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Proton Hexality in Local Grand Unification

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Abstract

Proton hexality is a discrete symmetry that avoids the problem of too fast proton decay in the supersymmetric extension of the standard model. Unfortunately it is inconsistent with conventional grand unification. We show that proton hexality can be incorporated in the scheme of "Local Grand Unification" discussed in the framework of model building in (heterotic) string theory.

1 Introduction

The question of proton stability is of great importance in elementary particle physics. Conserved baryon number (B) could be a reason for a stable proton but would be incompatible with the creation of a baryon asymmetry in the universe. In the standard $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ model of particle physics $\mathrm{U}(1)_B$ is a good symmetry at the renormalisable level (broken by non-perturbative effects and possibly dimension-6 operators) and proton decay is sufficiently suppressed. The phenomenon of B-violation is thus mainly concerned with physics beyond the standard model (SM). Indeed, a natural framework to address the question is grand unification (GUTs). Quarks and leptons appear in unified multiplets and $\mathrm{U}(1)_B$ is broken. The known stability of the proton requires the GUT scale to be rather large $M_{GUT} > 10^{16}$ GeV such that dimension-6 operators are sufficiently suppressed.

Over the years it appears that the framework of GUTs seems to require supersymmetry. This is then consistent with unification of gauge coupling constants at $M_{GUT} \sim 10^{16}$ but it leads to new complications with proton stability due to potentially dangerous dimension-4 and dimension-5 operators. New symmetries like R-parity [1] (or matter parity [2,3]) have been conjectured to forbid the dim-4 operators. Such a symmetry would lead to a new stable particle as the source for dark matter. Still, the dim-5 operators are problematic. Other discrete symmetries like baryon triality (B_3) [4, 5] might solve the problem. The most attractive symmetry is proton hexality (P_6) and has been identified by Dreiner, Luhn and Thormeier [6]. It forbids all the problematic dim-4 and -5 operators, but allows lepton number violation in form of Majorana neutrino masses: thus P_6 perfectly fits our needs.

Proton hexality is a beautiful symmetry and grand unification is a very attractive scheme: but unfortunately there is a clash between the two. P_6 is incompatible with a unified structure of quark and lepton multiplets [7]. This is in contrast to matter parity $\mathbb{Z}_2^{\text{matter}}$ which can be incorporated e.g. in an SO(10) GUT (it is a discrete subgroup of $\mathrm{U}(1)_{B-L}$ in SO(10)). This is not true for B_3 and P_6. An ultraviolet (UV) completion of theories with P_6 needs something more general than grand unification. String theory could be a candidate as it also includes a consistent description of gravitational interactions.[1]

Recent work towards string theoretic constructions of the minimal supersymmetric standard model (MSSM) [8–12] has revealed the concept of "Local Grand Unification" [8, 13,14], a variant of GUTs that addresses some of its problematic properties. It allows "split multiplets" that e.g. solve the doublet-triplet splitting in the Higgs-sector and simplifies the breakdown of the grand unified gauge groups. These incomplete or split multiplets make it possible that P_6 can become compatible with local grand unification.

Moreover, it has recently been argued that discrete symmetries appear abundantly in string model constructions [15–17], with important applications for particle physics model building [18–22]. We are thus in a situation that P_6 could originate from a string model as consistent UV completion and that such a symmetry is compatible with local grand

[1] Global symmetries might be broken by gravitational interactions. Therefore it is important to discuss these questions in theories where gravity is consistently incorporated.

unification. Within this top-down approach we can be confident that the global symmetries are respected by gravitational interactions.[2]

The present paper is devoted to the study of proton hexality within the framework of local grand unification. In section 2 we shall present P_6 followed in section 3 by a discussion of the incompatibility of P_6 with GUTs. In section 4 we shall try to incorporate P_6 via a bottom-up approach in extra-dimensional GUT-like theories and stress the geometric aspects of local grand unification. Section 5 presents the results of heterotic orbifold constructions towards the incorporation of P_6. We provide some toy models where P_6 appears in various different ways, anomalous or non-anomalous. A completely satisfactory model has not been found yet. Section 6 will discuss possible lines of future research in the direction of explicit model building.

2 Proton Hexality

In this section we will motivate proton hexality as a discrete symmetry in the supersymmetric extension of the SM in somewhat more detail. For derivations, however, we will refer to original literature.

Ensuring sufficient stability of the proton in theories beyond the SM often provides non-trivial restrictions. New fields can give rise to baryon or lepton number violating couplings. In supersymmetric extensions of the SM, for instance, the most general superpotential respecting renormalisability and gauge invariance is

$$\begin{aligned} W = \; & h^E_{ij}\, L_i H_d \overline{E}_j + h^D_{ij}\, Q_i H_d \overline{D}_j + h^U_{ij}\, Q_i H_u \overline{U}_j + \mu\, H_d H_u \\ & + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \kappa_i\, L_i H_u \\ & + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k, \end{aligned} \tag{1}$$

where i, j, k are family indices and gauge indices are suppressed. Terms in the first line encode Yukawa couplings needed for lepton and quark mass generation and the μ-term contribution to the Higgs potential. Terms in the second and third line of (1) violate lepton and baryon number, respectively. The lepton or baryon number violating couplings (λ, λ', κ and λ'') can be forbidden by imposing an additional discrete symmetry such as R-parity or matter parity. Compared to R-parity, matter parity is a $\mathbb{Z}_2$ symmetry under which all constituents of a chiral multiplet carry the same charge, *viz.* matter multiplets are odd whereas Higgs multiplets are even.

