

Logarithmic $O(\alpha_s^3)$ contributions to the DIS Heavy Flavor Wilson Coefficients at $Q^2 \gg m^2$

Isabella Bierenbaum*

Instituto de Física Corpuscular, CSIC-Universitat de València, Apartado de Correos 22085, E-46071 Valencia, Spain

E-mail: Isabella.Bierenbaum@ific.uv.es

Johannes Blümlein†‡

Deutsches Elektronen-Synchrotron, DESY, Platanenallee 6, D-15638 Zeuthen, Germany

E-mail: Johannes.Blumlein@desy.de

Sebastian Klein§

Institut für theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany

E-mail: sklein@physik.rwth-aachen.de

The logarithmic contributions to the massive twist-2 operator matrix elements for deep-inelastic scattering are calculated to $O(\alpha_s^3)$ for general values of the Mellin variable N .

*XVIII International Workshop on Deep-Inelastic Scattering and Related Subjects, DIS 2010
April 19-23, 2010
Firenze, Italy*

*Supported in part by Generalitat Valenciana Grant No. PROMETEO/2008/69.

†Speaker.

‡Supported in part by SFB-TR/9 and EU TMR network HEPTOOLS.

§Supported in part by SFB-TR/9.

1. Introduction

The heavy flavor contributions to deep-inelastic scattering (DIS) are rather large in the small x region. With the current DIS data a precision of $\sim 1\%$ is reached [1] for $F_2(x, Q^2)$. This requires to describe the heavy flavor corrections to 3-loop order, to perform a consistent next-to-next-to leading order (NNLO) analysis, to measure the strong coupling constant $\alpha_s(M_Z^2)$ and the unpolarized twist-2 parton distribution functions at highest precision possible, cf. [2]. In the region $Q^2/m^2 \geq 10$ one may compute all contributions except the power suppressed terms, $\propto (m^2/Q^2)^k, k \geq 1$ using the factorization theorem given in Ref. [3]. Here, the massive Wilson coefficients factorize into massive operator matrix elements (OMEs), A_{ij} , and the massless Wilson coefficients, which are known to $O(\alpha_s^3)$ [4]. The $O(\alpha_s^3)$ Mellin moments of the massive operator matrix elements up to $N = 10 \dots 14$, depending on the process, were computed in [5]. The calculation was performed relating the moments of the massive operator matrix elements to massive tadpoles and using MATAD [6]. In Ref. [5] also the complete renormalization for a single massive quark has been derived. Different other contributions, needed in the renormalization process, were computed at general values of N in Refs. [7]. For the structure function $F_L(x, Q^2)$ the asymptotic corrections to $O(\alpha_s^3)$ are known for general values of N [8]. They are, however, only applicable at scales $Q^2/m^2 \geq 800$. For transversity the matrix elements were computed for general N at 2-loop order and a series of moments at 3-loop order in [9]. Very recently, the general N results at $O(\alpha_s^3)$ for $F_2(x, Q^2)$ for the color coefficients $\propto n_f$ have been completed [10, 11]. These computations use modern summation technologies encoded in the package Sigma [12] and the results can be expressed in terms of nested harmonic sums [13]. From the single pole terms in the massive computations of Refs. [5, 9, 10, 11] the corresponding contributions to the 3-loop anomalous dimensions were derived, either for the respective moments [14], or at general values of N [16].

In this note we report on the logarithmic $O(\alpha_s^3)$ contributions to the massive operator matrix elements, cf. also [17]. They are known for general values of N and depend on the 3-loop anomalous dimensions and massive OMEs up to $O(\alpha_s^2)$.

