rho^{++}(0,0) = +1 \quad (\text{V.3})$$

within experimental errors which are not large. The highest energy (9.3 GeV photon energy) data on $\gamma N \rightarrow \omega N, \phi N$ also show [5] the same features, though the errors are larger than for $\gamma N \rightarrow \rho N$. Written in terms of the n- and u-type amplitudes, (V.3) reads as

$$(a-b)/(a+b) = +1, \quad (\text{V.4a})$$

where

$$\begin{aligned} a &= a_1 + a_2, \\ a_1 &= |n_{++}^{++}|^2 + |n_{-+}^{++}|^2, \\ a_2 &= |n_{+-}^{++}|^2 + |n_{--}^{++}|^2; \end{aligned} \quad (\text{V.4b})$$

$$\begin{aligned} b &= b_1 + b_2, \\ b_1 &= |u_{++}^{++}|^2 + |u_{-+}^{++}|^2, \\ b_2 &= |u_{+-}^{++}|^2 + |u_{--}^{++}|^2. \end{aligned} \quad (\text{V.4c})$$

Equations (V.4) imply that within the experimental errors,

$$u_{++}^{++} = u_{--}^{++} = u_{-+}^{++} = u_{+-}^{++} = 0 \quad (\text{V.5})$$

in $\gamma N \rightarrow \rho N$. This result is remarkable in that the single ratio (V.3) implies the vanishing of all the u-type amplitudes with a nonzero helicity of the vector meson. The n- and u- type amplitudes with a zero helicity of the vector meson are experimentally small, and do not provide such a clear-cut conclusion, though data are consistent with the vanishing of these u-amplitudes also.

The direct experimental evidence for

$$\text{Re } \rho^{+-}(0,0)/\rho^{++}(1,0) = +1 \quad (\text{V.6})$$

is weak since these density-matrix elements are small and consistent with zero. However, (V.5) implies (V.6). Similarly, (V.2) fully and the second and third equalities of (V.1) for also polarised targets ($k = 2$) are implied by (V.5). One may note that M-purity of only type 1 would not lead to the result (V.3).

Further experimental evidence for M-Purity in $\gamma N \rightarrow VN$ comes from data for the asymmetries P_σ and Σ defined as [5,8]

$$P_\sigma = (\sigma^N - \sigma^U)/(\sigma^N + \sigma^U), \quad (\text{V.7})$$

$$\begin{aligned} \Sigma &= (\sigma_{\parallel} - \sigma_{\perp})/(\sigma_{\parallel} + \sigma_{\perp}) \\ &= (\rho^{++}(1,0) + \text{Re } \rho^{+-}(1,0))/(\rho^{++}(0,0) + \text{Re } \rho^{+-}(0,0)) \end{aligned} \quad (\text{V.8})$$

where $\sigma^{N,U}$ are contributions to the cross-section from natural and unnatural parity exchanges respectively in the t-channel, $\sigma_{\parallel,\perp}$ are cross-sections for producing pseudoscalar meson-pairs (from V-decay) parallel and normal respectively to the photon polarisation. To leading order in energy,

$$P_\sigma = (2 \text{Re } \rho^{+-}(1,0) - \rho^{00}(1,0))/\text{tr } \rho(0,0). \quad (\text{V.9})$$

The second and third equalities of (V.1), if used in (V.8), lead to

$$\Sigma = M. \quad (\text{V.10})$$

From these definitions, and the fact that $\rho^{++}(1,0)$, $\text{Re } \rho^{+-}(0,0)$, $\rho^{00}(0,0)$ and $\rho^{00}(1,0)$ are experimentally small, it is clear that to a good approximation, P_σ and Σ depend primarily on only the large matrix-elements $\text{Re } \rho^{+-}(1,0)$ and $\rho^{++}(0,0)$ which occur also in (V.3).

The main experimental evidence for M-purity in $\gamma N \rightarrow VN$ is, therefore, based on (V.3) which implies $M = +1$ for all amplitudes with nonzero V-helicity.

We estimate the experimental errors associated with M-purity in $\gamma N \rightarrow VN$ using $\gamma N \rightarrow \rho^0 N$ data [5] for unpolarised targets at 9.3 GeV/c at a typical small $|t|$ value, $t = - (0.12 \rightarrow 0.18) \text{ (GeV/c)}^2$. Eq. (V.1) then reads as

$$- \frac{-0.05 \pm 0.04}{0.03 \pm 0.02} = \frac{0.01 \pm 0.03}{-0.02 \pm 0.04} = \frac{0.48 \pm 0.05}{0.485 \pm 0.01} = - \frac{-0.01 \pm 0.03}{0.03 \pm 0.02} = M \quad (\text{V.11a})$$

or

$$1.6 \pm 1.7 = -0.5 \pm 1.8 = 0.99 \pm 0.11 = 0.33 \pm 1.02 = M \quad (\text{V.11b})$$

where the correlations between the various errors have been dropped (also hereafter). Except for the ratio (V.3), the errors in (V.11) are too large to justify a firm conclusion about $M = +1$, though $M = +1$ is allowed. For the ratio (V.3), the quantity b/a is seen to be small. Neglecting second and higher powers of b/a , one gets from (V.11b),

$$b/a = 0.005 \pm 0.055 \quad (\text{V.12})$$

which shows that the $M = -1$ combination b is at most about 5% of the $M = +1$ combination a .

In fact, the numbers for $\text{Re } \rho^{+-}(1,0)$, $\rho^{++}(0,0)$ and $\text{Im } \rho^{+-}(2,0)$ imply [7] that any of the three combinations a_2 , b_1 and b_2 are at most a few percent of the dominant combination a_1 :

$$\begin{aligned} a_1 - b_1 &= (0.96 \pm 0.064) d, & b_1 + b_2 &= (0.005 \pm 0.051) d, \\ a_1 + a_2 &= (0.965 \pm 0.051) d, & a_2 + b_1 &= (0.005 \pm 0.041) d, \\ a_1 + b_2 &= (0.965 \pm 0.041) d, & a_2 - b_2 &= (0.0 \pm 0.064) d \end{aligned} \quad (\text{V.13})$$

where d is the overall normalisation. Taking $0.05d$ as the typical upper limit on b_1 and b_2 , the $M = -1$ amplitudes occurring in b are consistent with zero, and bounded in magnitude by about a fifth of the $M = +1$ combination $(|n_{++}^{++}|^2 + |n_{+-}^{++}|^2)^{1/2}$. It would be nice to have much smaller

errors in (V.12). We use $b/a = 0$ in the next subsection.