Assuming that the supersymmetric extension of the SM originates from a more fundamental theory, such as a GUT, we also have to discuss effective non-renormalisable couplings. The superpotential can contain dangerous terms respecting all the symmetries including matter parity,

$$\kappa^{(1)}_{ijkl}\, Q_i Q_j Q_k L_l + \kappa^{(2)}_{ijkl}\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_l. \tag{2}$$

[2] Earlier attempts in a bottom-up approach [4–7] tried to solve this problem through the notion of "anomaly free discrete symmetries". In the top-down approach we do not have to worry about these constraints as long as we are dealing with a consistent string model.

These terms lead to dimension five interactions violating baryon as well as lepton number. Such dimension five couplings are suppressed by just one power of the GUT scale and lead to too fast proton decay [23]. The authors of [4,5] proposed another discrete $\mathbb{Z}_3$ symmetry forbidding also the dimension five baryon number violating couplings. This symmetry was later dubbed baryon triality, B_3 [24,25]. The charges are defined modulo hypercharge (Y) transformation and listed in Table 1.

Matter parity as well as baryon triality can be obtained from a spontaneously broken additional U(1) gauge symmetry [4,5]. Anomaly cancellation restricts the spectrum of a gauge theory leading to a finite number of possible discrete symmetries, which can be obtained in such a way. The authors of [6] looked through all such symmetries and identified proton hexality, P_6, as the only other phenomenologically interesting discrete symmetry. On SM fields P_6, as defined by the charge assignments in Table 1, acts as a $\mathbb{Z}_6$ symmetry which is the product of baryon triality and matter parity. Proton hexality had been discussed before in [26–29]. It forbids all dangerous dimension four and five baryon or lepton number violating couplings while phenomenologically desirable couplings are allowed. That is, only terms in the first line of (1) are allowed at the renormalisable level. At dimension five level, only interactions coming from a superpotential LH_uLH_u (family indices are suppressed) are allowed. These respect baryon number and, moreover, provide Majorana mass terms for left-handed neutrinos after electroweak symmetry breaking. Dimension six interactions are suppressed by two powers of the GUT scale which results in a sufficiently stable proton for supersymmetric theories (with $M_{GUT} \sim 10^{16}$ GeV).

	Q	$\bar{U}$	$\bar{D}$	L	$\bar{E}$	H_u	H_d	$\bar{\nu}$
$6\,Y$	1	-4	2	-3	6	3	-3	0
$\mathbb{Z}_2^{\text{matter}}$	1	1	1	1	1	0	0	1
B_3	0	-1	1	-1	2	1	-1	0
P_6	0	1	-1	-2	1	-1	1	3

Table 1: Hypercharge and discrete charges of the MSSM particles under hypercharge Y, matter parity $\mathbb{Z}_2^{\text{matter}}$, baryon triality B_3 and proton hexality P_6. One can show that $P_6 = \mathbb{Z}_2^{\text{matter}} \times B_3$ up to a hypercharge shift. $\mathbb{Z}_2^{\text{matter}}$, B_3 and P_6 are defined just modulo 2, 3 and 6, respectively. A right-handed neutrino $\bar{\nu}$ has been included.

So far, our discussion did not include right-handed neutrinos. They can be included in a straightforward way (as shown in Table 1). Interactions including right-handed neutrinos do not introduce baryon number violating terms, since their B_3 charge is zero. All terms needed for the see-saw mechanism are allowed by P_6 [6].

3 Proton Hexality and Unified Gauge Groups

Since proton hexality forbids also dangerous dimension five couplings, it is desirable to embed P_6 into an underlying more fundamental theory. As a first example we look at grand unification. The embedding of P_6 into a GUT has been examined to some extent in [7]. There, the authors added an extra anomaly free U(1) to the unified gauge group and identified all possible discrete subgroups of that U(1). In this case, proton hexality does not work with any of the usual candidate unified gauge groups (Pati-Salam, SU(5), SO(10)).

3.1 Proton Hexality from Pati-Salam$\times$U(1)$_X$

An option which has been excluded in [7] is to break the unified gauge group times an extra $\mathrm{U}(1)_X$ simultaneously to the SM gauge group times P_6. For unified groups with rank larger than four P_6 can, in principle, be embedded diagonally into the extra $\mathrm{U}(1)_X$ times another U(1) originating from the GUT gauge group. Such a scheme works for Pati–Salam[3] with gauge group $\mathrm{SU}(4)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$. The colour SU(3) is embedded into the upper three times three block of the hermitian SU(4) generator and hypercharge transformations are generated by a combination of an SU(4) and $\mathrm{SU}(2)_R$ transformation

$$\mathrm{SU}(3)\colon \begin{pmatrix} & trace & & 0 \\ & less & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathrm{U}(1)_Y\colon \begin{pmatrix} \frac{1}{6} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_R . \tag{3}$$

The left-handed quarks and leptons merge into a $(\mathbf{4},\mathbf{2},\mathbf{1})$ while the right-handed quarks and leptons form a $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$ representation. The supersymmetric SM Higgs pair is combined into a $(\mathbf{1},\mathbf{2},\mathbf{2})$ representation. Now, we add an additional $\mathrm{U}(1)_X$ with generator X and denote the corresponding charges by subscripts. With the assignments

$$(\mathbf{4},\mathbf{2},\mathbf{1})_1, \quad (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{-1} \quad \text{and} \quad (\mathbf{1},\mathbf{2},\mathbf{2})_0 \tag{4}$$

we reproduce the correct P_6 charges if we take P_6 to be the $\mathbb{Z}_6$ subgroup of the U(1) generated by

$$P_6\colon \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_R - \frac{1}{2}X. \tag{5}$$

