

# Reconciling $J/\psi$ production at HERA, RHIC, Tevatron, and LHC with NRQCD factorization at next-to-leading order

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We calculate the cross section of inclusive direct  $J/\psi$  hadroproduction at next-to-leading order (NLO) within the factorization formalism of nonrelativistic quantum chromodynamics (NRQCD), including the full relativistic corrections due to the intermediate  $^1S_0^{[8]}$ ,  $^3S_1^{[8]}$ , and  $^3P_J^{[8]}$  color-octet states. We perform a combined fit of the color-octet (CO) long-distance matrix elements to the transverse-momentum ( $p_T$ ) distributions measured by CDF at the Fermilab Tevatron and H1 at DESY HERA and demonstrate that they also successfully describe the  $p_T$  distributions from PHENIX at BNL RHIC and CMS at the CERN LHC as well as the photon-proton c.m. energy and (with worse agreement) the inelasticity distributions from H1. This provides a first rigorous test of NRQCD factorization at NLO. In all experiments, the CO processes are shown to be indispensable.

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The factorization formalism of nonrelativistic QCD (NRQCD) [1] provides a rigorous theoretical framework for the description of heavy-quarkonium production and decay. This implies a separation of process-dependent short-distance coefficients, to be calculated perturbatively as expansions in the strong-coupling constant  $\alpha_s$ , from supposedly universal long-distance matrix elements (LDMEs), to be extracted from experiment. The relative importance of the latter can be estimated by means of velocity scaling rules; *i.e.*, the LDMEs are predicted to scale with a definite power of the heavy-quark ( $Q$ ) velocity  $v$  in the limit  $v \ll 1$ . In this way, the theoretical predictions are organized as double expansions in  $\alpha_s$  and  $v$ . A crucial feature of this formalism is that it takes into account the complete structure of the  $Q\bar{Q}$  Fock space, which is spanned by the states  $n = {}^{2S+1}L_J^{[a]}$  with definite spin  $S$ , orbital angular momentum  $L$ , total angular momentum  $J$ , and color multiplicity  $a = 1, 8$ . In particular, this formalism predicts the existence of color-octet (CO) processes in nature. This means that  $Q\bar{Q}$  pairs are produced at short distances in CO states and subsequently evolve into physical, color-singlet (CS) quarkonia by the nonperturbative emission of soft gluons. In the limit  $v \rightarrow 0$ , the traditional CS model (CSM) is recovered in the case of  $S$ -wave quarkonia. In the case of  $J/\psi$  production, the CSM prediction is based just on the  $^3S_1^{[1]}$  CS state, while the leading relativistic corrections, of relative order  $\mathcal{O}(v^4)$ , are built up by the  $^1S_0^{[8]}$ ,  $^3S_1^{[8]}$ , and  $^3P_J^{[8]}$  ( $J = 0, 1, 2$ ) CO states.

The greatest success of NRQCD was that it was able to explain the  $J/\psi$  hadroproduction yield at the Fermilab Tevatron [2], while the CSM prediction lies orders of magnitude below the data, even if the latter is evaluated at NLO [3, 4]. Also in the case of  $J/\psi$  photoproduction at DESY HERA, the CSM cross section at NLO significantly falls short of the data [5, 6]. Complete NLO calculations in NRQCD were performed for inclusive  $J/\psi$  production in two-photon collisions [7],  $e^+e^-$

annihilation [8], and direct photoproduction [6]. As for hadroproduction at NLO, the CO contributions due to intermediate  $^1S_0^{[8]}$  and  $^3S_1^{[8]}$  states [4] were calculated as well as the complete NLO corrections to  $\chi_J$  production, including the  $^3S_1^{[8]}$  contribution [9].

In order to convincingly establish the CO mechanism and the LDME universality, it is an urgent task to complete the NLO description of  $J/\psi$  hadroproduction by including the full CO contributions at NLO, which is actually achieved in this Letter. In fact, because of their high precision and their wide coverage and fine binning in  $p_T$ , the Tevatron data on inclusive  $J/\psi$  production have so far been the major source of information on the CO LDMEs [10], and the LHC data to come will be even more constraining. Previous NLO analyses of  $J/\psi$  hadroproduction [3, 4] were lacking the  $^3P_J^{[8]}$  contributions, for which there is no reason to be insignificant. This technical bottleneck, which has prevented essential progress in the global test of NRQCD factorization for the past fifteen years, is overcome here by further improving and refining the calculational techniques developed in Ref. [6].

