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by

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ELECTROMAGNETIC PROPERTIES OF HADRONS IN A RELATIVISTIC QUARK MODEL *

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Abstract : Results from a relativistic dynamical quark model are presented for:
the vector-meson-photon coupling constants for ordinary and heavy vector mesons;
the total e^+e^- annihilation into hadrons;
the pion form factor in the space- and time-like region.

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ELECTROMAGNETIC PROPERTIES OF HADRONS IN A RELATIVISTIC QUARK MODEL

From the quark model one gets predictions for the electromagnetic formfactors of the hadrons and deep inelastic scattering in the space- and time-like region. Sometimes one is really surprised about the quality of results derived from the nonrelativistic or the free quark model. Here, we want to present the predictions from a relativistic dynamical quark model, [1] which

has as framework [2]: general field theory,

uses as dynamical input: heavy spin $\frac{1}{2}$ quarks with a "smooth" interaction,

results in: the spectrum and Bethe-Salpeter wavefunctions of the hadrons as $q\bar{q}$, ... boundstates,

is leading to: spin-saturation of the superstrong $q\bar{q}$ - binding forces in the strong mesonic vertices.

The matrix elements of the electromagnetic current

$$j_\mu(x) = Z\left[\bar{\psi}(x), \gamma_\mu Q \psi(x)\right] \quad , \quad Q = \frac{1}{3}\begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix} \qquad (1)$$

are a good field to test the structure of this dynamical model, especially the interplay between boundstate structure and formfactor effects of the quarks. Since the vector meson boundstates play a prominent role as a concrete realization of generalized vector meson dominance [3,4] we first present their spectrum and quantum numbers:

$$M_N^2 = M_o^2 + 2c\cdot\sqrt{\beta}\cdot N \; ; \quad N = n + 2r \; ; \; n, \; r = 0, 1, 2, \ldots$$

$$c = 3, \quad \sqrt{\beta} = 0.2 \text{ GeV}^2 \qquad\qquad \ell \in (n, n-2, n-4, \ldots) \qquad (2)$$

$$M_o^2 = M_\rho^2 = 0.59 \text{ GeV}^2 \qquad\qquad = 0,2 \text{ for vector mesons}$$

The spectrum of the ρ-, ω- or ϕ-like vector mesons is linear in M^2 and shows a degeneracy d

$$d = N + 1 = 1, 3, 5, \ldots \qquad (3)$$

There are orbital excitations with $l = 0, 2$, excitations in the relative energy and in the relative space-time-distance. The model does not have a genuine $\rho'(1250)$ (no odd daughters!), however the degeneracy at higher masses may lead to interferences and mass shifts.

The matrix elements

$$\langle 0 | j_\mu^{e.m.}(0) \left| \begin{matrix} M & 1^{--} \\ P & s_3 \end{matrix} \right\rangle = (2\pi)^{-\frac{3}{2}} \frac{M_V^2}{2\gamma_V} \varepsilon_\mu^{s_3} \tag{4}$$

of the local current (1) can be expressed by the BS wavefunctions at zero distance

$$(2\pi)^{-\frac{3}{2}} \frac{M_V^2}{2\gamma_V} \varepsilon_\mu^{s_3} = (2\pi)^{-\frac{11}{2}} \operatorname{Tr} Z \gamma_\mu Q \int \chi(k,P)\, d^4k \tag{5}$$

Therefore only the hyperradially excited vector mesons (n=0,r=0,1,2,...) are coupled to the electromagnetic current with the strength [4)]

$$\frac{1}{\gamma_{V_r}} = (-1)^r \cdot Z \cdot \frac{4\sqrt{3}}{\pi} \cdot \frac{\sqrt{r+1}\cdot\sqrt{\beta}}{M_{V_r}^2} \cdot \langle Q_V \rangle \qquad \langle Q_V \rangle = \frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{-1}{3} \quad (\rho^0, \omega, \phi) \tag{6}$$

Since the BS amplitudes are normalized, we obtain [*)]

$$\frac{\gamma_\rho^2(765)}{4\pi} = 0.54 \qquad \text{Exp.}^{5)} = 0.56 \pm 0.06 \tag{7}$$

$$\frac{\gamma_{\rho'}^2(1650)/4\pi}{\gamma_\rho^2(765)/4\pi} \approx 10 \;; \quad \text{Exp.}^{6)} = \frac{4.25 + 1.25}{0.56 \pm 0.06} \text{ to } \frac{3.25 \pm 1.25}{0.56 \pm 0.06} \tag{8}$$