The spontaneous breaking to the SM gauge group times P_6 can be achieved by turning on a vacuum expectation value (VEV) in the upper $\mathrm{SU}(2)_R$ component of a $(\mathbf{4},\mathbf{1},\mathbf{2})_7$ and the lower $\mathrm{SU}(2)_R$ component of a $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{-7}$. With respect to SU(4) the VEV points always into the fourth direction such that the colour SU(3) remains unbroken. Since these components carry $\mathrm{U}(1)_{P_6}$ charges ± 6 (cf. eqn. (5)), their VEVs leave P_6 unbroken. However, in a supersymmetric theory, we encounter mixed $\mathrm{SU}(2)^2_{L/R}\mathrm{U}(1)_X$ anomalies. For three families, these can be cancelled e.g. with the additional multiplets

$$2\times(\mathbf{1},\mathbf{2},\mathbf{1})_{-6}, \quad 2\times(\mathbf{1},\mathbf{1},\mathbf{2})_6 \,. \tag{6}$$

[3]Other discrete symmetries suppressing proton decay within Pati–Salam and SO(10) have been identified in [30].

These can have $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times P_6$ invariant mass terms and hence decouple at the breaking scale.

3.2 Proton Hexality from SO(12)

As far as gauge coupling unification is concerned Pati-Salam is still characterised by three couplings and, in that sense, not quite a GUT yet. If we try to go one step further to SO(10), for instance, we are back at the problems discussed in [7]: the matter fields in eqn. (4) should merge into a **16** of SO(10). However, they cannot do that due to their opposite $\mathrm{U}(1)_X$ charges. The only way out, is to double the number of 16-dimensional representations. Then, obviously, only half of each representation gives rise to SM matter. However, for the remaining half representation not needed for matter a mechanism of *multiplet splitting* has to be invoked.

With that in mind we might as well consider gauge groups larger than SO(10) also accommodating the extra $\mathrm{U}(1)_X$. One canonical choice is SO(12). For this discussion it is more convenient to use Cartan–Weyl notation. There, a Lie algebra is given in the form

$$[H_i, H_j] = 0 \quad , \quad [H_i, E_p] = p_i E_p, \tag{7}$$

with $i = 1, \ldots \mathrm{rank}(G) = 6$ and the p's denote charge vectors, or roots, of the remaining generators. SO(12) has six Cartan generators $H_1, \ldots, H_6$ and the roots p are given by

$$\big(\underline{\pm 1, \pm 1, 0, 0, 0, 0}\big) \, . \tag{8}$$

Here, all 60 roots are generated by permuting underlined entries, resulting in the $60 + 6 = 66$ dimensional adjoint of SO(12). Apart from the adjoint, the following SO(12) representations, specified by their weights, will be of interest for us

$$\begin{array}{ll} \mathbf{32}\text{:} & \big(\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\big) \quad \text{(even number of } - \text{ signs)}, \\ \mathbf{32'}\text{:} & \big(\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\big) \quad \text{(odd number of } - \text{ signs)}, \\ \mathbf{12}\text{:} & \big(\underline{\pm 1, 0, 0, 0, 0, 0}\big) \, . \end{array} \tag{9}$$

The $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X$ gauge group can be embedded into the adjoint of SO(12) as follows

$$\begin{array}{llll} \mathrm{SU}(4): & \big(\underline{\pm 1, \pm 1, 0}, 0, 0, 0\big) \, , H_1, H_2, H_3, & \mathrm{SU}(2)_L: & \pm\,(0, 0, 0, 1, -1, 0)\, , H_4 - H_5, \\ \mathrm{SU}(2)_R: & \pm\,(0, 0, 0, 1, 1, 0)\, , H_4 + H_5, & \mathrm{U}(1)_X: & -2H_6. \end{array} \tag{10}$$

Quarks and leptons become part of **32** and **32′** representations,

$$\begin{array}{lll} (\mathbf{4}, \mathbf{2}, \mathbf{1})_1: & (\underbrace{\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}}_{\text{even } \# \, -}, \underbrace{\pm\tfrac{1}{2}, \pm\tfrac{1}{2}}_{\text{odd } \# \, -}, -\tfrac{1}{2}) & \subset \mathbf{32} \\ (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})_{-1}: & (\underbrace{\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}}_{\text{odd } \# \, -}, \underbrace{\pm\tfrac{1}{2}, \pm\tfrac{1}{2}}_{\text{even } \# \, -}, \tfrac{1}{2}) & \subset \mathbf{32'} \, , \end{array} \tag{11}$$

for each generation. Last, the electroweak Higgs sits inside a 12-dimensional representation

$$(\mathbf{1},\mathbf{2},\mathbf{2})_0: \quad (0,0,0,\underline{\pm 1,0},0) \subset \mathbf{12}\ . \tag{12}$$

Clearly, these multiplets have a fair amount of additional fields which need to decouple via *multiplet splitting*. The situation becomes much more dramatic for the remaining fields: $(\mathbf{4},\mathbf{1},\mathbf{2})_7$, $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{-7}$ and the fields in eqn. (6). These are embedded in SO(12) representations containing weights of the form $(\pm 7/2,\ldots,\pm 7/2)$ and $(0,\ldots,0,\pm 3)$, respectively. The big charges under one of the SO(12) Cartan generators can be accommodated only in rather high-dimensional representations.[4] Even with a working mechanism for multiplet splitting one would rather not add such representations to an underlying fundamental theory.[5] Therefore, we should also have the option of adding incomplete or *split multiplets*. In the next section we will demonstrate that both mechanisms can be naturally obtained within string theory.