Invoking the factorization theorems of the QCD parton model and NRQCD [1], the inclusive  $J/\psi$  hadroproduction cross section is evaluated from

$$d\sigma(AB \rightarrow J/\psi + X) = \sum_{i,j,n} \int dx dy f_{i/A}(x) f_{j/B}(y) \langle \mathcal{O}^{J/\psi}[n] \rangle d\sigma(ij \rightarrow c\bar{c}[n] + X), \quad (1)$$

where  $f_{i/A}(x)$  are the parton distribution functions of hadron  $A$ ,  $\langle \mathcal{O}^{J/\psi}[n] \rangle$  are the LDMEs, and  $d\sigma(ij \rightarrow c\bar{c}[n] + X)$  are the partonic cross sections. Working in the fixed-flavor-number scheme,  $i$  and  $j$  run over the gluon  $g$  and the light quarks  $q = u, d, s$  and anti-quarks  $\bar{q}$ . The counterpart of Eq. (1) for direct photoproduction emerges by replacing  $f_{i/A}(x)$  by the photon flux function  $f_{\gamma/e}(x)$  and fixing  $i = \gamma$ .

We checked analytically that all appearing singularities

$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	$(4.50 \pm 0.72) \times 10^{-2} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	$(3.12 \pm 0.93) \times 10^{-3} \text{ GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$	$(-1.21 \pm 0.35) \times 10^{-2} \text{ GeV}^5$

TABLE I: NLO fit results for the  $J/\psi$  CO LDMEs.

cancel. As for the ultraviolet singularities, we renormalize the charm-quark mass and the wave functions of the external particles according to the on-shell scheme, and the strong-coupling constant according to the modified minimal-subtraction scheme. The infrared (IR) singularities are canceled similarly as described in Ref. [6]. In particular, the  $^3P_J^{[8]}$  short-distance cross sections produce two new classes of soft singularities, named *soft #2* and *soft #3* terms, on top of the *soft #1* terms familiar from the  $S$ -wave channels. The *soft #2* terms do not factorize to LO cross sections; they cancel against the IR singularities of the virtual corrections left over upon the usual cancellation against the *soft #1* terms. The *soft #3* terms cancel against the IR singularities from the radiative corrections to the  $\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$  and  $\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$  LDMEs.

We now describe our theoretical input and the kinematic conditions for our numerical analysis. We set  $m_c = 1.5 \text{ GeV}$ , adopt the values of  $m_e$ ,  $\alpha$ , and the branching ratios  $B(J/\psi \rightarrow e^+e^-)$  and  $B(J/\psi \rightarrow \mu^+\mu^-)$  from Ref. [11], and use the one-loop (two-loop) formula for  $\alpha_s^{(n_f)}(\mu)$ , with  $n_f = 4$  active quark flavors, at LO (NLO). As for the proton parton distribution functions, we use set CTEQ6L1 (CTEQ6M) [12] at LO (NLO), which comes with an asymptotic scale parameter of  $\Lambda_{\text{QCD}}^{(4)} = 215 \text{ MeV}$  (326 MeV). We evaluate the photon flux function using Eq. (5) of Ref. [13] with the cut-off  $Q_{\text{max}}^2 = 2.5 \text{ GeV}^2$  [15] on the photon virtuality. As for the CS LDME, we adopt the value  $\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$  from Ref. [16]. Our default choices for the renormalization, factorization, and NRQCD scales are  $\mu_r = \mu_f = m_T$  and  $\mu_\Lambda = m_c$ , respectively, where  $m_T = \sqrt{p_T^2 + 4m_c^2}$  is the  $J/\psi$  transverse mass.

Our strategy for testing NRQCD factorization in  $J/\psi$  production at NLO is as follows. We first perform a common fit of the CO LDMEs to the  $p_T$  distributions measured by CDF in hadroproduction at Tevatron Run II [17] and by H1 in photoproduction at HERA1 [14] and HERA2 [15] (see Table I and Fig. 1). We then compare the  $p_T$  distributions measured by PHENIX at RHIC [18] and CMS at the LHC [19] as well as the  $W$  and  $z$  distributions measured by H1 at HERA1 [14] and HERA2 [15] with our respective NLO predictions based on these CO LDMEs (see Fig. 2).