2. The heavy flavor Wilson coefficients in the asymptotic region

The heavy flavor correction to the structure function $F_2(x, Q^2)$ with n_f massless and one heavy flavor reads, [5] :

$$\begin{aligned}
 F_{(2,L)}^{Q\bar{Q}}(x, n_f, Q^2, m^2) &= \sum_{k=1}^{n_f} e_k^2 \left\{ L_{q,(2,L)}^{\text{NS}} \left(x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes [f_k(x, \mu^2, n_f) + f_{\bar{k}}(x, \mu^2, n_f)] \right. \\
 &+ \frac{1}{n_f} \left[L_{q,(2,L)}^{\text{PS}} \left(x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes \Sigma(x, \mu^2, n_f) + L_{g,(2,L)}^{\text{S}} \left(x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes G(x, \mu^2, n_f) \right] \left. \right\} \\
 &+ e_Q^2 \left[H_{q,(2,L)}^{\text{PS}} \left(x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes \Sigma(x, \mu^2, n_f) + H_{g,(2,L)}^{\text{S}} \left(x, n_f + 1, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes G(x, \mu^2, n_f) \right],
 \end{aligned} \tag{2.1}$$

with boundaries for the Mellin integral $[x(1 + 4m^2/Q^2), 1]$, and e_i the quark charges. Here the different Wilson coefficients are denoted by L_i, H_i in case the photon couples to a light (L) or the

heavy (H) quark. For $Q^2 \gg m^2$ they can be expressed in terms of the massive OMEs A_{ij} and the massless Wilson coefficients C_j . To $O(\alpha_s^3)$ they read ($a_s = \alpha_s/(4\pi)$)

$$\begin{aligned}
L_{q,(2,L)}^{\text{NS}}(n_f+1) &= a_s^2 \left[A_{qq,Q}^{(2),\text{NS}}(n_f+1) \delta_2 + \hat{C}_{q,(2,L)}^{(2),\text{NS}}(n_f) \right] \\
&\quad + a_s^3 \left[A_{qq,Q}^{(3),\text{NS}}(n_f+1) \delta_2 + A_{qq,Q}^{(2),\text{NS}}(n_f+1) C_{q,(2,L)}^{(1),\text{NS}}(n_f+1) + \hat{C}_{q,(2,L)}^{(3),\text{NS}}(n_f) \right] \\
L_{q,(2,L)}^{\text{PS}}(n_f+1) &= a_s^3 \left[A_{qq,Q}^{(3),\text{PS}}(n_f+1) \delta_2 + A_{gq,Q}^{(2)}(n_f) n_f \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) + n_f \hat{C}_{q,(2,L)}^{(3),\text{PS}}(n_f) \right] \\
L_{g,(2,L)}^{\text{S}}(n_f+1) &= a_s^2 A_{gg,Q}^{(1)}(n_f+1) n_f \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) + a_s^3 \left[A_{qg,Q}^{(3)}(n_f+1) \delta_2 \right. \\
&\quad \left. + A_{gg,Q}^{(1)}(n_f+1) n_f \tilde{C}_{g,(2,L)}^{(2)}(n_f+1) + A_{gg,Q}^{(2)}(n_f+1) n_f \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) \right. \\
&\quad \left. + A_{Qg}^{(1)}(n_f+1) n_f \tilde{C}_{q,(2,L)}^{(2),\text{PS}}(n_f+1) + n_f \hat{C}_{g,(2,L)}^{(3)}(n_f) \right], \\
H_{q,(2,L)}^{\text{PS}}(n_f+1) &= a_s^2 \left[A_{Qq}^{(2),\text{PS}}(n_f+1) \delta_2 + \tilde{C}_{q,(2,L)}^{(2),\text{PS}}(n_f+1) \right] + a_s^3 \left[A_{Qq}^{(3),\text{PS}}(n_f+1) \delta_2 \right. \\
&\quad \left. + \tilde{C}_{q,(2,L)}^{(3),\text{PS}}(n_f+1) + A_{gq,Q}^{(2)}(n_f+1) \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) \right. \\
&\quad \left. + A_{Qq}^{(2),\text{PS}}(n_f+1) C_{q,(2,L)}^{(1),\text{NS}}(n_f+1) \right], \\
H_{g,(2,L)}^{\text{S}}(n_f+1) &= a_s \left[A_{Qg}^{(1)}(n_f+1) \delta_2 + \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) \right] + a_s^2 \left[A_{Qg}^{(2)}(n_f+1) \delta_2 \right. \\
&\quad \left. + A_{Qg}^{(1)}(n_f+1) C_{q,(2,L)}^{(1),\text{NS}}(n_f+1) + A_{gg,Q}^{(1)}(n_f+1) \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) \right. \\
&\quad \left. + \tilde{C}_{g,(2,L)}^{(2)}(n_f+1) \right] + a_s^3 \left[A_{Qg}^{(3)}(n_f+1) \delta_2 + A_{Qg}^{(2)}(n_f+1) C_{q,(2,L)}^{(1),\text{NS}}(n_f+1) \right. \\
&\quad \left. + A_{gg,Q}^{(2)}(n_f+1) \tilde{C}_{g,(2,L)}^{(1)}(n_f+1) + A_{Qg}^{(1)}(n_f+1) \left\{ C_{q,(2,L)}^{(2),\text{NS}}(n_f+1) \right. \right. \\
&\quad \left. \left. + \tilde{C}_{q,(2,L)}^{(2),\text{PS}}(n_f+1) \right\} + A_{gg,Q}^{(1)}(n_f+1) \tilde{C}_{g,(2,L)}^{(2)}(n_f+1) + \tilde{C}_{g,(2,L)}^{(3)}(n_f+1) \right], \tag{2.2}
\end{aligned}$$