Using data on $\rho^{00}(0,0)$ and $\rho^{00}(1,0)$, one similarly gets

$$|n_{++}^{+0}|^2 + |n_{-+}^{+0}|^2 = (0.04 \pm 0.022) d \quad (V.14a)$$

$$|u_{++}^{+0}|^2 + |u_{-+}^{+0}|^2 = (-0.01 \pm 0.022) d \quad (V.14b)$$

It is, therefore, difficult to make a firm statement about M-purity of these small (helicity-flip) amplitudes. Because the remaining available [5] density-matrix-elements depend on unknown relative phases between different helicity amplitudes, it is hard to get further numerical estimates for M-purity.

5.2) Implications for $\gamma N \rightarrow \gamma N$

The result (V.5) is interesting also for $\gamma N \rightarrow \gamma N$. The vanishing of u_{-+}^{++} and u_{+-}^{++} is M-purity ($M = +1$) of type 1, as given by T-invariance in $\gamma N \rightarrow \gamma N$, but vanishing of the amplitudes u_{++}^{++} and u_{--}^{++} means M-purity ($M = +1$) of type 2 which is not required by T-invariance in $\gamma N \rightarrow \gamma N$. One may regard M-purity ($M = +1$) of type 1 in $\gamma N \rightarrow \gamma N$ as support¹² for the vector-meson dominance of the electromagnetic current, since the corresponding purity in $\gamma N \rightarrow \gamma N$ is already guaranteed by T-invariance. The smallness of the $\gamma N \rightarrow \gamma N$ amplitudes with zero V-helicity may be regarded as another support for vector meson dominance, since these amplitudes are absent in $\gamma N \rightarrow \gamma N$. With this support from data, one may turn the argument around: One can use the vector meson dominance model to deduce that the vanishing of u_{++}^{++} and u_{--}^{++} in $\gamma N \rightarrow \gamma N$ implies the vanishing of these amplitudes also in $\gamma N \rightarrow \gamma N$. Since u_{++}^{++} and u_{--}^{++} are the only independent u-amplitudes in T-invariant $\gamma N \rightarrow \gamma N$, the result (V.5) coupled with vector dominance implies that there are no u-type amplitudes in $\gamma N \rightarrow \gamma N$.

This full M-purity in even $\gamma N \rightarrow \gamma N$ has obvious implications for the comparison between M-purity relations and T-invariance relations, considered in subsec (4.3). An experimental test of these consequences of full M-purity would throw light also on the vector dominance model, as the above argument shows. One should, however, note that at present there are some experimental errors associated with (V.5) on which our argument for full M-purity in $\gamma N \rightarrow \gamma N$ is based. It would be nice to reduce these experimental errors.

6. M-PURITY RELATIONS IN $\gamma N \rightarrow VN$; COMPARISON WITH $\gamma N \rightarrow \gamma N$

Subsec. (5.1) shows that there is good evidence for M-purity ($M = +1$) for amplitudes with nonzero V-helicity in $\gamma N \rightarrow VN$, while that for amplitudes with zero V-helicity is not so good. These latter amplitudes are relatively small [5], making M-purity a good approximation in $\gamma N \rightarrow VN$. The extra (= zero) V-helicity in $\gamma N \rightarrow VN$ as compared to $\gamma N \rightarrow \gamma N$ allows one to study the effect of inelasticity on M-purity relations as a question of principle. As we shall see, this feature of inelasticity does not modify some classes of M-purity relations, exemplifying their generality.

For $\gamma N \rightarrow VN$, one needs to consider the vector meson density-matrix instead of the final photon polarisation of $\gamma N \rightarrow \gamma N$. Using the relation of this polarisation to the photon density-matrix, one translates the relations involving these polarisations into the corresponding $\gamma N \rightarrow VN$ relations. The relations (IV.7 to 10), for example, now become [7] respectively

$$\tilde{\zeta}_2^+(0,0) = 2 \operatorname{Re} \rho^{+-}(1,2) - \rho^{00}(1,2) \quad (\text{VI.1})$$

$$\tilde{\zeta}_2^+(1,0) = 2 \operatorname{Re} \rho^{+-}(0,2) - \rho^{00}(0,2) \quad (\text{VI.2})$$

$$\tilde{\zeta}_2^+(0,2) = 2 \operatorname{Re} \rho^{+-}(1,0) - \rho^{00}(1,0) \quad (\text{VI.3})$$

$$\text{and} \quad \tilde{\zeta}_2^+(1,2) = 2 \operatorname{Re} \rho^{+-}(0,0) - \rho^{00}(0,0) \quad (\text{VI.4})$$

where the elements ρ^{00} represent the modification due to amplitudes with zero V-helicity; these relations now depend on only parity-conservation (I.1).

We now consider the $\gamma N \rightarrow \gamma N$ analogues of the M-purity relations of sec. 4. The M-purity analogues of (IV.29 to 32) can be obtained by combining (VI.1 to 4) with the first three equalities of (V.1):

$$\zeta_2^+(0,0) = 2 \operatorname{Re} \rho^{+-}(1,2) - \rho^{00}(1,2) = M C \sigma(0,2) , \quad (\text{VI.5})$$

$$\zeta_2^+(1,0) = 2 \operatorname{Re} \rho^{+-}(0,2) - \rho^{00}(0,2) = M C \sigma(1,2) , \quad (\text{VI.6})$$

$$\zeta_2^+(0,2) = 2 \operatorname{Re} \rho^{+-}(1,0) - \rho^{00}(1,0) = M C \sigma(0,0) , \quad (\text{VI.7})$$

$$\zeta_2^+(1,2) = 2 \operatorname{Re} \rho^{+-}(0,0) - \rho^{00}(0,0) = M C \sigma(1,0) . \quad (\text{VI.8})$$

The way in which the cross-section coefficients appear in these relations is the same as in $\gamma N \rightarrow \gamma N$. For full M-purity, the relation between recoil nucleon polarisations and these coefficients is also unchanged.