The  $p_T$  distribution of  $J/\psi$  hadroproduction measured experimentally flattens at  $p_T < 3 \text{ GeV}$  due to nonperturbative effects, a feature that cannot be faithfully described by fixed-order perturbation theory. We, therefore, exclude the CDF data points with  $p_T < 3 \text{ GeV}$

from our fit. We checked that our fit results depend only feebly on the precise location of this cut-off. We also verified that exclusion of the H1 data points with  $p_T < 2.5 \text{ GeV}$ , which might require power corrections neglected here, is inconsequential for our fit. The fit results for the CO LDMEs corresponding to our default NLO NRQCD predictions are collected in Table I. In Figs. 1(a) and (d), the latter (solid lines) are compared with the CDF [17] and H1 [14, 15] data, respectively. For comparison, also the default predictions at LO (dashed lines) as well as those of the CSM at NLO (dot-dashed lines) and LO (dotted lines) are shown. In order to visualize the size of the NLO corrections to the hard-scattering cross sections, the LO predictions are evaluated with the same LDMEs. The yellow and blue (shaded) bands indicate the theoretical errors on the NLO NRQCD and CSM predictions, respectively, due to the lack of knowledge of corrections beyond NLO, which are estimated by varying  $\mu_r$ ,  $\mu_f$ , and  $\mu_\Lambda$  by a factor 2 up and down relative to their default values. The  $\mu_r$ ,  $\mu_f$ , and  $\mu_\Lambda$  dependencies of  $\alpha_s$ , the parton distribution functions, and the LDMEs, respectively, induced by the renormalization group are canceled only partially, namely through the order of the calculation, by linearly logarithmic terms appearing in the NLO corrections. Data-over-theory representations of Figs. 1(a) and (d) are given in Figs. 1(b) and (e), respectively. In Figs. 1(c) and (f), the default NLO NRQCD predictions of Figs. 1(a) and (d), respectively, are decomposed into their  $^3S_1^{[1]}$ ,  $^1S_0^{[8]}$ ,  $^3S_1^{[8]}$ , and  $^3P_J^{[8]}$  components. We observe from Fig. 1(c) that the  $^3P_J^{[8]}$  short-distance cross section of hadroproduction (excluding the negative LDME) receives sizable NLO corrections that even turn it negative at  $p_T \gtrsim 7 \text{ GeV}$ . This is, however, not problematic because a particular CO contribution represents an unphysical quantity depending on the choices of renormalization scheme and scale  $\mu_\Lambda$  and is entitled to become negative as long as the full cross section remains positive. Such features are familiar, e.g., from inclusive heavy-hadron production at NLO [20]. In contrast to the situation at LO, the line shapes of the  $^1S_0^{[8]}$  and  $^3P_J^{[8]}$  contributions significantly differ at NLO, so that  $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$  and  $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$  may now be fitted independently (see Table I). Besides that, the injection of HERA data into the fit also supports the independent determination of  $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$  and  $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$ . Notice that  $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$  comes out negative, which is not problematic by the same token as above. In compliance with the velocity scaling rules of NRQCD [1], the CO LDMEs in Table I are approximately of order  $\mathcal{O}(v^4)$  relative to  $\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$ . We read off from Fig. 1(a) and (d) that the NLO correction ( $K$ ) factors for hadro- and photoproduction range from 1.30 to 2.28 and from 0.54 to 1.80, respectively, in the  $p_T$  intervals considered.

We observe from Fig. 2 that our NLO NRQCD predictions nicely describe the  $p_T$  distributions from PHENIX [18] (a) and CMS [19] (b) as well as the  $W$  distribu-

