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News on Ambre and CSectors*

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Mellin-Barnes and sector decomposition methods are used to evaluate tensorial Feynman diagrams in the Euclidean kinematical region. Few software packages are shortly described and few examples demonstrate their use.

1. Introduction

Mellin-Barnes (MB) methods (see e.g. [1] and references therein) and sector decomposition (SD) methods (see e.g. [2] and references therein) are frequently used in theoretical high energy physics calculations. We will restrict ourselves here mainly to Feynman loop integrals in d dimensions. There are nowadays public software packages which may help to automatize these calculations. On the MB Tools webpage [3] there are useful codes which realize: analytic continuations of Mellin-Barnes integrals – MB.m [4] and MBresolve.m [5]; parametric expansions of Mellin-Barnes integrals – expansion.m (author M. Czakon) and barnesroutines.m (author D. Kosower). For the creation of MB representations and later calls of other tools, AMBRE.m can be used [6,7], which we describe here. AMBRE v1.x generates multiloop scalar [planar] and one-loop Feynman integrals. Here we extend the package to v2.0, which may treat also tensor [planar] structures. For the sector decomposition method, there are two public programs. FIESTA [8] and FIESTA2 [9] calculate scalar Feynman integrals with Mathematica, while Bogner/Weinzierl's (BW) program [10] allows to calculate basic polynomials needed for the calculation of any Feynman integral with the c++ based computer algebra system Ginac [11]. Here we describe the Mathematica program CSectors.m which may serve as a kind of user interface to BW and Ginac in order to calculate fully automatically tensor Feynman integrals.

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2. General tensor structure

We consider the L loop Feynman momentum integrals with N propagators P_n and tensor structure $T(k)$:

$$
\begin{aligned}
G_L[T(k)] &\equiv \frac{1}{(i\pi^{d/2})^L}\int\frac{d^dk_1\ldots d^dk_L T(k)}{P_1\ldots P_N}\\
&= \frac{1}{(i\pi^{d/2})^L}\int\frac{d^dk_1\ldots d^dk_L\ T(k)}{(q_1^2-m_1^2)^{\nu_1}\ldots(q_N^2-m_N^2)^{\nu_N}}\\
&= \frac{(-1)^{N_\nu}}{\Gamma(\nu_1)\ldots\Gamma(\nu_N)}\\
&\quad\times\int\prod_{j=1}^N dx_j x_j^{\nu_j-1}\,\delta\left(1-\sum_{i=1}^N x_i\right)\\
&\quad\times\sum_{r\le m}\frac{\Gamma\left(N_\nu-\frac{d}{2}L-\frac{r}{2}\right)}{(-2)^{\frac{r}{2}}}\frac{U^{N_\nu-\frac{d}{2}(L+1)-m}}{F^{N_\nu-\frac{d}{2}L-\frac{r}{2}}}\\
&\quad\times\left\{\mathscr{A}_r P^{m-r}\right\}^{[\mu_1,\ldots,\mu_m]}. \qquad (1)
\end{aligned}
$$

The momenta q_i are linear combinations of internal momenta k_j and external momenta p_n, and $T(k)$ is a tensor structure, e.g. $(1,k_l^\mu,k_l^\mu k_n^\nu,\ldots)$. Further, $N_\nu=\sum\nu_n$. The general scalar L loop integrals $G_L[1]$ are implemented in AMBRE.m v1.x, however tensor structures were allowed only for the one-loop case, $G_1[T(k)]$. The symbol $\{R\}^{[a_1\cdots a_r]}$ means the totally symmetrized sum of all possible combinations in the argument. We give as an explicit example the rank $R=3$ tensor:

$$
\begin{aligned}
&\sum_{r\le 3}\left\{\mathscr{A}_r P^{3-r}\right\}^{[\mu_1\mu_2\mu_3]} \qquad (2)\\
&= \left\{A_0P^3+A_1P^2+A_2P^1+A_3P^0\right\}^{[\mu_1\mu_2\mu_3]}\\
&= P^{\mu_1}P^{\mu_2}P^{\mu_3}+\tilde{g}^{\mu_1\mu_2}P^{\mu_3}+\tilde{g}^{\mu_2\mu_3}P^{\mu_1}\\
&\quad+\tilde{g}^{\mu_3\mu_1}P^{\mu_2}.
\end{aligned}
$$

