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The Chiral Magnetic Effect and chiral symmetry breaking in SU(3) quenched lattice gauge theory *

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We study some properties of the non-Abelian vacuum induced by strong external magnetic field. We perform calculations in the quenched SU(3) lattice gauge theory with tadpole-improved Lüscher-Weisz action and chirally invariant lattice Dirac operator. The following results are obtained: The chiral symmetry breaking is enhanced by the magnetic field. The chiral condensate depends on the strength of the applied field as a power function with exponent $\nu = 1.6 \pm 0.2$. There is a paramagnetic polarization of the vacuum. The corresponding susceptibility and other magnetic properties are calculated and compared with the theoretical estimations. There are non-zero local fluctuations of the chirality and electromagnetic current, which grow with the magnetic field strength. These fluctuations can be a manifestation of the Chiral Magnetic Effect (CME).

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1. Introduction

The modern experiments provide a possibility to discover new physical effects caused by presence of the strong (hadronic scale) magnetic field. At the Relativistic Heavy Ion Collider (RHIC) at the first moments ($\tau \sim 1$ fm/c) of noncentral collision the very strong ($B \sim 10^{15}$T, $\sqrt{eB} \sim 300$ MeV) magnetic fields appear[1, 4]. Such strong magnetic fields can be also created in ALICE experiment at LHC, at the Facility for Antiproton and Ion Research (FAIR) at GSI and in the experiment NICA in Dubna. The additional motivation for the study of the effects induced by the strong magnetic field could also come from the physics of the early Universe, where the strong fields ($B \sim 10^{16}$T, $\sqrt{eB} \sim 1$ GeV) could have been produced after the electroweak phase transition[6]. Due to the nonperturbative nature of the effects we perform the calculations in the lattice gauge theory. We use quenched approximation and show that for some problems it provides rather reasonable values of the physical quantities.

This work has been done analogously to the previous SU(2) lattice studies[7, 8, 9, 10]. The list of considered effects induced by the magnetic field is the following.

The strong magnetic field can enhance the chiral symmetry breaking. There are various models (see Sec.3) which predict the growing of the chiral condensate.

The second effect is the chiral magnetization of the QCD vacuum. This effect has a paramagnetic nature. The vacuum magnetization is related to the nucleon magnetic moments[21] and other nonperturbative effects of hadrons[23]. We calculate the magnetic susceptibility and other quantities in Sec.4.

The quarks develop an electric dipole moment along the field due to the local fluctuations of the topological charge[9]. We study this effect in Sec.5.

Finally, the fluctuations of the topological charge can be a source of the asymmetry between numbers of quarks with different chiralities created in heavy-ion collisions. The so called "event-by-event P- and CP-violation"[1] can be explained by this asymmetry and observed at RHIC. So, our aim is also to see any evidences of this effect in SU(3) lattice simulations, nevertheless they are similar to SU(2) lattice results[10].

2. Technical details

We use the quenched $SU(3)$ lattice gauge theory with tadpole-improved Lüscher-Weisz action [11]. To generate the statistically independent gauge field configurations we use the Cabibbo-Marinari heat bath algorithm. The lattice size is 14^4, and lattice spacing $a = 0.105\,fm$. All observables we discuss later have a similar structure: $\langle \bar{\Psi}\mathcal{O}\Psi \rangle$ for VEV of a single quantity or $\langle \bar{\Psi}\mathcal{O}_1\Psi\ \bar{\Psi}\mathcal{O}_2\Psi \rangle$ for dispersions or correlators. Here $\mathcal{O}$, $\mathcal{O}_1$, $\mathcal{O}_2$ are some operators in spinor and color space. These expectation values can be expressed through the sum over M low-lying[1] but non-zero eigenvalues $i\lambda_k$ of the chirally invariant Dirac operator D (Neuberger's overlap Dirac