with $\delta_2 = 0(1)$ for $F_L(F_2)$ and $\hat{f}(n_f) = f(n_f+1) - f(n_f)$, $\tilde{f}(n_f) = f(n_f)/n_f$. The massive OMEs depend on the ratio m^2/μ^2 , while the scale ratio of the massless Wilson coefficients is μ^2/Q^2 . The latter are pure functions of the momentum fraction z , or the Mellin variable N , if one sets $\mu^2 = Q^2$. The massive OMEs obey then the general structure

$$A_{ij}^{(3)} \left(\frac{m^2}{Q^2} \right) = a_{ij}^{(3),3} \ln^3 \left(\frac{m^2}{Q^2} \right) + a_{ij}^{(3),2} \ln^2 \left(\frac{m^2}{Q^2} \right) + a_{ij}^{(3),1} \ln \left(\frac{m^2}{Q^2} \right) + a_{ij}^{(3),0}. \tag{2.3}$$

3. The matrix element $A_{Qg}^{(3)}(N)$

In the following we present, as an example, the logarithmic expansion coefficients of Eq. (2.3), $a^{(3),k}$, $k \geq 1$, for the massive OME $A_{Qg}^{(3)}(N)$ in the $\overline{\text{MS}}$ -scheme. They are given by :

$$\begin{aligned}
a_{Qg}^{(3),3} &= \frac{8(N^2+N+2)T_F}{9N(N+1)(N+2)} \left[T_F n_f \left(C_F \left(\frac{P_1}{(N-1)N^2(N+1)^2(N+2)} - 4S_1 \right) \right. \right. \\
&\quad \left. \left. + C_A \left(4S_1 - \frac{8(N^2+N+1)}{(N-1)N(N+1)(N+2)} \right) \right) - 8T_F^2 + C_A^2 \left(-\frac{(11N^4+22N^3-59N^2-70N-48)S_1}{(N-1)N(N+1)(N+2)} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& -12S_1^2 + \frac{2(N^2 + N + 1)(11N^4 + 22N^3 - 35N^2 - 46N - 24)}{(N-1)^2N^2(N+1)^2(N+2)^2} \Big) + C_A T_F \left(-\frac{56(N^2 + N + 1)}{(N-1)N(N+1)(N+2)} \right. \\
& + 28S_1 \Big) + C_F^2 \left(-3\frac{(3N^2 + 3N + 2)^2}{4N^2(N+1)^2} + \frac{6S_1(3N^2 + 3N + 2)}{N(N+1)} - 12S_1^2 \right) + C_F T_F \left(-16S_1 \right. \\
& + \frac{2P_2}{(N-1)N^2(N+1)^2(N+2)} \Big) + C_A C_F \left(24S_1^2 - \frac{(N^2 + N + 6)(7N^2 + 7N + 4)S_1}{(N-1)N(N+1)(N+2)} \right. \\
& \left. \left. - \frac{(3N^2 + 3N + 2)(11N^4 + 22N^3 - 59N^2 - 70N - 48)}{4(N-1)N^2(N+1)^2(N+2)} \right) \right]. \tag{3.1}
\end{aligned}$$