It is $A_0 = P^0 = 1$, and A_r is zero for r odd, while $\mathscr{A}_r = \tilde{g}^{[\mu_1\mu_2}\cdots\tilde{g}^{\mu_{r-1}\mu_r]}$ for r even. Further, $\tilde{g}^{\mu_i\mu_j} = (\tilde{M}^{-1})_{ab} g^{\mu_i\mu_j}$ and $P_a^{\mu_i} = \sum_l [\tilde{M}_{al} Q_l]^{\mu_i}$. The $\tilde{M}$ and Q_l are defined from $\sum_{i=1}^n P_i x_i = \sum_{i=1}^N (q_i^2 - m_i^2) x_i = \sum_{i,j=1}^L k_i^T M_{ij} k_j - 2\sum_{j=1}^L k_j^T Q_j + J$ and $\tilde{M} = \det(M) M^{-1}$. The U and F polynomials were defined e.g. in [6,7].

3. The package AMBRE.m

In [12] it is shown how the tensor structures introduced in section 2 lead, in the loop-by-loop approach, to a list of Mellin-Barnes representations. For that purpose, the following basic function of the AMBRE.m package is invoked: †

MBrepr[{numerator},{propagators}, {internal momenta}].

Let us take the example of figure 1, and sketch a sample math file. ‡

First, packages must be loaded:

```
« MBv1.2.m
« AMBRE.m
« MBnum.m
```
(3)

or

```
« MBresolve.m
« barnesroutines.m
```
(4)

As indicated, `barnesroutines.m` works with version 1.2 of MB.m. If we want to use this package, the MB integrals up to the needed power in ε must be prepared first (by analytic continuation), and the second part of the sample MB_SE5l0m.m file may look as follows:

```
invariants = {p1^2 -> s};
repr = MBrepr[{k1*p1, k1*p1, k2*p1},
 {PR[k1,0,n1]*PR[k2,0,n2]*PR[k2-k1,0,n3]
 *PR[k1+p1,0,n4]*PR[k2+p1,0,n5]},
  {k2,k1}];
```

in the case of the MBnum.m package, followed by:

```
SetOptions[MBnum, Analytical -> True,
 Numerical -> False, ShowMBrep -> True];
MBanalytic = MBnum[repr, 1, {s -> -11},
 {n1->1, n2->1, n3->1, n4->1, n5->1}, 2];
```
(5)

or, in the case of the MBresolve.m package, followed by:

```
step1=MBresolve[#/.powers,eps]&/@repr;
step2=MBexpand[step1,
       Exp[2*eps*EulerGamma],{eps,0,1}];
MBanalytic=MBmerge[step2];
```
(6)

Finally, after getting the MB integrals at the ε level needed (here ε^1, second argument of the MBnum function), we "Simplify" the result using Barnes lemmas with the internal MB.m functions MBmerge and MBintegrate:

```
after =
Process[MBanalytic, Range[10]]//MBmerge;
SEnumMB=MBintegrate[after,{s->-11}]
```

The phrase `Range[10]` means that we assume to get up to 10 dimensional MB representations; in fact, they are maximally 2-dimensional here. §

The final result is:

$$\begin{aligned}\mathrm{SEnumMB} = {} & -7.5625/\varepsilon^2 - 20.4506/\varepsilon \\ & -178.18 \pm 0.0171936 \\ & +(18.3642 \pm 0.0248465)\varepsilon. \end{aligned} \quad (7)$$

Singularities are here at most 1-dimensional integrals, so MB.m solves them within Mathematica very accurately and no error is given.