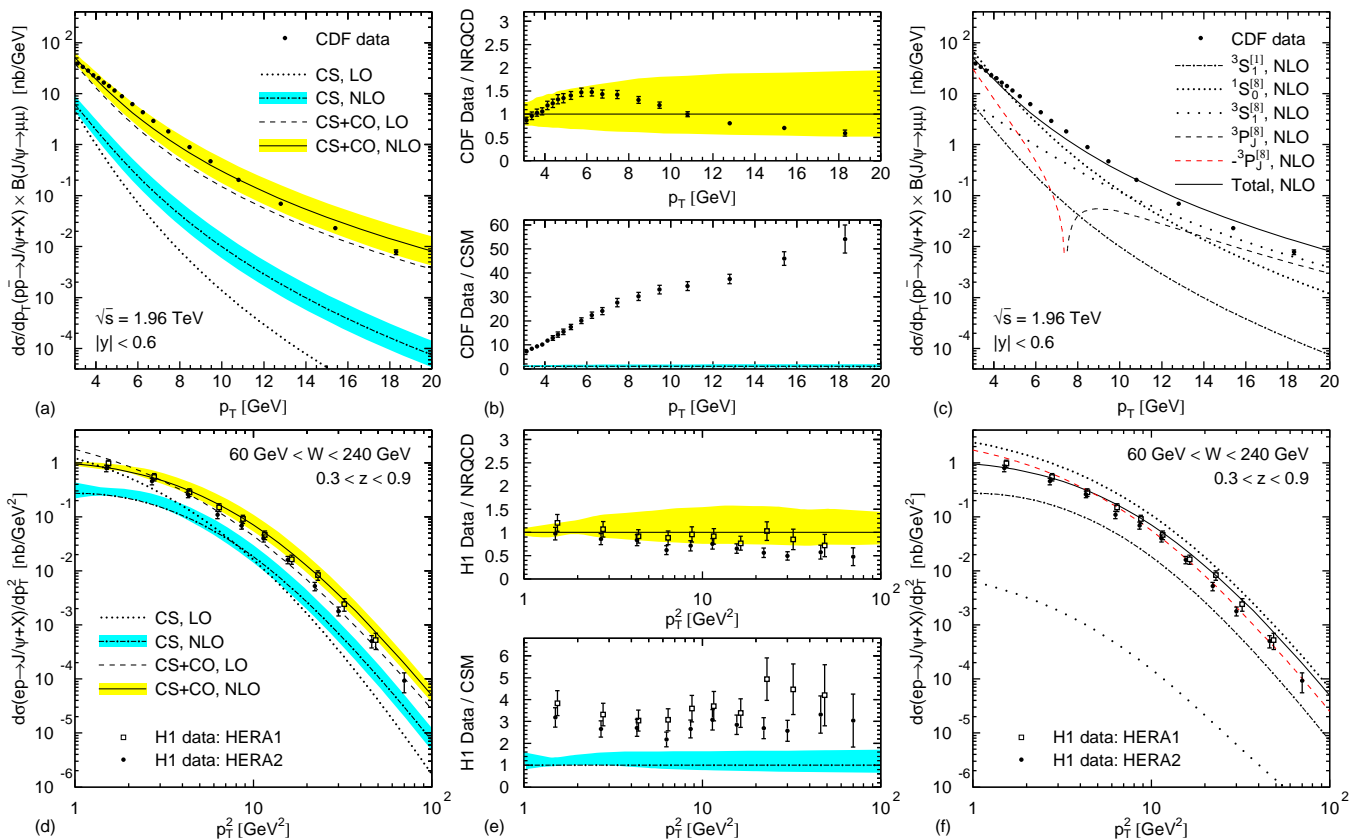


FIG. 1: NLO NRQCD predictions of  $J/\psi$  hadro- and photoproduction resulting from the fit compared to the CDF [17] and H1 [14, 15] input data. The coding of the lines in part (f) of the figure is the same as in part (c). The seeming singularity of the  ${}^3P_J^{[8]}$  contribution in part (c) is an artifact of the logarithmic scale on the vertical axis.

tions from H1 [14, 15] (c), with most of the data points falling inside the yellow (shaded) error band. In all these cases, inclusion of the NLO corrections tends to improve the agreement. The NLO NRQCD prediction of the  $z$  distribution (d) agrees with the H1 data in the intermediate  $z$  range, but its slope appears to be somewhat too steep at first sight. However, the contribution due to resolved photoproduction, which is not yet included here, is expected to fill the gap in the low- $z$  range, precisely where it is peaked; the overshoot of the NRQCD prediction in the upper endpoint region, which actually turns into a breakdown at  $z = 1$ , is an artifact of the fixed-order treatment and may be eliminated by invoking soft collinear effective theory [21]. We conclude from Figs. 1 and 2 that all experimental data sets considered here significantly overshoot the NLO CSM predictions, by many experimental standard deviations. Specifically, the excess amounts to 1–2 orders of magnitude in the case of hadroproduction [see Fig. 1(b)] and typically a factor of 3 in the case of photoproduction [see Fig. 1(e)]. On the other hand, these data nicely agree with the NLO NRQCD predictions, apart from well-understood deviations in the case of the  $z$  distribution of photoproduction [see Fig. 2(d)]. This constitutes the most rigorous evidence for the existence of CO processes in nature and the LDME universality since the introduction of the NRQCD