$$\begin{aligned}
a_{Qg}^{(3),2} &= 4T_F^2 n_f \left(C_F \left(-\frac{4(N^2 + N + 2)}{3N(N+1)(N+2)} (S_2 + S_1^2) + \frac{8(5N^3 + 8N^2 + 19N + 6)S_1}{9N^2(N+1)(N+2)} \right. \right. \\
& \left. \left. - \frac{P_3}{9(N-1)N^4(N+1)^4(N+2)^3} \right) + C_A \left(-\frac{8(5N^4 + 20N^3 + 47N^2 + 58N + 20)S_1}{9N(N+1)^2(N+2)^2} \right. \right. \\
& \left. \left. + \frac{4(N^2 + N + 2)}{3N(N+1)(N+2)} (2S_{-2} + S_2 + S_1^2) - \frac{2P_4}{9(N-1)N^2(N+1)^3(N+2)^3} \right) \right) + 2C_A^2 T_F \left(\right. \\
& \frac{8(N^2 + N + 2)}{N(N+1)(N+2)} (2S_{-2,1} - S_1^3 - 3S_2S_1 - S_{-3} - 4S_{-2}S_1 - S_3) - \frac{2P_5S_1^2}{3(N-1)N^2(N+1)^2(N+2)^2} \\
& + \frac{4P_6S_1}{9(N-1)^2N^3(N+1)^3(N+2)^3} - \frac{4(N^2 + N + 2)(11N^4 + 22N^3 - 59N^2 - 70N - 48)S_{-2}}{3(N-1)N^2(N+1)^2(N+2)^2} \\
& \left. + \frac{8P_7}{9(N-1)^2N^4(N+1)^4(N+2)^4} - \frac{2(N^2 + N + 2)(11N^4 + 22N^3 - 83N^2 - 94N - 72)S_2}{3(N-1)N^2(N+1)^2(N+2)^2} \right) \\
& + 4C_A T_F^2 \left(\frac{4(N^2 + N + 2)}{N(N+1)(N+2)} (S_1^2 + S_2 + 2S_{-2}) + \frac{8(5N^4 + 20N^3 - N^2 - 14N + 20)S_1}{9N(N+1)^2(N+2)^2} \right. \\
& \left. - \frac{2P_8}{9(N-1)N^3(N+1)^3(N+2)^3} \right) + 2C_F^2 T_F \left(\frac{8(N^2 + N + 2)}{N(N+1)(N+2)} (3S_2S_1 - S_1^3 + 4S_{-2}S_1 + 2S_3 - 4S_{-2,1}) \right. \\
& - \frac{16(N^2 + N + 2)S_{-2}}{N^2(N+1)^2(N+2)} - \frac{6(N^2 + N + 2)(3N^2 + 3N + 2)S_2}{N^2(N+1)^2(N+2)} + \frac{2(3N^4 + 14N^3 + 43N^2 + 48N + 20)S_1^2}{N^2(N+1)^2(N+2)} \\
& \left. - \frac{4P_9S_1}{N^3(N+1)^3(N+2)} + \frac{P_{10}}{2N^4(N+1)^4(N+2)} + \frac{16(N^2 + N + 2)S_{-3}}{N(N+1)(N+2)} \right) + 4C_F T_F^2 \left(\right. \\
& \frac{4(N^2 + N + 2)}{3N(N+1)(N+2)} (S_2 - 3S_1^2) + \frac{8(5N^3 + 14N^2 + 37N + 18)S_1}{9N^2(N+1)(N+2)} - \frac{P_{11}}{9(N-1)N^4(N+1)^4(N+2)^3} \Big) \\
& + 2C_F C_A T_F \left(\frac{4(N^2 + N + 2)}{N(N+1)(N+2)} (4S_1^3 - 2S_3 + 4S_{-2,1} - 2S_{-3} - 3S_{-2}) \right. \\
& \left. + \frac{4P_{12}S_1^2}{3(N-1)N^2(N+1)^2(N+2)^2} - \frac{4P_{13}S_1}{9(N-1)N^3(N+1)^3(N+2)^3} \right. \\
& \left. + \frac{P_{14}}{18(N-1)N^3(N+1)^3(N+2)^3} + \frac{4(N^2 + N + 2)(N^4 + 2N^3 + 8N^2 + 7N + 18)S_2}{3(N-1)N^2(N+1)^2(N+2)^2} \right). \tag{3.2}
\end{aligned}$$