Unlike $\gamma N \rightarrow \gamma N$, there is no T-invariance analogue of (VI.5 and 8) now, but we consider M-purity of only type 1 to illustrate how the inelastic component (i.e. amplitudes with zero V-helicity) behaves so as to leave the relations between ζ' and σ 's for full M-purity unmodified. For M-purity of only type 1, the interesting elements are $\operatorname{Re} \rho^{+-}(1,2)$ and $\operatorname{Re} \rho^{+-}(0,0)$. The relevant relations (VI.5, 8) then become

$$\zeta_2^+(0,0) = 2 \operatorname{Re} \rho^{+-}(1,2) - \rho^{00}(1,2) = M C \sigma(0,2) - [M \rho^{00}(0,2) + \rho^{00}(1,2)] , \quad (\text{VI.9})$$

$$\zeta_2^+(1,2) = 2 \operatorname{Re} \rho^{+-}(0,0) - \rho^{00}(0,0) = M C \sigma(1,0) - [M \rho^{00}(1,0) + \rho^{00}(0,0)] . \quad (\text{VI.10})$$

If M-purity holds also for amplitudes with zero V-helicity with the same value of M as for the other amplitudes, the first equality of

(V.1) shows that the square brackets in (VI.9, 10) vanish and one recovers, for full M-purity, the analogues (VI.5, 8) of the T-invariance relations (IV.13, 14) for even the inelastic reaction $\gamma N \rightarrow VN$. A similar remark applies to analogues (VI.6, 7) of the full M-purity relations (IV.30, 31).

Because the above relations between recoil nucleon polarisations and cross-section coefficients are the same as in $\gamma N \rightarrow \gamma N$, the relations (IV.33, 35, 37) which determine recoil nucleon polarisations hold also for $\gamma N \rightarrow VN$; so does the M-purity version of (IV.15). Similarly, the relations (IV.39, 40) between recoil nucleon polarisations and cross-section asymmetries remain unmodified. Also, the relation (IV.42) resembling the depolarisation relation for πN scattering is not modified.

Using the definition of \tilde{P}_j^i and (III.14), one sees that the $\gamma N \rightarrow VN$ analogues of the M-purity relations¹³ (IV.44 to 47) are all contained in (V.2). Similarly, the M-purity relations (like IV.34, 36, 38, 43) concerning \tilde{P}_1^i all get translated into the $\gamma N \rightarrow VN$ case by using the second and third equalities of (V.1).

The M-purity analogues of (IV.21 to 28) continue to be null identities, as in $\gamma N \rightarrow \gamma N$.

Some comments on these ρ^{00} modifications: They occur already in the relevant relations (VI. 1 to 4) following from only parity-invariance, but appear systematically in such a way that for full M-purity, the form of the relations (VI.5 to 8) between recoil nucleon polarisations and cross-section coefficients is not changed as compared to $\gamma N \rightarrow \gamma N$. The M-purity equalities relating $\text{Re } \rho^{+-}(i,k)$ to $\rho^{++}(j,k)$, $(i,j) = (0,1)$, $i \neq j$, $k = (0,2)$, embodied in (V.1) do not get modified¹³ in going over to $\gamma N \rightarrow VN$. It is the relation of ρ^{++} to $\text{tr } \rho$ (and therefore, to σ) that gets modified by the ρ^{00} contributions. The

interesting point is that the ρ^{00} modifications occur in such a way that under full M-purity, the trace of the final photon density matrix ρ_Y , in $\gamma N \rightarrow \gamma N$ gets replaced by exactly the trace of the vector meson density matrix ρ in $\gamma N \rightarrow VN$.

There is thus a "vertex-dependence" in the effects of inelasticity on M-purity relations: The relations (IV.29 to 32) between recoil nucleon polarisations and cross-section coefficients are not modified in going over to $\gamma N \rightarrow VN$. The corresponding relations (IV.29 to 32)

$$2 \operatorname{Re} \rho_Y^{+-}(i,k) = M.C. \sigma(j,k), i \neq j, (i,j) = (0,1), k = (0,2) \quad (VI.11)$$

of $\gamma N \rightarrow \gamma N$ become (VI. 5 to 8)

$$2 \operatorname{Re} \rho^{+-}(i,k) = M.C. \sigma(j,k) + \rho^{00}(i,k) \quad (VI.12)$$

for $\gamma N \rightarrow VN$. In contrast to the "unexcited vertex" (nucleon \rightarrow nucleon), there is a modification at the "inelastic vertex" (photon \rightarrow vector meson).

7. SUMMARY AND DISCUSSION

We have considered M-purity relations for spin-effects in elastic and in inelastic scattering; the emphasis was on relations which resemble those following from T-invariance in elastic scattering- in particular, the asymmetry-polarisation equality of sec. 2. The clue to why some M-purity relations resemble T-invariance ones is the fact that T-invariance forbids some $M = -1$ amplitudes in elastic scattering, Eq. (I.5). The M-purity analogues of the asymmetry-polarisation theorem hold, however, for spin configurations much more general than that for the theorem, as discussed in sec. 2. It is interesting that any relations resembling those following from T-invariance do exist between spin effects also for inelastic reactions.

Several reasons were given in subsec. (1.2) why M-purity is experimentally interesting, but perhaps the cleanest case where there is already experimental evidence (sec. 5) of M-purity (with $M = +1$, especially for amplitudes with nonzero V-helicity) is $\gamma N \rightarrow \gamma N$ for which M-purity relations were given in sec. 6 and compared with the corresponding ones for $\gamma N \rightarrow \gamma N$. The M-purity relations for $\gamma N \rightarrow \gamma N$ were compared with the corresponding T-invariance relations in sec. 4, the results being summarised in table 1.