The ratio (8) would imply for the photoproduction cross sections in the additive quark model [4)]

$$\frac{\sigma(\gamma N \to \rho'(1650)N)}{\sigma(\gamma N \to \rho(765)N)} \approx 0.1 \tag{9}$$

which value is consistent with the experimental ratio [7)] at 9.3 GeV:

$$\frac{(1.6 \pm 0.4)\,\mu b}{(13.5 \pm 0.5)\,\mu b}$$

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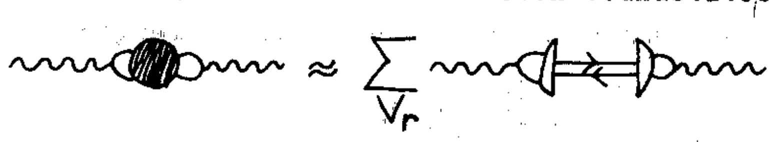
*) Here we have used $Z = 0.73$ as determined from the normalization of the π form factor $F_\pi(0) = 1$ [see the discussion following Eq. (19)].

The total annihilation cross section for $e^+e^- \to$ hadrons is related to the quark four-point function $\langle 0 | j_\mu^{e.m.}(x)\, j_\nu^{e.m.}(0) | 0 \rangle$ The parton model (= free, pointlike, light quarks) predicts

$$\frac{\sigma_{tot}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \equiv R = \sum Q_i^2 \tag{10}$$

i.e. R should be independent of the four momentum squared q^2. In our model of strongly bound heavy quarks this Green's function should be approximated by the one-particle intermediate states with the correct quantum numbers, i.e. the vector meson boundstates



(11)

Under the assumption that the individual vector mesons dominate locally ($\Gamma \ll M$), the cross section is given by [8]

$$\sigma_{tot}(e^+e^- \to \text{hadrons}) \underset{\lim \Gamma \to 0}{=} \frac{4\pi^2\alpha^2}{(q^2)^2} \sum_V \frac{M_V^4}{\gamma_V^2} \cdot \frac{M_V \Gamma_V}{(q^2 - M_V^2)^2 + M_V^2 \Gamma_V^2} \tag{12}$$

With the mass spectrum Eq. (2) and the couplings Eq. (6) we obtain[1,4] for Gell-Mann-Zweig quarks

$$\frac{\sigma_{tot}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = z^2 \cdot \frac{2}{3} \cdot \left(1 + \frac{1.6\ \text{GeV}^2}{q^2}\right) \underset{\text{large } q^2}{\longrightarrow} z^2 \cdot \frac{2}{3} \tag{13}$$

At the same time when we found this result, we were very proud about it, because we had shown that scaling does occur not only with point-like quarks, but also in a dynamical quark model with resonance excitation, suggesting duality between "current quarks" and "constituent quarks" for e^+e^--annihilation.

Obviously, now we have the same difficulty with the data from CEA and SPEAR like other people, and we also see a simple way out, which is still in accordance with our basic assumptions leading to Eq. (12). For this we introduce an anomalous magnetic moment term in the current [9]

$$j_\mu(x) = Z \left[\bar{\psi}(x) \left(\gamma_\mu + \frac{\kappa}{2 m_q} \sigma_{\mu\nu} \overleftrightarrow{\partial}^\nu \right) Q\, \psi(x) \right] \tag{1'}$$

which leads to the following modification of the VDM coupling constants:

$$\left(\frac{1}{\gamma_{V_r}}\right)^{an.m.} = \left(\frac{1}{\gamma_{V_r}}\right)\cdot\left(1 + K\cdot\frac{M_V^2}{4m_q^2}\right)$$
$$R^{an.m.} = R\cdot\left(1 + K\cdot\frac{q^2}{4m_q^2}\right)^2 \qquad (14)$$

Since the deviation of R from constancy seems to be rather large, we have to see whether the effect obtainable from Eq. (14) is consistent with a large quark mass, our primary dynamical assumption. By relating the quark magnetic moment to the nucleon moment via

$$K\cdot\frac{1}{2m_q} = \mu_N - \frac{1}{2m_q} \approx \frac{2.79}{2M_N} \qquad (15)$$

and by making an "eyeball" fit to the data, as shown in Figure 1, we get $m_q \sim 6$ GeV, a reasonable value.