[1]We believe that the IR quantities are insensitive to the UV cutoff realized by selecting some finite number of the eigenmodes[13]

operator[12]):

$$\langle \bar{\Psi} \mathscr{O} \Psi \rangle = \sum_{|k|<M} \frac{\psi_k^\dagger \mathscr{O} \psi_k}{i\lambda_k + m} \tag{2.1}$$

and

$$\langle \bar{\Psi} \mathscr{O}_1 \Psi \; \bar{\Psi} \mathscr{O}_2 \Psi \rangle = \sum_{k,p} \frac{\langle k|\mathscr{O}_1|k\rangle\langle p|\mathscr{O}_2|p\rangle - \langle p|\mathscr{O}_1|k\rangle\langle k|\mathscr{O}_2|p\rangle}{(i\lambda_k + m)(i\lambda_p + m)}, \tag{2.2}$$

where all spinor and color indices are contracted and we omit them for simplicity. The λ_k are defined by the equation

$$D\psi_k = i\lambda_k \psi_k, \tag{2.3}$$

where ψ_k are the corresponding eigenfunctions and the uniform magnetic field $F_{12} = B_3 \equiv B$ is introduced as described in[7]. To perform calculations in the chiral limit one calculates the expression (2.1) or (2.2) for some non-zero m and averages it over all configurations of the gauge fields. Then one repeats the procedure for other quark masses m and extrapolates the VEV to $m \to 0$ limit.

3. Chiral condensate

In this section we present our results for the chiral condensate

$$\Sigma \equiv -\langle 0|\bar{\Psi}\Psi|0\rangle, \tag{3.1}$$

as a function of the magnetic field B. The general tendency for Σ to grow with B was already obtained in various models: in the chiral perturbation theory [14, 15] ($\Sigma \propto B$ for weak fields, $\Sigma \propto B^{3/2}$ for strong fields), in the Nambu-Jona-Lasinio model [16] ($\Sigma \propto B^2$), in a confining deformation of the holographic Karch-Katz model [17] ($\Sigma \propto B^2$), in D3/D7 holographic system [18] ($\Sigma \propto B^{3/2}$ for low temperatures, $\Sigma \propto B$ for high temperatures) and in SU(2) lattice calculations [7]($\Sigma \propto B$). Here our aim is to see how the chiral condensate behaves in the $SU(3)$ quenched gluodynamics.

We use the Banks-Casher formula [19], which relates the condensate (3.1) with the density $\rho(\lambda)$ of near-zero eigenvalues of the Dirac operator:

$$\Sigma = \lim_{\lambda \to 0} \frac{\pi \rho(\lambda)}{V}, \tag{3.2}$$

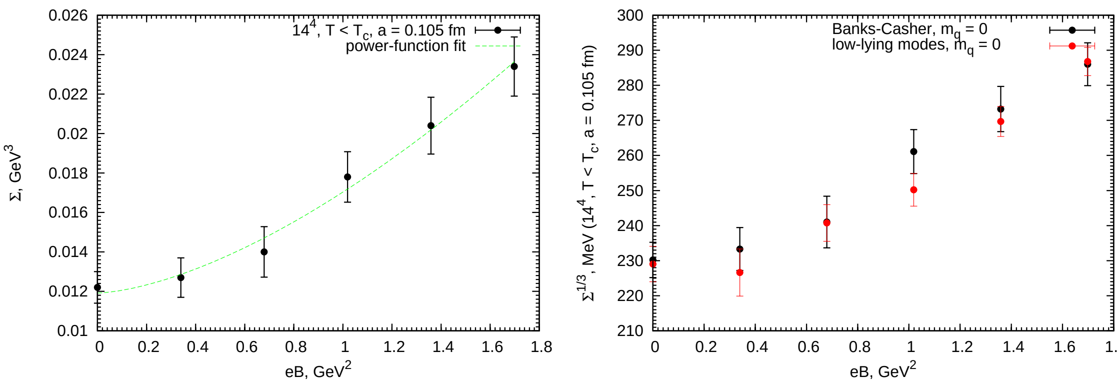
where V is the four-volume of the Euclidean space-time. The result is shown in Fig.1(a).

We perform the fit of the results by the following function:

$$\Sigma^{fit}(B) = \Sigma_0 \left[1 + \left(\frac{eB}{\Lambda_B^2} \right)^{\nu} \right], \tag{3.3}$$

where $\Sigma_0 \equiv \Sigma(0)$. The obtained fitting parameters are

$$\Sigma_0 = [(228 \pm 3) MeV]^3, \qquad \Lambda_B = (1.31 \pm 0.04)\, GeV, \qquad \nu = 1.57 \pm 0.23. \tag{3.4}$$



(a) (b)