Our examples of the T-invariance relations could be divided into three classes: a) relations between different final state photon polarisations, (IV.17, 18), b) relations between recoil nucleon polarisations in the reaction plane and final photon polarisations (IV.21 to 24) and c) relations between recoil nucleon polarisations normal to the reaction plane and final photon polarisations, (IV.7 to 10). It is to class c) that the two examples (IV.15, 16) of the standard asymmetry-polarisation theorem in $\gamma N \rightarrow \gamma N$ belong; in fact, the M-purity analogues and extensions (IV.29 to 32) and (VI.5 to 8) of the class c) all resemble the theorem. The M-purity analogues of the class b) are only null-identities; those of the class a) are the relations (IV.44 to 47) for $\gamma N \rightarrow \gamma N$ and (V.2) for $\gamma N \rightarrow \gamma N$ (and $\gamma N \rightarrow \gamma N$). But the M-purity analogues for the classes a) and b) do not reveal interesting information about the transition from an elastic to an inelastic reaction; they do not involve the inelastic component (i.e. amplitudes with zero V-helicity), and go over unchanged to $\gamma N \rightarrow \gamma N$. The M-purity analogues of the class c) do offer this information; we discuss this class now.

Consider first the recoil nucleon polarisations and their relation to the corresponding cross-section asymmetries. From T-invariance, one

gets the standard theorem (IV.13) for $\gamma N \rightarrow \gamma N$. The corresponding M-purity relations for $\gamma N \rightarrow \gamma N$ and $\gamma N \rightarrow V N$ are, respectively¹⁴, (IV.29) and (VI.5). The interesting point is that not only do the u-amplitudes forbidden by T-invariance in $\gamma N \rightarrow \gamma N$ appear in the proper way so as to give the relations (IV.29) and (VI.5), but also do the extra amplitudes (of both n- and u- types) with a zero V-helicity appear properly so as to give (VI.5). The relations (IV.30) and (VI.6) provide examples of M-purity relations having no T-invariance analogues; these relations between cross-section asymmetries and recoil polarisations have the "other" particle polarised. The comment about the extra n- and u- amplitudes with a zero V-helicity appearing properly so as to leave the form (IV.30) unchanged applies again. Similarly for the transition from (IV.31,32) to (VI.7,8).

While the form of the relation of cross-section asymmetries to recoil nucleon polarisations is not changed in going from $\gamma N \rightarrow \gamma N$ to $\gamma N \rightarrow V N$, there is a change in the form of the corresponding relation to recoil photon polarisations \tilde{P}_1' written as $2 \operatorname{Re} \rho_{\gamma'}^{+-}$. These changes due to the inelastic component are conveniently summarised in Eqs. (VI.11,12) where an extra positive semi-definite quantity ρ^{00} adds on to the appropriate cross-section coefficient.

This implies a "vertex-dependence" in the effects of inelasticity on the M-purity analogues and extensions of the asymmetry-polarisation theorem: When one considers the recoil polarisation and the cross-section asymmetry corresponding to the unexcited vertex (nucleon \rightarrow nucleon), there is no effect of inelasticity. For the excited vertex (photon \rightarrow vector meson), there is a modification due to inelasticity. While the example of $\gamma N \rightarrow \gamma N$, $V N$ is interesting, it provides only a rather simple form of

introducing inelasticity. On the basis of only this example, it is difficult to make statements about the changes in M-purity analogues of the asymmetry-polarisation theorem in going over to a more generally excited vertex (for example, a spin-change: $J^P = 1^-$ to $J^P = 2^+$).

The corresponding relations for the unexcited vertex (nucleon \rightarrow nucleon) are just an example of sec. 2 where these relations were illustrated for the case when the only restriction on the unexcited vertex was the equality of the initial and final spins. The $\gamma \rightarrow V$ vertex goes only a little farther than this. Because the case of a more general excited vertex has no direct T-invariance analogue, we do not consider the M-purity analogues of the theorem for such a vertex.

In table 2 (which is only another version of table 1) is given a summary of our illustrations of the T-invariance and the M-purity relations for $\gamma N \rightarrow \gamma N$; and in table 3, a summary of the modifications in these M-purity relations in going over to $\gamma N \rightarrow VN$.

The information to be obtained from cross-section asymmetry measurements in $\gamma N \rightarrow \gamma N$, VN has been considered in the Appendix. The question: "which asymmetries are necessary (and which determined therefrom by parity conservation)?" has been answered, see remarks (especially no: 1) there. A simple observation concerning the asymmetry-polarisation theorem is that though T-invariance relates the recoil polarisation for an unpolarised initial state to the cross-section asymmetry with the "other" particle unpolarised, parity invariance equates this asymmetry also to the corresponding asymmetry with some special nonzero polarisations of the "other" particle; the various asymmetries in Eqs. (A.15 and 16) of the Appendix provide the relevant examples.

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Appendix : Cross section Asymmetries in $\gamma N \rightarrow VN$ and $\gamma N \rightarrow \gamma N$

Some consequences of parity-invariance for cross-section asymmetries in vector meson photoproduction and Compton scattering are now considered.