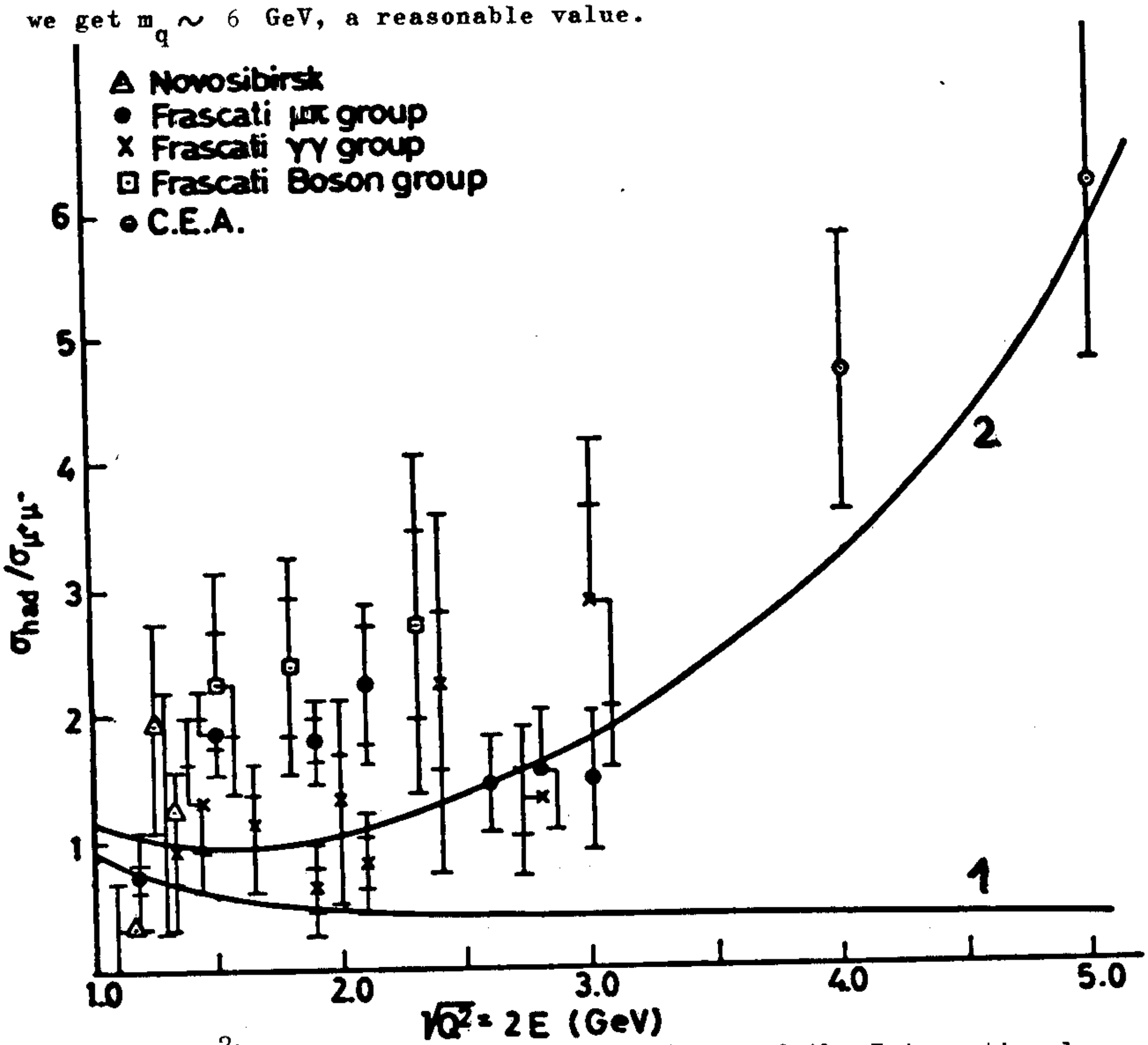
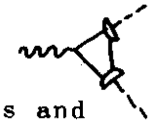


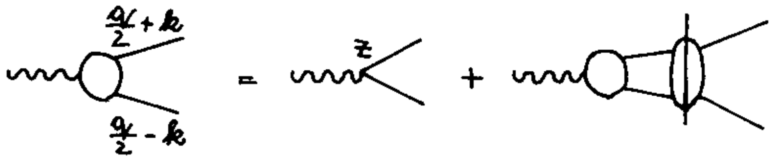
Fig. 1. R(q^2) according to K. Strauch; Proc. of the International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973.
Curve 1 and 2 refer to Eq. (13) and (14) respectively.