$$a_{Qg}^{(3),1} = \frac{1}{2} \hat{\gamma}_{qg}^{(2)}(n_f) - \frac{n_f}{2} \hat{\gamma}_{qg}^{(2)}(n_f) + 4T_F^2 n_f \left(C_F \left(\frac{4(N^2 + N + 2)}{9N(N+1)(N+2)} (4S_3 - S_1^3 - 3S_2S_1) + \frac{4(3N+2)S_1^2}{3N^2(N+2)} \right. \right.$$

$$\begin{aligned}
& + \frac{4(N^4 - N^3 - 20N^2 - 10N - 4)S_1}{3N^2(N+1)^2(N+2)} + \frac{2P_{15}}{3(N-1)N^5(N+1)^5(N+2)^4} + \frac{4P_{16}S_2}{3(N-1)N^3(N+1)^3(N+2)^2} \\
& + C_A \left(\frac{4(N^2 + N + 2)}{9N(N+1)(N+2)} \left(S_1^3 + 9S_2S_1 + 6S_{-3} + 12S_{-2}S_1 + 8S_3 - 12S_{-2,1} \right) - \frac{4P_{17}S_1}{3N(N+1)^3(N+2)^3} \right. \\
& - \frac{4(N^3 + 8N^2 + 11N + 2)S_1^2}{3N(N+1)^2(N+2)^2} + \frac{4P_{18}}{3(N-1)N^4(N+1)^4(N+2)^4} - \frac{4P_{19}S_2}{3(N-1)N^2(N+1)^2(N+2)^2} \\
& \left. + \frac{16(N^2 - N - 4)S_{-2}}{3(N+1)^2(N+2)^2} \right) + 2C_A^2 T_F \left(\frac{8(N^2 + N + 2)}{3N(N+1)(N+2)} \left(12S_{-2,1}S_1 - S_1^4 - 9S_2S_1^2 - 8S_3S_1 - 6S_{-3}S_1 \right. \right. \\
& - 12S_{-2}S_1^2 \left. \right) - \frac{2P_{20}S_1^3}{9(N-1)N^2(N+1)^2(N+2)^2} + \frac{2P_{21}S_1^2}{3(N-1)N^2(N+1)^3(N+2)^3} \\
& - \frac{2P_{22}S_1}{3(N-1)N^4(N+1)^4(N+2)^4} - \frac{2P_{23}S_2S_1}{(N-1)N^2(N+1)^2(N+2)^2} - \frac{2P_{24}}{3(N-1)^2N^5(N+1)^5(N+2)^5} \\
& + \frac{4(N^2 + N + 2)(11N^4 + 22N^3 - 35N^2 - 46N - 24)}{9(N-1)N^2(N+1)^2(N+2)^2} \left(6S_{-2,1} - 4S_3 - 3S_{-3} \right) \\
& - \frac{8(N^2 - N - 4)(11N^4 + 22N^3 - 35N^2 - 46N - 24)S_{-2}}{3(N-1)N(N+1)^3(N+2)^3} - \frac{8P_{25}S_{-2}S_1}{3(N-1)N^2(N+1)(N+2)^2} \\
& + \frac{2(11N^4 + 22N^3 - 35N^2 - 46N - 24)P_{19}S_2}{3(N-1)^2N^3(N+1)^3(N+2)^3} \left. \right) + 4C_A T_F^2 \left(\frac{8(N^2 + N + 2)}{9N(N+1)(N+2)} \left(S_1^3 + 8S_3 - 12S_{-2,1} \right. \right. \\
& + 9S_2S_1 + 12S_{-2}S_1 + 6S_{-3} \left. \right) - \frac{8(N^3 + 8N^2 + 11N + 2)S_1^2}{3N(N+1)^2(N+2)^2} - \frac{8P_{19}S_2}{3(N-1)N^2(N+1)^2(N+2)^2} \\
& + \frac{2P_{26}}{27(N-1)N^4(N+1)^4(N+2)^4} + \frac{32(N^2 - N - 4)S_{-2}}{3(N+1)^2(N+2)^2} + \frac{4P_{27}S_1}{27N(N+1)^3(N+2)^3} \left. \right) + 2C_F^2 T_F \left(\right. \\