Using an expansion like (III.6) for the complete set of the actual cross-sections $\sigma(P_i, \zeta_k)$, one gets

$$\sigma(P_0, \zeta_0) = \sigma(P_0, \zeta_1) = \sigma(P_0, \zeta_3) = \sigma(P_2, \zeta_0) = \sigma(P_3, \zeta_0) = \sigma(0, 0) \quad (A.1)$$

$$\sigma(P_0, \zeta_2) = \sigma(P_2, \zeta_2) = \sigma(P_3, \zeta_2) = \sigma(0, 0) + \zeta_2 \sigma(0, 2) \quad (A.2)$$

$$\sigma(P_1, \zeta_0) = \sigma(P_1, \zeta_1) = \sigma(P_1, \zeta_3) = \sigma(0, 0) + P_1 \sigma(1, 0) \quad (A.3)$$

$$\sigma(P_1, \zeta_2) = \sigma(0, 0) + P_1 \sigma(1, 0) + \zeta_2 \sigma(0, 2) + P_1 \zeta_2 \sigma(1, 2) \quad (A.4)$$

$$\sigma(P_2, \zeta_1) = \sigma(0, 0) + P_2 \zeta_1 \sigma(2, 1) \quad (A.5)$$

$$\sigma(P_2, \zeta_3) = \sigma(0, 0) + P_2 \zeta_3 \sigma(2, 3) \quad (A.6)$$

$$\sigma(P_3, \zeta_1) = \sigma(0, 0) + P_3 \zeta_1 \sigma(3, 1) \quad (A.7)$$

$$\sigma(P_3, \zeta_3) = \sigma(0, 0) + P_3 \zeta_3 \sigma(3, 3) \quad (A.8)$$

where use has been made of the vanishing [7] (because of parity-invariance) of eight coefficients out of the sixteen possible ones:

$$\sigma(0, 1) = \sigma(1, 1) = \sigma(2, 0) = \sigma(3, 0) = \sigma(2, 2) = \sigma(3, 2) = \sigma(0, 3) = \sigma(1, 3) = 0. \quad (A.9)$$

The cross-section asymmetries are

$$A(P_i, \pm \zeta_k) = \frac{\sigma(P_i, \zeta_k) - \sigma(P_i, -\zeta_k)}{\sigma(P_i, \zeta_k) + \sigma(P_i, -\zeta_k)}, \quad (A.10)$$

$$A(\pm P_i, \zeta_k) = \frac{\sigma(P_i, \zeta_k) - \sigma(-P_i, \zeta_k)}{\sigma(P_i, \zeta_k) + \sigma(-P_i, \zeta_k)}, \quad (A.11)$$

$$\text{and } A(\pm P_i, \pm \zeta_k) = \frac{\sigma(P_i, \zeta_k) - \sigma(-P_i, -\zeta_k)}{\sigma(P_i, \zeta_k) + \sigma(-P_i, -\zeta_k)} \quad (A.12)$$

where (i and k) can be (0, 1, 2 and 3), but the subscript of the polarisation appearing with both signs in the argument of an asymmetry cannot be zero; the asymmetries (A.10) and (A.11) can be called

"single" asymmetries in contrast to the "double" asymmetry (A.12). As shown below, the double asymmetry $A(\pm P_1, \pm \zeta_2)$ is interesting, but the other nonzero double asymmetries are very simply given by some single asymmetry because of parity-invariance.

The parity-conservation results (A.1 to 8) give, for the single asymmetries,

$$A(P_0, \pm \zeta_1) = A(P_0, \pm \zeta_3) = A(P_1, \pm \zeta_1) = A(P_1, \pm \zeta_3) = 0 , \quad (A.13)$$

$$A(\pm P_2, \zeta_0) = A(\pm P_2, \zeta_2) = A(\pm P_3, \zeta_0) = A(\pm P_3, \zeta_2) = 0 , \quad (A.14)$$

$$A(P_0, \pm \zeta_2) = A(P_2, \pm \zeta_2) = A(P_3, \pm \zeta_2) = \zeta_2 \sigma(0,2)/\sigma(0,0) , \quad (A.15)$$

$$A(\pm P_1, \zeta_0) = A(\pm P_1, \zeta_1) = A(\pm P_1, \zeta_3) = P_1 \sigma(1,0)/\sigma(0,0) , \quad (A.16)$$

$$A(\pm P_2, \zeta_1) = A(P_2, \pm \zeta_1) = P_2 \zeta_1 \sigma(2,1)/\sigma(0,0) , \quad (A.17)$$

$$A(\pm P_2, \zeta_3) = A(P_2, \pm \zeta_3) = P_2 \zeta_3 \sigma(2,3)/\sigma(0,0) , \quad (A.18)$$

$$A(\pm P_3, \zeta_1) = A(P_3, \pm \zeta_1) = P_3 \zeta_1 \sigma(3,1)/\sigma(0,0) , \quad (A.19)$$

$$A(\pm P_3, \zeta_3) = A(P_3, \pm \zeta_3) = P_3 \zeta_3 \sigma(3,3)/\sigma(0,0) , \quad (A.20)$$

$$A(P_1, \pm \zeta_2) = \zeta_2 [\sigma(0,2) + P_1 \sigma(1,2)] / [\sigma(0,0) + P_1 \sigma(1,0)] , \quad (A.21)$$

$$A(\pm P_1, \zeta_2) = P_1 [\sigma(1,0) + \zeta_2 \sigma(1,2)] / [\sigma(0,0) + \zeta_2 \sigma(0,2)] , \quad (A.22)$$

$$1 + A(P_1, \pm \zeta_2) = [1 + A(\pm P_1, \zeta_2)] / [1 + A(P_0, \pm \zeta_2)] / [1 + A(\pm P_1, \zeta_0)] , \quad (A.23)$$

and, for the double asymmetries,

$$A(\pm P_2, \pm \zeta_1) = A(\pm P_2, \pm \zeta_3) = A(\pm P_3, \pm \zeta_1) = A(\pm P_3, \pm \zeta_3) = 0 , \quad (A.24)$$

$$A(\pm P_1, \pm \zeta_1) = A(\pm P_1, \pm \zeta_3) = P_1 \sigma(1,0)/\sigma(0,0) , \quad (A.25)$$

$$A(\pm P_2, \pm \zeta_2) = A(\pm P_3, \pm \zeta_2) = \zeta_2 \sigma(0,2)/\sigma(0,0) , \quad (A.26)$$

$$A(\pm P_1, \pm \zeta_2) = [P_1 \sigma(1,0) + \zeta_2 \sigma(0,2)] / [\sigma(0,0) + P_1 \zeta_2 \sigma(1,2)] . \quad (A.27)$$