4 Proton Hexality and Local Grand Unification

The picture of local grand unification has drawn considerable attention in the recent past in particular in the context of heterotic model building (for recent reviews, see [32, 33]). Before discussing orbifold compactifications of the heterotic string, let us demonstrate how the previous problems are solved in a bottom-up approach with two extra dimensions[6]. For simplicity, we construct the previously described Pati–Salam times $\mathrm{U}(1)_X$ model in four dimensions. When discussing actual string models, later on, we will be interested in four-dimensional gauge symmetry of the form: SM gauge group times P_6 (times hidden sector gauge group). Since the step from Pati–Salam times $\mathrm{U}(1)_X$ to the SM times P_6 is comparatively simple there is no conceptual difference.

Two extra dimensions are compactified on a torus obtained by modding the complex plane with a quadratic lattice spanned by the vectors e_1 and e_2. Further we mod out a $\mathbb{Z}_4$ symmetry generated by $\pi/2$ rotations in the plane. The fixed point structure is depicted in Figure 1. As a six-dimensional theory we take an SO(12) gauge theory. The $\pi/2$ rotation is embedded into SO(12) by the adjoint action of $e^{2\pi \mathrm{i} H_j V_j}$ and lattice shifts by e_1 or e_2 by $e^{2\pi \mathrm{i} H_j W_j}$. (Shifts by e_1 and e_2 have to be embedded identically since e_1 is mapped onto e_2 by the $\pi/2$ rotation.) We choose

$$V = \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},0,0,0\right) \quad , \quad W = \left(0,0,0,0,0,\frac{1}{2}\right). \tag{13}$$

[4]The corresponding highest weights are $\left(\frac{7}{2},\frac{7}{2},\frac{7}{2},\frac{7}{2},\frac{7}{2},\frac{7}{2}\right)$ and $(3,0,0,0,0,0)$ with Dynkin labels $[0,0,0,0,0,7]$ and $[3,0,0,0,0,0]$. The dimensions can be computed using the Weyl formula (see e.g. [31]) and are 2,617,472 and 352.

[5]Note that in the case of matter parity as a subgroup of SO(10) we would need a 126-dimensional representation to get the desired symmetry breakdown.

[6]In the spirit of [14] this may correspond to a compactification of the heterotic string on $T^6/\mathbb{Z}_M \times \mathbb{Z}_N$, where our T^2 appears as a fixed torus under the $\mathbb{Z}_N$ factor. For a list of possible $\mathbb{Z}_M \times \mathbb{Z}_N$ orbifolds see e.g. [34].

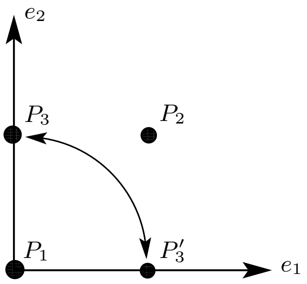

Figure 1: Fixed point structure of $T^2/\mathbb{Z}_4$ orbifold. The origin P_1 is fixed under any rotation. P_2 is fixed under a $\pi/2$ rotation followed by a shift by e_1. P_3 and P_3' are fixed under a π rotation and shifts by e_2 and e_1, respectively. P_3 and P_3' are related by a $\pi/2$ rotation and thus identical on the orbifold.

The unbroken gauge group in four dimensions is the subgroup of SO(12) which is invariant under the orbifold action and lattice shifts, i.e. $p{\cdot}V = 0 \bmod 1$ and $p{\cdot}W = 0 \bmod 1$, p being a root of SO(12). This yields Pati-Salam times $\mathrm{U}(1)_X$ embedded into SO(12) as discussed in the previous section eqn. (10). To avoid enormous representations, the $(\mathbf{4},\mathbf{1},\mathbf{2})_7$ and $(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{-7}$ and fields in (6) should be localised to points where $\mathrm{U}(1)_X$ factorises and a big charge does not need a large representation. The gauge group geography showing the local projections of SO(12) at the fixed points is depicted in Figure 2. At the fixed point(s)

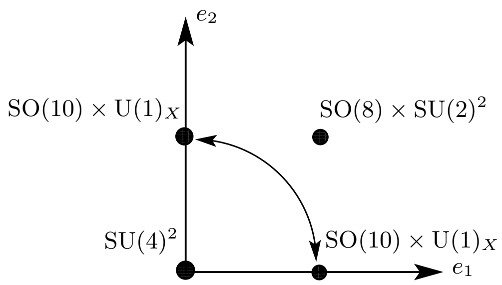

Figure 2: Gauge group geography of $T^2/\mathbb{Z}_4$ orbifold. The gauge group in the bulk is SO(12).