Figure 1: Chiral condensate

It is interesting to compare quantitatively the condensate obtained by the Banks-Casher formula and that one calculated by the expression (2.1) with $\mathscr{O} = \mathbb{1}$. The result is shown in Fig.1(b). The value of the condensate in absence of the magnetic field equals $\Sigma(0) = [(230 \pm 5)\, MeV]^3$ which is not so far away from the value, which can be estimated by the Gell-Mann-Oakes-Renner formula[20]:

$$\Sigma(0) = \frac{F_\pi^2 m_\pi^2}{2(m_u + m_d)} \simeq [(240 \pm 10)\, MeV]^3 . \tag{3.5}$$

4. Chiral magnetization and susceptibility

In this section we calculate the quantity

$$\langle \bar{\Psi} \sigma_{\alpha\beta} \Psi \rangle = \chi(F) \langle \bar{\Psi}\Psi \rangle q F_{\alpha\beta} , \tag{4.1}$$

where $\sigma_{\alpha\beta} \equiv \frac{1}{2i}\left[\gamma_\alpha, \gamma_\beta\right]$ and $\chi(F)$ is some coefficient of proportionality (susceptibility), which depends on the field strength.

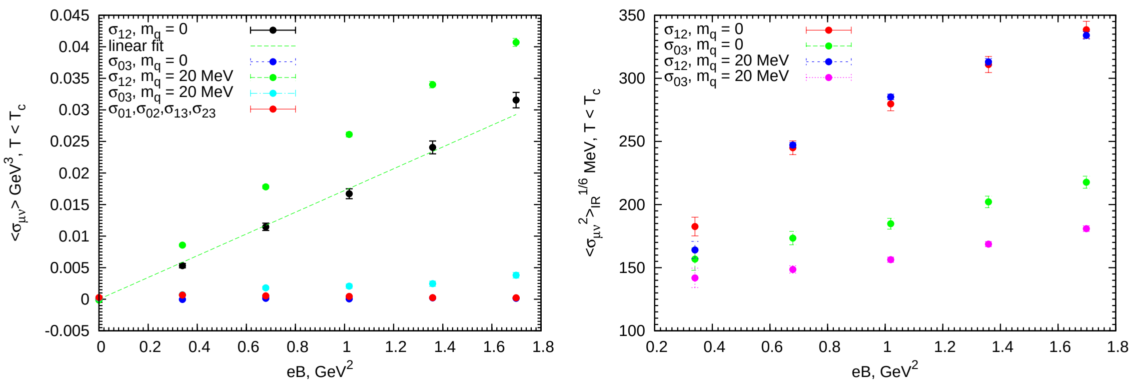
This quantity was introduced in[21] and can be used to estimate the spin polarization of the quarks in external magnetic field. The magnetization can be described by the dimensionless quantity $\mu = \chi \cdot qB$, so that

$$\langle \bar{\Psi} \sigma_{12} \Psi \rangle = \mu \langle \bar{\Psi}\Psi \rangle . \tag{4.2}$$

The expectation value (4.1) can be calculated on the lattice by (2.1) with $\mathscr{O} = \sigma_{\alpha\beta}$. The result is shown in Fig.2(a) (here for comparison we also plot series for some finite quark mass). We can see, that the 12-component grows linearly with the field, which agrees with[21]. This allows us to find the chiral susceptibility $\chi(0) \equiv \chi_0$. After making a linear approximation $\langle \bar{\Psi}\sigma_{12}\Psi \rangle = \Omega^{fit} eB$, where[2]

$$\Omega^{fit} \equiv -\frac{1}{3} \chi_0^{fit} \Sigma_0 , \tag{4.3}$$

[2] in our simulation we calculate the magnetization of the d-quark condensate, thus $q = \left|-\frac{e}{3}\right|$



(a) (b)

Figure 2: Expectation values of $\bar{\Psi}\sigma_{\alpha\beta}\Psi$ and their square

we obtain $\Omega^{fit} = (172.3 \pm 0.5) MeV$ and χ_0^{fit} (see below).