The double asymmetries (A.25, 26) are related to the single asymmetries (A.16, 15) which are interesting for T-invariance in $\gamma N \rightarrow \gamma N$. One gets

$$A(\pm P_1, \pm \zeta_1) = A(\pm P_1, \zeta_1) = A(\pm P_1, \zeta_0) \quad (A.28a)$$

$$= A(\pm P_1, \pm \zeta_3) = A(\pm P_1, \zeta_3) = A(\pm P_1, \zeta_0) , \quad (A.28b)$$

$$A(\pm P_2, \pm \zeta_2) = A(P_2, \pm \zeta_2) = A(P_0, \pm \zeta_2) \quad (A.29a)$$

$$= A(\pm P_3, \pm \zeta_2) = A(P_3, \pm \zeta_2) = A(P_0, \pm \zeta_2) . \quad (A.29b)$$

The remaining double asymmetry (A.27) can also be related to single asymmetries, but the relation is not equally simple. One gets

$$A(\pm P_1, \zeta_0) + A(P_0, \pm \zeta_2) = A(\pm P_1, \pm \zeta_2) [1 + P_1 \zeta_2 \sigma(1,2)/\sigma(0,0)] ; \quad (A.30)$$

since the factor $P_1 \zeta_2 \sigma(1,2)/\sigma(0,0)$ also occurs in the single asymmetries (A.21, 22), one gets

$$A(P_1, \pm \zeta_2) [1 + A(\pm P_1, \zeta_0)] - A(P_0, \pm \zeta_2) = -1 + [A(\pm P_1, \zeta_0) + A(P_0, \pm \zeta_2)] / A(\pm P_1, \pm \zeta_2), \quad (A.31),$$

used in (IV.39, 40). Because of (A.23), a corresponding relation holds between $A(\pm P_1, \zeta_2)$ and $A(\pm P_1, \pm \zeta_2)$.

SOME REMARKS

1) Out of the eight independent nonvanishing cross-section coefficients, the seven independent ratios

$$\sigma(1,0)/\sigma(0,0), \sigma(2,1)/\sigma(0,0), \sigma(2,3)/\sigma(0,0), \sigma(3,1)/\sigma(0,0), \sigma(3,3)/\sigma(0,0), \sigma(1,2)/\sigma(0,0) \text{ and } \sigma(0,2)/\sigma(0,0) \quad (A.32)$$

represent the information obtainable from asymmetry measurements. This information is contained in seven independent asymmetries - for example, the set

$$[A(P_0, \pm \zeta_2), A(P_{2,3}, \pm \zeta_1), A(P_{2,3}, \pm \zeta_3)] \text{ and } A(\pm P_1, \zeta_0), \quad (A.33)$$

or equivalently, the set

$$[A(\pm P_1, \zeta_{0,2}), A(\pm P_2, \zeta_{1,3}), A(\pm P_3, \zeta_{1,3})] \text{ and } A(P_0, \pm \zeta_2). \quad (A.34)$$

2A) Asymmetries requiring a reversal of target polarisation in the reaction plane may be regarded unnecessary because a) some of these vanish, as in (A.13), and b) the others are related to asymmetries requiring a reversal of the photon (but not nucleon) polarisations, as in (A.17 to 20).

2B) One can make a corresponding statement for asymmetries requiring a reversal of photon polarisations which are either circular, or at $45^\circ(135^\circ)$ to the reaction plane because a) some of these vanish, as in (A.14), and b) the others are related to asymmetries requiring a reversal of the nucleon (but not photon) polarisations in the reaction plane, as in (A.17 to 20).

Of course, either photon or nucleon polarisation reversal is necessary to measure the asymmetries (A.17 to 20). The same remark applies to (A.21, 22) though the polarisation directions now relevant are different.

3A) The asymmetry (A.15) requiring a reversal of nucleon polarisation normal to the reaction plane remains the same whether the "other" particle (photon) is polarised circularly or linearly at $45^\circ(135^\circ)$ to the reaction plane or not polarised at all. While, T-invariance relates - (IV.15) - only $A(P_0, \pm \zeta_2)$ to the recoil nucleon polarisation normal to the reaction plane with an unpolarised initial state, the relation (A.15) shows that that recoil polarisation is related also to the corresponding cross-section asymmetries for some special¹⁵ nonzero polarisations of the "other" particle; these special polarisations behave as "inactive spectators".

3B) A corresponding statement holds in the case of the asymmetries (A.16) requiring a reversal of photon polarisations normal (or parallel) to the reaction plane; the initial nucleon may be unpolarised or polarised in the reaction plane; T-invariance relates - (IV.16) - only $A(\pm P_1, \zeta_0)$ to the final state photon polarisation normal (or parallel) to the reaction plane with an unpolarised initial state.

To obtain $\sigma(0,2)/\sigma(0,0)$, it is not necessary to use photon polarisations which are circular or linear at $45^\circ(135^\circ)$ to the reaction plane; an unpolarised photon beam would do; this supplements the remark 2B) above. Similarly, initial nucleon polarisations in the reaction plane do not go beyond an unpolarised target in determining $\sigma(1,0)/\sigma(0,0)$; this supplements the remark 2A) above.

4) Under M-purity, the asymmetries (A.17 to 20) vanish [7], leaving (A.15, 16 and 21) as the only nonzero independent ones; these occur in Eqs. (IV.29 to 32) for $\gamma N \rightarrow \gamma N$ and (VI.5 to 8) for $\gamma N \rightarrow \gamma N$.