with $\mathrm{SO}(10) \times \mathrm{U}(1)_X$ gauge group we localise a $2 \times \mathbf{10}_6$, $2 \times \mathbf{10}_{-6}$, $\mathbf{16}_7$, $\overline{\mathbf{16}}_{-7}$. From an SO(12) perspective these are *split multiplets* of dramatically lower dimension than complete multiplets containing fields of the same $\mathrm{U}(1)_X$ charges. We demonstrate the mechanism of *multiplet splitting* by getting SM matter and Higgses from complete SO(12) multiplets in the bulk. For each family of quarks and leptons we add a **32** and a $\mathbf{32}'$-plet and for the electroweak Higgs a **12**-plet to the bulk. The action of orbifold and lattice shifts is again given by $e^{2\pi \mathrm{i} H_j V_j}$ and $e^{2\pi \mathrm{i} H_j W_j}$ in the corresponding representation. In addition it is accompanied by phase factors called orbifold parities. (For a discussion within the context of heterotic orbifolds, see e.g. [35].) In our bottom-up approach we pick the phases by hand. With an appropriate choice, it is easy to project the multiplets exactly to the desired $\mathrm{SU}(4){\times}\mathrm{SU}(2)_L{\times}\mathrm{SU}(2)_R{\times}\mathrm{U}(1)_X$ multiplets in the four-dimensional effective theory.

There are still six-dimensional bulk anomalies to worry about. The relevant formulæ can be found e.g. in [36]. To cancel the bulk anomalies, we have to add 16 **12**-plets to the bulk theory if all three families originate from bulk multiplets. We will not get further into the details of our illustrative example. After all, this theory is not UV complete and what we are really after is a fully-fledged string model.

5 Embedding Proton Hexality in Heterotic Orbifolds

5.1 Proton Hexality from local GUTs

In this section, we are interested in how to obtain string compactifications furnished with gauged proton hexality. We follow the strategy depicted in the previous section, i.e. we search for MSSM-like constructions in which P_6 arises as a subgroup of $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X \subset \mathrm{SO}(12)$ and the matter generations reside in (split) representations **32** and **32′** of the six-dimensional SO(12) GUT. With this purpose, we consider the $T^6/\mathbb{Z}_4 \times \mathbb{Z}_4$ orbifold with torus lattice $\mathrm{SO}(5)^3$. The $\mathbb{Z}_4 \times \mathbb{Z}_4$ action is generated by the twists

$$v_1 = \left(\tfrac{1}{4}, 0, -\tfrac{1}{4}\right) \quad \text{and} \quad v_2 = \left(0, \tfrac{1}{4}, -\tfrac{1}{4}\right) . \tag{14}$$

This orbifold allows for one Wilson line of order 2 per SO(5) torus, hence three independent Wilson lines. Note that, since the twist v_1 (v_2) leaves the second (first) SO(5) torus invariant, it defines a $T^4/\mathbb{Z}_4$ orbifold.

Along the lines of the heterotic Mini-Landscape [10, 11], we apply a search strategy based on local GUTs to find a fertile region of heterotic orbifolds endowed with proton hexality. The search strategy reads as follows. To start with, we consider the first $\mathbb{Z}_4$. From all its possible gauge embeddings V_1 [37, 38], we choose the shift vector (denoted by (IVg) in [38])

$$V_1 = \left(\tfrac{3}{4}, \tfrac{1}{4}, 0^6\right)\left(0^8\right) , \tag{15}$$

which results in a $T^4/\mathbb{Z}_4$ model with SO(12) gauge group and twisted matter states transforming as **32** or **32′**-plets. We look for all admissible gauge embeddings V_2 associated to the second $\mathbb{Z}_4$ action, resulting in 164 possibilities and select only those that break SO(12) to Pati-Salam,

$$\mathrm{SO}(12) \quad \xrightarrow{V_2} \quad \mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X . \tag{16}$$

This reduces the number of promising shifts V_2 dramatically from 164 to 30. In addition, we want SM matter representations arising from **32** and **32′**-plets, i.e. quarks and leptons shall stem from the twisted sectors of the underlying $T^4/\mathbb{Z}_4$ orbifold. Hence, we choose only those 12 shifts V_2 out of the 30 where the $(i, 0)$-twisted sector ($i = 1, 2, 3$) of the full $T^6/\mathbb{Z}_4 \times \mathbb{Z}_4$ contains some Pati-Salam matter representations.

The final step is to select models with up to two Wilson lines that yield the gauge group and matter spectrum of the MSSM plus vector-like exotics. With the help of the methods developed in [12], we find more than 850 heterotic orbifolds with the desired properties. In many of these models, there is a U(1) that by construction, leads to (at least some) quarks and leptons with the correct P_6 charges and, therefore, suppression of proton decay occurs automatically. However, the models we were able to construct suffer from exotics that are not vector-like w.r.t. P_6. In what follows we present an example of these constructions.

An example The model is defined by the shift V_1 of eqn. (15), the second shift

$$V_2 = \left(-\tfrac{3}{4}, \tfrac{1}{2}, -\tfrac{1}{2}, 0^3, \tfrac{1}{4}, \tfrac{1}{2}\right)\left(-1, 0^3, \tfrac{1}{4}^4\right) , \tag{17}$$

and one Wilson line W_2 associated to the e_2 direction

$$W_2 = \left(-\tfrac{3}{4}, \tfrac{9}{4}, -\tfrac{5}{4}, -\tfrac{3}{4}, -\tfrac{1}{4}, \tfrac{5}{4}, \tfrac{1}{4}, -\tfrac{3}{4}\right)\left(\tfrac{1}{2}^4, 0^3, 2\right) . \tag{18}$$

Schematically, the gauge symmetry is broken as expected:

$$\mathrm{E}_8 \xrightarrow{V_1} \mathrm{SO}(12) \xrightarrow{V_2} \mathrm{PS} \times \mathrm{U}(1)_X \xrightarrow{W_2} \mathrm{SM} \tag{19}$$

resulting in the four-dimensional gauge group

$$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times \mathrm{U}(1)^4 \times [\mathrm{SU}(4) \times \mathrm{SO}(10)] . \tag{20}$$

One of the U(1)'s, orthogonal to $\mathrm{U}(1)_Y$, appears anomalous.