factorization formalism 15 years ago [1].

We should remark that our theoretical predictions refer to direct  $J/\psi$  production, while the CDF and CMS data include prompt events and the H1 and PHENIX data even non-prompt ones, but the resulting error is small against our theoretical uncertainties and has no effect on our conclusions. In fact, the fraction of  $J/\psi$  events originating from the feed-down of heavier charmonia only amounts to about 30% [22] for hadroproduction and 15% [15] for photoproduction, and the fraction of  $J/\psi$  events from  $B$  decays is negligible at HERA [15] and RHIC energies.

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*Note added.* At the final stage of preparing this manuscript, after our results were presented at an international conference [23], a preprint [24] appeared that also reports on a NLO calculation of  $J/\psi$  hadroproduction in full NRQCD. Adopting their inputs, we find agreement with their results for the  ${}^3S_1^{[1]}$ ,  ${}^1S_0^{[8]}$ ,  ${}^3S_1^{[8]}$ , and  ${}^3P_J^{[8]}$  contributions.

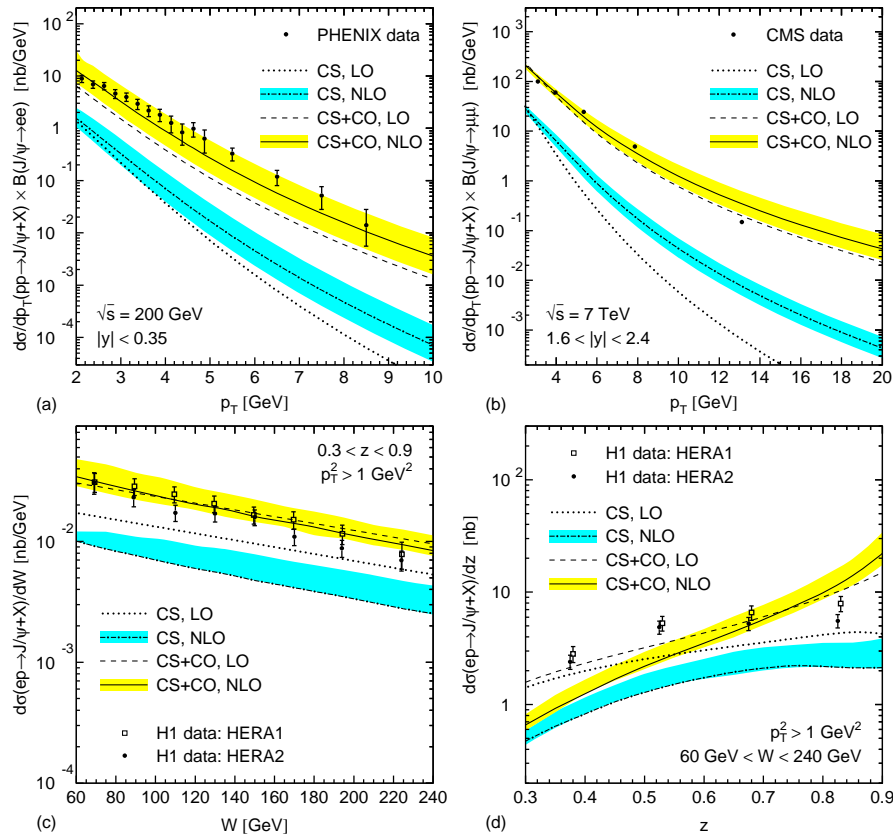


FIG. 2: NLO NRQCD predictions of  $J/\psi$  hadro- and photoproduction resulting from the fit compared to PHENIX [18], CMS [19], and H1 [14, 15] data not included in the fit.

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