Of course we consider this model only an interesting and illuminating speculation. The estimate of m_q depends on the assumption Eq. (15) and on the form of our BS amplitudes in first order of $\frac{1}{m_q}$ (see Table VII in Ref. 1). On the other hand it is clear that the one-photon annihilation cross section alone cannot rise indefinitely for unitarity reasons. In our model Eq. (14) will be no longer valid for energies close to the $q\bar{q}$-threshold. In this picture we would expect that the ratio no longer increases beyond this energy.

As a last application let us discuss the pion form factor, which is related to the quark-six-point function: $\langle {\pi^+ \atop P} | j_\mu(0) | {\pi^+ \atop P'} \rangle$. In general, form factors are the classical test for wavefunctions. Thus it is interesting to see what a model dealing with Gaussian wavefunctions, resulting from the approximation of the interaction by harmonic forces, can say about form factors.

A pointlike coupling of the e.m. current to the quarks  corresponds to a simple convolution of the pion wavefunctions and leads to a Gaussian form factor $F_\pi(q^2) \sim \exp(q^2/\text{const.})$ This simple procedure for calculating form factors, however, does not take into account the appearance of vector mesons in the time-like region.

In our field theoretical model the quark form factor satisfies an inhomogeneous BS equation [10])



(16)

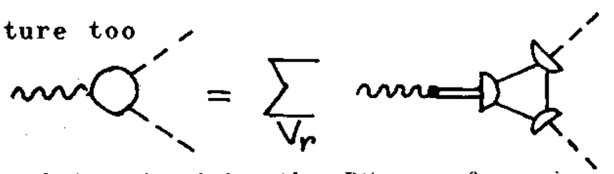
The solution leads to a generalized vector dominance structure for the quark form factor

radial
vector mesons

(17)

The residues factorize into the vector meson-photon couplings $g_{V_r\gamma} = M_r^2/2\gamma_{V_r}$ and the $q\bar{q}$-vector meson BS vertex functions $\Gamma_\mu(k, M_r)$. - From the mathematical point of view a function is determined by its poles and residues up to an additive entire function only. Following duality arguments, we make the assumption that the poles without a background term build up the quark form factor correctly. -

As a consequence the pion form factor has a generalized vector dominance structure too



(18)

The residues are determined by the BS wavefunctions of the pion and the heavy vector meson, i.e. the $\rho^o_r \to \pi^+\pi^-$ couplings [11]. Explicitly we obtain [10]

$$F_\pi(q^2) = g_{\rho^o\gamma} \cdot g_{\rho\pi\pi} \cdot \sum_{r=0}^{\infty} \frac{(-1/3)^r \cdot e^{\frac{r}{2}} \cdot L^1_r(r+0.21)}{M^2_{\rho^o} + 12r\cdot\sqrt{\beta} - q^2} \qquad (19)$$

For comparison with experiment [12], shown in Figure 2, we have used Breit Wigner forms with Γ_ρ = 150 MeV and $\Gamma_{\rho'}$ = 500 MeV. The only free parameter, the renormalization constant Z is determined from $F_\pi(o) = 1$: Z = 0.73