Using the $\mathrm{U}(1)_X$ direction orthogonal to Pati-Salam, but inside SO(12) we can define a (non-anomalous) generator along the lines of eqn. (5). Explicitly, it reads

$$t_{P_6} = \big(0, 0, 2, -2, 2, -2, 2, -2\big)\big(0^8\big) . \tag{21}$$

This is our prime choice for a U(1) proton hexality. In fact, the spectrum contains three generations of quarks and leptons with the right charge assignments and four SM singlets with $\mathrm{U}(1)_{P_6}$ charge 6 that can be used to break $\mathrm{U}(1)_{P_6}$ to its discrete subgroup P_6,

$$3(\mathbf{3},\mathbf{2})_{\frac{1}{6},0} + 3(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3},1} + 3(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3},-1} + 3(\mathbf{1},\mathbf{2})_{-\frac{1}{2},4} + 3(\mathbf{1},\mathbf{1})_{1,1} + 3(\mathbf{1},\mathbf{1})_{0,-3} + 4(\mathbf{1},\mathbf{1})_{0,6} . \tag{22}$$

See Table 2 for the full massless matter spectrum. The spectrum contains in addition exotics that are vector-like with respect to the SM. Unfortunately, these exotics turn out to be chiral with respect to P_6. Hence, they can only decouple once P_6 is broken. One would have to make sure that this breakdown is compatible with the desired proton stability. These questions will be studied in a future publication.

5.2 Proton Hexality as Accidental Symmetry

We have focused so far on exact gauge symmetries of orbifold compactifications and how they can be broken to give rise to P_6. Alternatively, one can consider the large set of approximate U(1) symmetries that arise naturally in orbifold models and lead to solutions of certain issues, such as the strong CP-problem [39, 40].

The general procedure to address this question is as follows. Considering the effective superpotential $\mathcal{W}$ truncated at a given order, one has to identify the U(1) symmetries (other than the gauged ones) under which $\mathcal{W}$ is invariant. Then one must require that a linear combination $\mathrm{U}(1)_{P_6}$ of these U(1)s provide the correct charge assignments for all the SM-fields (see Table 1) and that there be at least one SM-singlet χ with charges ± 6 whose VEV breaks $\mathrm{U}(1)_{P_6}$ down to P_6. One must further enforce that the spectrum be three generations plus vector-like matter also w.r.t. $\mathrm{U}(1)_{P_6}$.

We have performed a search of such symmetries among the models of the $\mathbb{Z}_6$-II Mini-Landscape with two and three Wilson lines and found no example displaying three generations with the standard P_6 charges. If one relaxes this condition and requires correct P_6 only for the first two generations, there are some examples. However, they do not satisfy all the requirements listed before. In the following we present a model of this type. It is not clear whether better models are possible in this scenario. In fact the strategy employed in the construction of the Mini-Landscape [10,11] aiming preferentially at complete multiplets could be incompatible with the incorporation of P_6 in a satisfactory way.

An example Let us consider the $\mathbb{Z}_6$–II orbifold model described by the shift embedding

$$V^{\mathrm{E}_6,1} = \left(\tfrac{1}{6},\, -\tfrac{1}{3},\, -\tfrac{1}{2},\, 0,\, 0,\, 0,\, 0,\, 0\right)(0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0) \tag{23}$$

and the Wilson lines

$$\begin{aligned}
W_3 &= \left(-\tfrac{5}{6},\, -\tfrac{7}{6},\, \tfrac{1}{2},\, \tfrac{1}{2},\, \tfrac{1}{2},\, -\tfrac{1}{2},\, -\tfrac{1}{2},\, -\tfrac{1}{2}\right)\left(0,\, 0,\, \tfrac{1}{3},\, \tfrac{1}{3},\, \tfrac{1}{3},\, 0,\, 1,\, \tfrac{2}{3}\right)\,, && \text{(24a)}\\
W_2 &= \left(1,\, \tfrac{1}{2},\, 0,\, \tfrac{1}{2},\, \tfrac{1}{2},\, -\tfrac{1}{2},\, -1,\, 0\right)\left(-\tfrac{1}{4},\, \tfrac{3}{4},\, \tfrac{1}{4},\, \tfrac{1}{4},\, \tfrac{3}{4},\, -\tfrac{3}{4},\, -\tfrac{3}{4},\, \tfrac{3}{4}\right)\,, && \text{(24b)}\\
W_2' &= \left(\tfrac{3}{4},\, \tfrac{3}{4},\, -\tfrac{1}{4},\, -\tfrac{1}{4},\, -\tfrac{1}{4},\, \tfrac{3}{4},\, \tfrac{1}{4},\, \tfrac{1}{4}\right)\left(-\tfrac{1}{4},\, -\tfrac{1}{4},\, -\tfrac{1}{4},\, -\tfrac{1}{4},\, -\tfrac{1}{4},\, \tfrac{1}{4},\, \tfrac{1}{4},\, \tfrac{3}{4}\right)\,. && \text{(24c)}
\end{aligned}$$

The four-dimensional gauge group is $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times [\mathrm{SU}(6) \times \mathrm{U}(1)^7]$ with $\mathrm{U}(1)_Y \subset \mathrm{SU}(5)$, and the massless spectrum includes three SM generations plus vector-like exotics with respect to the SM gauge group.