& \frac{8(N^2 + N + 2)}{3N(N+1)(N+2)} \left(4S_3S_1 - S_1^4 - 3S_2S_1^2 \right) + \frac{2(3N^4 + 42N^3 + 107N^2 + 92N + 28)S_1^3}{3N^2(N+1)^2(N+2)} \\
& + \frac{2P_{28}S_1^2}{N^3(N+1)^2(N+2)} + \frac{2P_{29}S_1}{N^4(N+1)^4(N+2)} + \frac{2(7N^4 + 74N^3 + 79N^2 - 12N - 4)S_2S_1}{N^2(N+1)^2(N+2)} \\
& - \frac{8(N^2 + N + 2)(3N^2 + 3N + 2)S_3}{3N^2(N+1)^2(N+2)} - \frac{2(3N^2 + 3N + 2)(N^4 + 17N^3 + 17N^2 - 5N - 2)S_2}{N^3(N+1)^3(N+2)} \\
& \left. - \frac{P_{30}}{N^5(N+1)^5(N+2)} \right) + 4C_F T_F^2 \left(\frac{8(N^2 + N + 2)}{9N(N+1)(N+2)} \left(4S_3 - S_1^3 - 3S_2S_1 \right) \right. \\
& + \frac{P_{31}}{3(N-1)N^5(N+1)^5(N+2)^2} + \frac{8(N^4 - N^3 - 20N^2 - 10N - 4)S_1}{3N^2(N+1)^2(N+2)} + \frac{8(N^4 + 17N^3 + 17N^2 - 5N - 2)S_2}{3N^2(N+1)^2(N+2)} \\
& + \frac{8(3N+2)S_1^2}{3N^2(N+2)} \left. \right) + 2C_F C_A T_F \left(\frac{8(N^2 + N + 2)}{3N(N+1)(N+2)} \left(2S_1^4 + 12S_2S_1^2 + 4S_3S_1 - 12S_{-2,1}S_1 + 6S_{-3}S_1 \right. \right. \\
& + 12S_{-2}S_1^2 \left. \right) + \frac{4P_{32}S_1^3}{9(N-1)N^2(N+1)^2(N+2)^2} - \frac{4P_{33}S_1^2}{3(N-1)N^3(N+1)^3(N+2)^3} \\
& - \frac{4P_{34}S_1}{3(N-1)N^4(N+1)^4(N+2)^4} - \frac{4P_{35}S_2S_1}{3(N-1)N^2(N+1)^2(N+2)^2} - \frac{P_{36}}{3(N-1)N^4(N+1)^5(N+2)^4} \\
& - \frac{8P_{37}S_{-2}S_1}{N^2(N+1)^2(N+2)^2} + \frac{4(N^2 + N + 2)(3N^2 + 3N + 2)}{N^2(N+1)^2(N+2)} \left(2S_{-2,1} - S_{-3} \right) \\
& - \frac{8(N^2 - N - 4)(3N^2 + 3N + 2)S_{-2}}{N(N+1)^3(N+2)^2} - \frac{2P_{38}S_2}{3(N-1)N^3(N+1)^3(N+2)^2}
\end{aligned}$$

$$-\frac{8(N^2 + N + 2)(29N^4 + 58N^3 - 41N^2 - 70N - 48)S_3}{9(N-1)N^2(N+1)^2(N+2)^2} \Bigg). \quad (3.3)$$

Here, $S_{\bar{a}} \equiv S_{\bar{a}}(N)$, P_k denote some polynomials in N , cf. [17], and $\gamma_{ij}^{(2)}$ are the 3-loop anomalous dimensions. The expansion coefficients given above depend on harmonic sums up weight $w = 3$. Numerical studies show, that within the kinematic region of HERA the constant terms to (2.3) are as important as the logarithmic contributions. Further details will be given in [17].

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