The corresponding theoretical value can be expressed as[22]:

$$\chi_0^{th} = -\frac{c_\chi N_c}{8\pi^2 F_\pi^2}, \tag{4.4}$$

where c_χ is a dimensionless parameter, according to the pion dominance[22] it can be chosen as $c_\chi = 2$. $F_\pi = 130.7 MeV$ is the pion decay constant for $N_c = 3$. Comparing this value with our result we find a good agreement:

$$\chi_0^{th} \simeq -4.46 GeV^{-2}, \qquad \chi_0^{fit} \simeq -4.24 GeV^{-2}, \tag{4.5}$$

The other interesting phenomenological quantity is the product of the chiral susceptibility χ and the condensate $\langle \bar{\Psi}\Psi \rangle$[23]. In our calculations it is equal to

$$-\chi_0^{fit} \langle \bar{\Psi}\Psi \rangle \simeq 52 MeV, \tag{4.6}$$

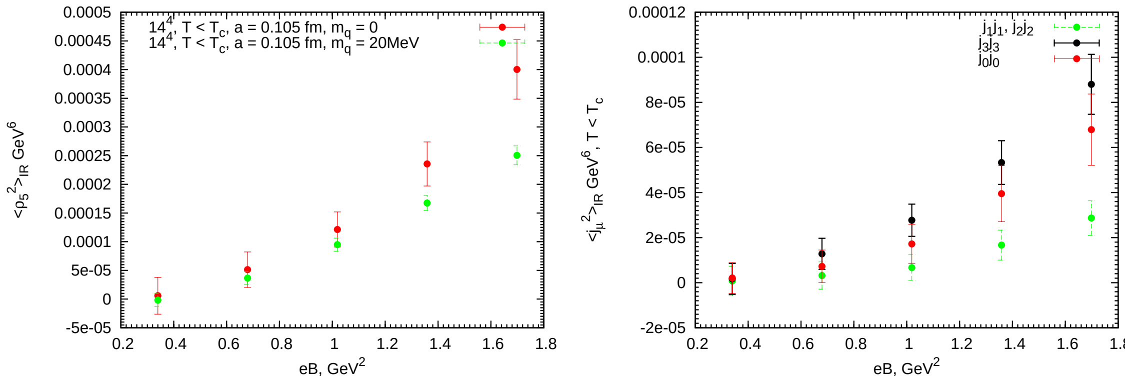
while from the QCD sum rules one can estimate this quantity as approximately 50 MeV [24], which is also close to our value.

5. Electric dipole moment

The other interesting effect due to the magnetic field is a quark local electric dipole moment along the field[9]. This quantity corresponds to the $i0$-components of the (4.1):

$$d_i(x) \equiv \bar{\Psi}(x)\sigma_{i0}\Psi(x), \qquad i = \overline{1,3} \tag{5.1}$$

In the real CP-invariant vacuum the VEV of this quantity should be zero: $\langle d_i(x) \rangle = 0$, that we actually see in our results (Fig.2(a)). At the same time the fluctuations of $d_i(x)$ can be sufficiently strong. We measure VEV's (2.2) with $\mathcal{O}_1 = \mathcal{O}_2 = \sigma_{\alpha\beta}$. In the case of $i0$-components it corresponds to dispersions of $\vec{d}$. The result is shown in Fig.2(b), we see that the longitudinal fluctuations of the local dipole moment grow with the field strength, while transverse fluctuations are absent. Here



(a) (b)

Figure 3: Fluctuations of the chirality and electromagnetic current/charge

and after we use the “IR” subscript to emphasize, that we subtract from the quantity its value at $B = 0$:

$$\langle Y\rangle_{IR}(B) = \frac{1}{V}\int_V d^4x\langle Y(x)\rangle_B - \frac{1}{V}\int_V d^4x\langle Y(x)\rangle_{B=0} \tag{5.2}$$

6. Some evidences of the Chiral Magnetic Effect

The nontrivial topological structure of QCD is due to some nontrivial effects in the presence of the strong magnetic field. One example of a such effect is the Chiral Magnetic Effect (CME), which generates an electric current along the field in the presence of the nontrivial gluonic background[1, 2]. This effect was probably been observed by the STAR collaboration at RHIC[3, 5] in heavy-ion collisions. A lattice evidence of the effect can be found in[10, 25, 26]. Here we implement the same procedure for the $SU(3)$ case and study the local chirality

$$\rho_5(x) = \bar{\Psi}(x)\gamma_5\Psi(x) \equiv \rho_L(x) - \rho_R(x) \tag{6.1}$$

and the electromagnetic current

$$j_\mu(x) = \bar{\Psi}(x)\gamma_\mu\Psi(x). \tag{6.2}$$

The expectation value of the first quantity can be computed by (2.1) with $\mathscr{O} = \gamma_5$ and with $\mathscr{O} = \gamma_\mu$ for the second quantity. The both VEV’s are zero, as expected, but the corresponding fluctuations obtained from (2.2) are finite and grow with the field strength (see Fig.3).

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