At order four in the superpotential, apart from the gauge U(1)s, there appear 82 accidental U(1)s. A linear combination of all the available U(1)s renders the following properties (cf. Table 3):

a) there is a set of SM singlets χ that break $\mathrm{U}(1)_{P_6}$ to P_6;

b) two SM families (including right-handed neutrinos) have proper P_6 charges;

c) a pair of Higgs fields h_u, h_d have proper P_6 charges; and

d) apart from a third generation with unwanted P_6 charges, all other exotic states are vector-like.

This model has the potential to forbid proton decay. However, there are still some questions to be analysed. First, in this specific model, operators of order five in the superpotential break explicitly $\mathrm{U}(1)_{P_6}$. Secondly, a large mass for the top-quark is forbidden. Finally, $\mathrm{U}(1)_{P_6}$ exhibits anomalies such as $\mathrm{tr}\, Q_{P_6}^3, \mathrm{tr}\, Q_{P_6}^2 Q_Y, \mathrm{tr}\, Q_Y^2 Q_{P_6} \neq 0$, that would have to be cancelled. A search for a more realistic model is under way.

6 Conclusions

Proton decay is a crucial question in grand unified theories. The proton should decay, but not too fast. In supersymmetric theories we have to forbid dim-4 and -5 operators. Usual matter parity is not enough as it still allows dangerous dim-5 operators. In this respect proton hexality is perfect. It forbids all the couplings that we do not want and allows those we need.

For a long time, its incompatibility with grand unification was thought to be a problem. The concept of "local grand unification" discussed in string theories (and theories of extra dimensions), however, changes the picture. Split multiplets, solving already the problem of doublet-triplet splitting and the question of breakdown of the grand unified gauge group, come to rescue and make hexality potentially compatible with models that have been constructed in the framework of (heterotic) string theory.

Still the search for a fully realistic model requires more work and perhaps a dedicated search strategy. Models as discussed e.g. in the Mini-Landscape [10, 11] are not so well suited here by construction, as they were based on the desire to have unified multiplets for the first two families. Hexality would require a different approach and it would be desirable to set up a general geometric picture that naturally incorporates P_6.

So far we have learned some lessons from string theory. Hexality can come from various sources. It could be

- a subgroup of a non-anomalous symmetry,
- a subgroup of an anomalous symmetry,
- an accidental symmetry.

In fact, P_6 could just be an approximate symmetry that is valid at the level of lower-dimensional operators or valid only for part of the spectrum (like the first and second family).

It is worthwhile to explore these questions further, both from the bottom-up and top-down approaches. The present model building attempts just scratch the surface of the landscape. More dedicated model building is needed to construct fully realistic models. We have here presented some toy models that illustrate the potential marriage of hexality with local grand unification. We are confident to report about the construction of more realistic models in the near future.

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A Spectra of Toy Models

#	Irrep	
4	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1})_{(1/6,0)}$	q
1	$(\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1})_{(1/6,1)}$	q'
2	$(\overline{\mathbf{3}},\mathbf{2},\mathbf{1},\mathbf{1})_{(-1/6,2)}$	$\bar{q}'$
3	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})_{(-2/3,1)}$	$\bar{u}$
3	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})_{(1/3,-1)}$	$\bar{d}$
3	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})_{(1/3,1)}$	$\bar{d}'$
3	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})_{(1/3,-2)}$	$\bar{d}'$
1	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})_{(1/3,2)}$	$\bar{d}'$

#	Irrep	
1	$(\overline{\mathbf{3}},\mathbf{1},\mathbf{1},\mathbf{1})_{(1/3,-4)}$	$\bar{d}'$
2	$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})_{(-1/3,2)}$	d'
3	$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})_{(-1/3,-2)}$	d'
3	$(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})_{(-1/3,-1)}$	d'
4	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(-1/2,4)}$	ℓ
7	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(-1/2,-2)}$	ℓ
1	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(-1/2,2)}$	ℓ'
1	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(-1/2,-1)}$	ℓ'

#	Irrep	
8	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(1/2,2)}$	$\bar{\ell}'$
1	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(1/2,4)}$	$\bar{\ell}'$
1	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(1/2,-2)}$	$\bar{\ell}'$
9	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(1/2,-1)}$	h_u
9	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})_{(-1/2,1)}$	h_d
3	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(1,1)}$	$\bar{e}$
18	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,-3)}$	$\bar{\nu}$
15	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,3)}$	

3	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,6)}$	χ
1	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,-6)}$	χ
18	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,0)}$	s^0
8	$(\mathbf{1},\mathbf{2},\overline{\mathbf{4}},\mathbf{1})_{(0,-1/2)}$	m
2	$(\mathbf{1},\mathbf{2},\mathbf{4},\mathbf{1})_{(0,-5/2)}$	m'
6	$(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{1})_{(-1/2,3/2)}$	s^-
2	$(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{1})_{(-1/2,-3/2)}$	s^-

4	$(\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})_{(-1/2,-5/2)}$	s^-
2	$(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{1})_{(1/2,5/2)}$	s^+
4	$(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{1})_{(1/2,-7/2)}$	s^+
4	$(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{1})_{(1/2,-1/2)}$	s^+
2	$(\mathbf{1},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})_{(1/2,1/2)}$	s^+
5	$(\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})_{(0,2)}$	x
6	$(\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})_{(0,1)}$	

1	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})_{(0,2)}$	y
2	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})_{(0,-2)}$	
2	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})_{(0,1)}$	
2	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,2)}$	$\tilde{s}$
7	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,-2)}$	
3	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,1)}$	
9	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})_{(0,-1)}$	

Table 2: Massless spectrum of a model with gauged P_6. Quantum numbers w.r.t. $[\mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L}] \times [\mathrm{SU}(4) \times \mathrm{SO}(10)]$ (bold) and $\mathrm{U}(1)_Y \times \mathrm{U}(1)_{P_6}$ (subscripts) are given.