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FOOTNOTES

1. This notation should not be confused with the Toller quantum number 'M'. For example, Toller poles with the quantum number 'M' = 1 conspire, and do not have M-purity in our sense.
2. For elastic scattering and for $\gamma N \rightarrow VN$, the n and u amplitudes refer to natural and unnatural parity contributions respectively.
3. There are known instances where M-purity is not a good approximation; for example, high energy two-body processes involving significant pion-exchange contributions.
4. If the Pomeron factorises, it must have M-purity, even if it be not a Regge pole [4].
5. This uses an argument based on the vector-dominance model; the available [5] density-matrix data for $\gamma N \rightarrow VN$ indicate that in $\gamma N \rightarrow \gamma N$, even the unnatural parity contributions allowed by T-invariance are zero, or at most very small; sec. 5.
6. The discussion of cross-section asymmetries with general initial state polarisations, given in the Appendix, is relevant in this context. Many useful predictions about these asymmetries follow from only parity-conservation, and further ones from M-purity; see remarks 1 to 4 in the Appendix.
7. See, however, the remark 3) in the Appendix; for some special polarisations of a, the asymmetry may be the same as for unpolarised a.
8. This notation will be followed also for the vector meson density-matrix ρ .
9. One may note that while for some choice of m and n, $\text{tr } \rho(m,n)$ can be zero, $\text{tr } \rho(P_m, \zeta_n)$ is nonvanishing for all (m,n) because the unpolarised term $\text{tr } \rho(0,0)$ is always nonzero. The same remark applies to the other density-matrices: $\rho_\gamma, \rho_{N'}, \rho_{N',V}$. In (III.5), the argument of the trace is (P_m, ζ_n) .

10. The forms (IV.37 and 38) are suitable for generalisation to $\gamma N \rightarrow VN$. For $\gamma N \rightarrow \gamma N$, taking into account the T-invariance relations (IV.11 and 12), one gets

$$\zeta_2'(P_1, \zeta_2 = \pm 1) = \pm M \sigma(MP_1, \zeta_2 = \pm M) / \sigma(P_1, \zeta_2 = \pm 1)$$

$$\text{and } P_1'(P_1 = \pm 1, \zeta_2) = \pm M \sigma(P_1 = \pm M, M\zeta_2) / \sigma(P_1 = \pm 1, \zeta_2)$$

where $M = +1$ has been taken for purity of the type 1; in these relations, the right hand side does depend on P_1 and ζ_2 for $M = -1$ (but not for $M = +1$), M now referring to (IV.30 and 31).

11. For a summary, see ref. (9).
12. We note that our considerations involve only ratios of density-matrix elements; these ratios will remain unaffected even if the overall normalisation [5] provided by the vector dominance model is not correct.
- In the vector dominance model, the effective ρ^- contribution to the Compton amplitude is an order of magnitude larger than the corresponding ω^- and ϕ^- contributions so that the errors in $\gamma N \rightarrow \omega N$, ϕN data in connection with (V.3) are not very important.
13. These relations do not involve amplitudes with zero V-helicity, and go over from $\gamma N \rightarrow \gamma N$ to $\gamma N \rightarrow VN$ unchanged.
14. Of course, (IV.29) is interesting mainly for its comparison with $\gamma N \rightarrow VN$ because for $\gamma N \rightarrow \gamma N$, $M = +1$ is given by T-invariance, leading to (IV.13).
15. Here, for example, this is not true for photon polarisations normal to the reaction plane.

TABLE I

Summary of the T-Invariance and the M-Purity Relations for $\gamma N \rightarrow \gamma N$ Given in sec. 4

No:	Equation; Its Source	Quantities Related	Remarks
1	(IV.7 to 12); T-invariance constraint (IV.4) which overlaps with parity- conservation	Final nucleon polarisations normal to the reaction plane TO Final photon linear polarisa- tions normal or parallel to the reaction plane when the initial polarisations are also along these directions, or are zero.	Not necessary to invoke T- invariance
2	(IV.13, 15); T-invariance constraint (IV.3)	Cross-section asymmetry with initial nucleon polarised normal to the reaction plane and photon unpolarised TO recoil nucleon polarisation normal to the reac- tion plane, the initial nucleon and photon being unpolarised.	Standard Asymmetry-Polar- isation equality for elastic scattering
3	(IV.14, 16) T-invariance constraint (IV.3)	Same as in No: 2) with nucleon \leftrightarrow photon and with polarisation normal to reaction plane \rightarrow linear polarisation normal or parallel to reaction plane	
4	(IV.17 to 20); T-invariance constraint (IV.3)	Different final photon polarisa- tions either linear at 45° (135°) to the reaction plane or circular, when the initial photon is also polarised along one of these directions, but the initial nucleon is either unpolarised or polarised normal to the reaction plane	M-purity ana- logues are in (IV.44 and 45)
5	(IV.21 to 28); T-invariance constraint (IV.3)	Final nucleon polarisations in the reaction plane, initial nucleon being unpolarised and photon being polarised either linearly at 45° (135°) to the reaction plane or circularly TO Final photon polar- isation, either linear at 45° (135°) to the reaction plane or circular, the initial photon being unpolarised and the nucleon polarised in the reaction plane	M-purity analo- gues are null identities because both sides are an interference between ampli- tudes of the n- and u- types

contd.

TABLE I continued

6a	(IV.29, 32); M-purity	Cross section coefficients for initial nucleon polarisation zero or normal to the reaction plane and photon either unpolarised or polarised linearly normal or parallel to the reaction plane TO Recoil nucleon polarisations normal (or photon linear polarisations normal or parallel) to the reaction plane when the initial polarisations are the same as those for a particular cross section coefficient.	M-purity analogues of (IV.13, 14)
6b	(IV.30, 31); M-purity		No T-invariance analogue.
7	(IV.39, 40); The M-purity relation (IV.30)	Recoil nucleon polarisation normal to the reaction plane for initial nucleon unpolarised and photons polarised linearly normal or parallel to the reaction plane TO Cross section asymmetries with photon and nucleon polarisation either zero or along the above directions	Cross section asymmetry version of (IV.30); Can be expressed in terms of recoil photon (instead of nucleon) polarisation; No T-invariance analogue
8a	(IV.42); The M-purity relation (IV.31)	Final nucleon polarisation when initial nucleon is polarised TO that when initial nucleon is unpolarised and TO the initial nucleon polarisation, all polarisations being normal to the reaction plane, and photons unpolarised	Resembles the Depolarisation Relation in $\pi N \rightarrow \pi N$; No T-invariance analogue; Similar relations follow from (IV.29, 30, 32)
8b	(IV.43); The M-purity relation (IV.31)	Same as in no: 8a), with nucleon \leftrightarrow photon, and nucleon polarisations normal to the reaction plane \rightarrow linear photon polarisations normal or parallel to the reaction plane	
9	(IV.44, 45); M-purity	Same as in no: 4)	T-invariance version is in (IV.17, 18)
10	(IV.46, 47); M-purity		No T-invariance analogue