4. The package CSectors.m

The package CSectors.m allows to derive integral representations for Feynman integrals, which are decomposed into so-called sectors with a well-defined singularity structure. The invariants should be of the Euclidean type. The structure of the main function is analogous to that in Ambre.m:

† Kinematics in the form of a list must be also properly defined.

‡ See also the complete MB_SE5l0m.m source at [7].

§ Actually the use of barnesroutines.m does not help much here, and it takes most of the running time: the time of the total calculation is 17 seconds without applying Barnes Lemmas.

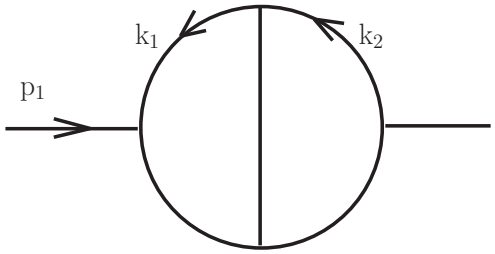

Figure 1. A 2-loop self energy topology.

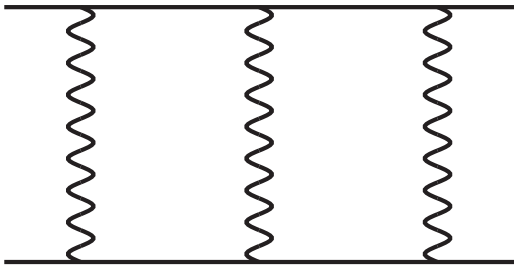

Figure 2. The planar 2-loop Bhabha topology B1.

DoSectors[{numerator},{propagators}, {internal momenta}][low,des]

The sample executable file might look as follows: [¶]

```
#!/bin/bash
math << \FILE

<< CSectors.m

invariants = {p1^2->s}

SetOptions[DoSectors, SetStrategy -> C,
TextDisplay->True, SourceName->SE5l0m_num,
TempFileDelete->False];

max=0; s=-11;
invariants = {p1^2 -> s};

SEnumSD = DoSectors[{k1*p1,k1*p1,k2*p1},
 {PR[k1,0,1]*PR[k2,0,1]*PR[k2-k1,0,1]*
  PR[k1+p1,0,1]*PR[k2+p1,0,1]},{k2,k1}]
  [-2,max]

FILE
```

The numerical result in the example is (see also file SD_LL2010.out at [13]):

$$\begin{aligned}\text{SEnumSD} &= -178.1927 - 7.56258/\varepsilon^2 \\ &-20.4505/\varepsilon + 18.394000000000005\varepsilon, \\ &\{0.011130644628528934, 0.00029563/\varepsilon^2, \\ &0.00302109/\varepsilon, 0.06629380324170578 * \varepsilon\}. \quad (8)\end{aligned}$$

The second bracket gives an error estimation. Due to the tensor structure (1), a number A of integrals I_i appear at a given ε level. An error for the numerical result I_ε at ε level is estimated in a standard way:

$$\Delta I_\varepsilon = \sqrt{\sum_{i=1}^{A} (\Delta I_i)^2}. \quad (9)$$

In the MB.m package the error ΔI is, for the single MB integral I, controlled by switching on an appropriate MB.m option: `Debug->True`. Let us just note that the error can be underestimated, see [4] and MB.m for details.

5. Mellin-Barnes approach versus Sector Decomposition approach

All the results given in the last section in (7) and (8) have been obtained with default parameters needed for numeric defined by the WB and MB.m packages. The total time of calculation on a Xeon PC with 32 GB RAM was in the MB case 17s and about 5 minutes for both the C- and X-strategies in the SD case.

Calculational times can differ substantially depending on methods and integrals. In table 1 and table 2, results for the massless and massive planar 2-loop topology B1 are given, see Fig. 2 and [14,15] for the topology's definition.