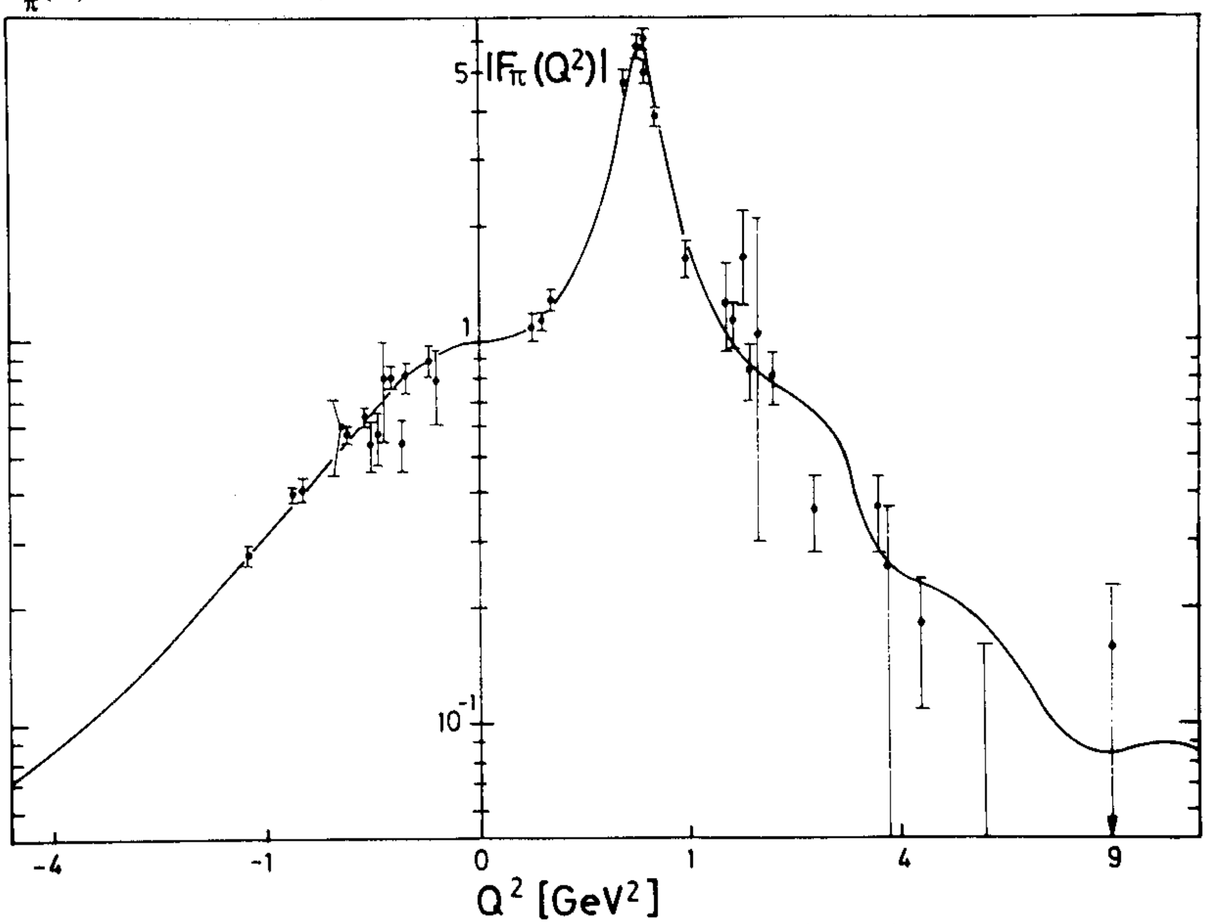


Fig. 2. Comparison of our result for the pion form factor with experiment.

The sum of the residues in Eq. (19) converges, we therefore obtain for large spacelike values of q^2 a simple pole behaviour:

$$F_\pi(q^2) \sim \frac{0.33\ \mathrm{GeV}^2}{-q^2} \tag{20}$$

This concludes the explicit results for the electromagnetic properties of hadrons we have obtained so far from this relativistic dynamical quark model. Obviously, many things are still to be done, like the study of inclusive processes $e^+e^- \rightarrow \pi + X$, etc. Since the subject of this meeting are the leptonic interactions at high energies, you may ask what you have learned about them from this talk. Let me answer this by saying: If there is any truth in a world built up from heavy quarks, why should these objects not be produceable finally? Electromagnetic and weak processes, for instance $e^+e^- \rightarrow q\bar{q}$ would be a good means to do so.If one would take seriously our explanation of the CEA and SPEAR data, then quarks could be seen above $q^2 = (2E)^2 \sim (2\times 6\ \ \mathrm{GeV})^2$. Maybe, next time at the Rencontre de Moriond, we should give an estimate for the production cross section.

References and Footnotes

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