TABLE II

Summary of Illustrations of the Three Classes of T-Invariance and M-purity Relations for $\gamma N \rightarrow \gamma N$. The examples A-1 and A-2 are the standard asymmetry polarisation theorem

Class	Relations due to		Examples	Reference to Text (Equation Number)	Reference to Table 1 (Item Number)
	T-invariance	M-purity			
A	Exist (use $M = +1$)	Exist	1. $\tilde{P}'_1(0,0) = M C \sigma(1,0)$	(IV.14, 32)	3, 6a
			2. $\tilde{\zeta}'_2(0,0) = M C \sigma(0,2)$	(IV.13, 29)	2, 6a
			3. $\tilde{P}'_2(2,2) = M \tilde{P}'_3(3,2)$ $\tilde{P}'_2(3,0) = -M \tilde{P}'_3(2,0)$	(IV.17, 44) (IV.18, 45)	4, 9 4, 9
B	Exist	None (Only Null-Identities)	1. $\tilde{P}'_2(0,1) = \tilde{\zeta}'_1(2,0)$	(IV.21)	5
			$\tilde{P}'_2(0,3) = -\tilde{\zeta}'_3(2,0)$	(IV.22)	5
			$\tilde{P}'_3(0,3) = \tilde{\zeta}'_3(3,0)$	(IV.23)	5
			$\tilde{P}'_3(0,1) = -\tilde{\zeta}'_1(3,0)$	(IV.24)	5
C	None	Exist	1. $\tilde{P}'_1(0,2) = M C \sigma(1,2)$	(IV.30)	6b, 7
			$\tilde{P}'_1(1,0) = M C \sigma(0,0)$	(IV.31)	6b, 8b
			2. $\tilde{\zeta}'_2(1,0) = M C \sigma(1,2)$	(IV.30)	6b, 7
			$\tilde{\zeta}'_2(0,2) = M C \sigma(0,0)$	(IV.31)	6b, 8a
			3. $\tilde{P}'_2(2,0) = M \tilde{P}'_3(3,0)$ $\tilde{P}'_2(3,2) = -M \tilde{P}'_3(2,2)$	(IV.46) (IV.47)	10 10

TABLE III

Effect of Inelasticity $(\gamma N \rightarrow \gamma N) \rightarrow (\gamma N \rightarrow \gamma N)$ on M-purity relations of the types A and C of table 2. For the type B, the corresponding relations are only null-identities for both these processes.

Class and Example	Form for $\gamma N \rightarrow \gamma N$	Form for $\gamma N \rightarrow \gamma N$	Reference to Text (Equation number)
A	1. $\tilde{P}_1^+(0,0) \equiv 2 \operatorname{Re} \rho_{\gamma}^{+-}(0,0)$ $= M C \sigma(1,0)$	$2 \operatorname{Re} \rho^{+-}(0,0) = M C \sigma(1,0) + \rho^{00}(0,0)$	(IV.32), (VI.8,10)
	2. $\tilde{P}_2^+(0,0) = M C \sigma(0,2)$	No change	(IV.29), (VI.5,9)
	3. $\tilde{P}_2^+(2,2) = M \tilde{P}_3^+(3,2)$ or $- \operatorname{Im} \rho_{\gamma}^{+-}(2,2) = M \rho_{\gamma}^{++}(3,2)$	No change: $- \operatorname{Im} \rho^{+-}(2,2) = M \rho^{++}(3,2)$	(IV.44), (V.2)
C	$\tilde{P}_2^+(3,0) = -M \tilde{P}_3^+(2,0)$ or $\operatorname{Im} \rho_{\gamma}^{+-}(3,0) = M \rho_{\gamma}^{++}(2,0)$	No change: $\operatorname{Im} \rho^{+-}(3,0) = M \rho^{++}(2,0)$	(IV.45), (V.2)
	1. $\tilde{P}_1^+(0,2) \equiv 2 \operatorname{Re} \rho_{\gamma}^{+-}(0,2)$ $= M C \sigma(1,2)$	$2 \operatorname{Re} \rho^{+-}(0,2) = M C \sigma(1,2) + \rho^{00}(0,2)$	(IV.30), (VI.6)
	$\tilde{P}_1^+(1,0) \equiv 2 \operatorname{Re} \rho_{\gamma}^{+-}(1,0)$ $= M C \sigma(0,0)$	$2 \operatorname{Re} \rho^{+-}(1,0) = M C \sigma(0,0) + \rho^{00}(1,0)$	(IV.31), (VI.7)
2.	$\tilde{P}_2^+(1,0) = M C \sigma(1,2)$	No change	(IV.30), (VI.6)
	$\tilde{P}_2^+(0,2) = M C \sigma(0,0)$	No change	(IV.31), (VI.7)
	3. $\tilde{P}_2^+(2,0) = M \tilde{P}_3^+(3,0)$ or $- \operatorname{Im} \rho_{\gamma}^{+-}(2,0) = M \rho_{\gamma}^{++}(3,0)$	No change: $- \operatorname{Im} \rho^{+-}(2,0) = M \rho^{++}(3,0)$	(IV.46), (V.2)
3.	$\tilde{P}_2^+(3,2) = -M \tilde{P}_3^+(2,2)$ or $\operatorname{Im} \rho_{\gamma}^{+-}(3,2) = M \rho_{\gamma}^{++}(2,2)$	No change: $\operatorname{Im} \rho^{+-}(3,2) = M \rho^{++}(2,2)$	(IV.47), (V.2)