We can see that in the case of the massless box B1, the MB method can be much faster than the SD method. In the massive case the difference is smaller, moreover, the SD method can be often faster. In addition results obtained by FIESTA2 [9] are presented. Note a difference in sign between FIESTA2 and MB/SD, see [9] for FIESTA2 conventions.

For completeness, the FIESTA2 sample Mathematica file for getting the numbers in Table 2 is repro-

[¶] See file SD_SE5l0m.sh at [13].

Table 1
The scalar massless 2-loop planar double box $B1$, shown in figure 2. The scalar case allows for direct comparisons of the packages discussed here with FIESTA2. For source and output files see [7] and [13].

massless	AMBRE and MB
ε^0	$-0.1034 \pm 6 \cdot 10^{-6}$
ε^{-1}	-0.10907
ε^{-2}	-0.00966
ε^{-3}	0.083188
ε^{-4}	-0.022857
T [s]	22
massless	CSectors, X-strat.
ε^0	-0.1035 ± 0.0002
ε^{-1}	-0.10915 ± 0.00008
ε^{-2}	-0.00966 ± 0.00001
ε^{-3}	0.083191 ± 0.000005
ε^{-4}	$-0.0228574 \pm 1 \cdot 10^{-6}$
T [s]	1712
massless	FIESTA2
ε^0	0.103267 ± 0.001117
ε^{-1}	0.109049 ± 0.000348
ε^{-2}	0.009656 ± 0.000082
ε^{-3}	-0.08319 ± 0.000012
ε^{-4}	0.022858
T [s]	350
$s = -5, t = -7$	

duced:

```
<< FIESTA_2.0.0.m;
CIntegratePath="./CIntegrateMP"
QLinkPath = "./QLink64"

invariants =
{p1^2->m^2,p2^2->m^2,p3^2->m^2,p4^2->m^2,
 p1*p2->1/2*s-m^2,p3*p4->1/2*s-m^2,
 p1*p3->1/2*t-m^2,p2*p4->1/2*t-m^2,
 p2*p3->1/2*u-m^2,p1*p4->1/2*u-m^2}
 /.u->4*m^2-s-t/.m->1//Simplify;
kinematics = {s -> -5, t -> -7};

integral=PR[k1,1,n1]*PR[k1+p1,0,n2]*
 PR[k1+p1+p2,1,n3]*PR[k1-k2,0,n4]*
 PR[k2,1,n5]*PR[k2+p1+p2,1,n6]*
 PR[k2 + p1 + p2 + p4, 0, n7];
```

Table 2
The scalar massive 2-loop planar double box $B1$.

massive	AMBRE and MB
ε^0	0.2246
ε^{-1}	0.06359
ε^{-2}	-0.023524
T [s]	42
massive	CSectors, C-strat.
ε^0	0.2246 ± 0.0001
ε^{-1}	0.06357 ± 0.00003
ε^{-2}	$-0.023524 \pm 4 \cdot 10^{-6}$
T [s]	362
massive	FIESTA2
ε^0	-0.224756 ± 0.000485
ε^{-1}	-0.063622 ± 0.000102
ε^{-2}	0.023528 ± 0.00001
T [s]	35
$s = -5, t = -7, m = 1$	

```
fintegral=(List @@ integral)
  /.PR[q_,m_,n_]->-(q^2-m^2);

SDEvaluate[UF[{k1,k2},fintegral,
        Join[invariants, kinematics]],
         {1,1,1,1,1,1,1},0]
```

6. Summary

The CSectors.m package has been prepared in order to allow an easy use of the SD package by Bogner/Weinzierl for the evaluation of Feynman tensor integrals. The AMBRE package has been enlarged to involve tensor structures for multiloop integrals in an automatic way. We are planning for the near future to automatize the construction of MB representations for nonplanar diagrams, what deserves not to use the loop-by loop approach. Further, we will foresee to build MB representations for special forms of linear propagators which are present e.g. in the HQET theory or in calculations of the QCD static potential (see the talk by V. Smirnov at this conference [16]).

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