#	Irrep		#	Anti-irrep	
2	$(\mathbf{3},\mathbf{2};\mathbf{1})_{(1/6,0)}$	$q_{1,2}$			
1	$(\mathbf{3},\mathbf{2};\mathbf{1})_{(1/6,25/2)}$	q_3			
1	$(\mathbf{3},\mathbf{2};\mathbf{1})_{(1/6,7/2)}$	q'	1	$(\overline{\mathbf{3}},\mathbf{2};\mathbf{1})_{(-1/6,-7/2)}$	$\bar{q}'$
2	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(-1/2,-2)}$	$\ell_{1,2}$			
1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(-1/2,0)}$	ℓ_3			
1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(-1/2,1)}$	h_d	1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(1/2,-1)}$	h_u
2	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(-1/2,2)}$	ℓ'	2	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(1/2,-2)}$	$\bar{\ell}'$
1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(-1/2,-1/2)}$		1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(1/2,1/2)}$	
1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(-1/2,3)}$		1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(1/2,3)}$	
2	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(1/3,-1)}$	$\bar{d}_{1,2}$			
1	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(1/3,0)}$	$\bar{d}_3$			
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(1/3,2)}$	$\bar{d}'$	3	$(\mathbf{3},\mathbf{1};\mathbf{1})_{(-1/3,-2)}$	d'
1	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(1/3,0)}$		1	$(\mathbf{3},\mathbf{1};\mathbf{1})_{(-1/3,6)}$	
1	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(1/3,1/2)}$		1	$(\mathbf{3},\mathbf{1};\mathbf{1})_{(-1/3,-1/2)}$	
2	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(-2/3,1)}$	$\bar{u}_{1,2}$			
1	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(-2/3,9/2)}$	$\bar{u}_3$			
1	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(-2/3,-7/9)}$	$\bar{u}'$	1	$(\mathbf{3},\mathbf{1};\mathbf{1})_{(2/3,7/9)}$	u'
3	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(1,1)}$	$\bar{e}_{1,2,3}$			
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(1,0)}$	$\bar{e}'$	1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(-1,12)}$	e'
1	$(\mathbf{3},\mathbf{1};\mathbf{1})_{(1/6,25/31)}$	v	1	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1})_{(-1/6,-25/31)}$	$\bar{v}$
1	$(\mathbf{1},\mathbf{1};\mathbf{6})_{(1/2,40/27)}$	w^+	1	$(\mathbf{1},\mathbf{1};\mathbf{6})_{(-1/2,-40/27)}$	w^-
3	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(1/2,2/27)}$	s^+	3	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(-1/2,-2/27)}$	s^-
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(1/2,-2)}$		2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(-1/2,8)}$	
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(1/2,0)}$		1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(-1/2,0)}$	
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(1/2,1/31)}$		1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(-1/2,-1/31)}$	
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(1/2,-1/2)}$		1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(-1/2,1/2)}$	
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(1/2,2)}$		1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(-1/2,-2)}$	
2	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(0,0)}$	m			
2	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(0,3/31)}$	m'	2	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(0,-3/31)}$	$\bar{m}'$
1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(0,1)}$		1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(0,-1)}$	
1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(0,-3)}$		1	$(\mathbf{1},\mathbf{2};\mathbf{1})_{(0,3)}$	

#	Irrep	
11	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,0)}$	s^0
4	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 6)}$	χ
6	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,3)}$	$\bar{\nu}$
4	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 21/31)}$	$\tilde{s}$
4	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 1/2)}$	
4	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 3/2)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 7/2)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 2/9)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 7/9)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 52/27)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 31/27)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 22/27)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 14/27)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 2/27)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 125/27)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 244/27)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 8/31)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 1/31)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,\pm 16/31)}$	
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,-34/9)}$	
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,-20/9)}$	
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,29/3)}$	
1	$(\mathbf{1},\mathbf{1};\mathbf{1})_{(0,-11/3)}$	
2	$(\mathbf{1},\mathbf{1};\mathbf{6})_{(0,0)}$	x
2	$(\mathbf{1},\mathbf{1};\mathbf{6})_{(0,\pm 1/2)}$	x'
2	$(\mathbf{1},\mathbf{1};\overline{\mathbf{6}})_{(0,0)}$	$\bar{x}$
2	$(\mathbf{1},\mathbf{1};\overline{\mathbf{6}})_{(0,\pm 1/2)}$	$\bar{x}'$

Table 3: Massless spectrum of a model with P_6 as accidental symmetry. Quantum numbers w.r.t. $[\mathrm{SU}(3)_C \times \mathrm{SU}(2)_\mathrm{L}] \times [\mathrm{SU}(6)]$ (bold) and $\mathrm{U}(1)_Y \times \mathrm{U}(1)_{P_6}$ (subscripts) are given.

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