A Mellin Space Program for $W^{\pm}$ and Z^0 Production at NNLO

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We present a program for the evaluation of full unpolarized cross sections for the $W^{\pm}$ and Z^0 production in the narrow width approximation at NNLO in perturbative QCD using Mellin space techniques.

1 Introduction

The Drell-Yan process, originally described in the context of the parton model [1], concerns the production of a lepton pair of large invariant mass in hadron-hadron collisions. With the increase of the centre of mass energy at particle accelerators, the Drell-Yan process led to the discovery of $W^{\pm}$ and Z^0 bosons at UA1 and UA2 experiments [2, 3]. Since then the properties of massive vector bosons have been studied in great detail. At present the production of $W^{\pm}$ and Z^0 provides an important benchmark for the LHC and a test of the Standard Model (SM) in a new range of centre of mass energies [4].

As guaranteed by the factorisation theorem [5], one can separate the physics of soft energy scales from the physics at hard energy scales where perturbation theory applies. The higher order QCD corrections to the Drell-Yan process have been calculated up to next-to-next-to-leading order (NNLO), see [6–8] and references therein. The full cross section is obtained as a convolution with the parton distribution functions (PDFs) that encode the non-perturbative information.

In this paper, we present a program for evaluation of the full inclusive cross section for $W^{\pm}$ and Z^0 production in a fast and accurate way using a Mellin space approach. After a brief description of the basic ingredients of the calculation we give formulae for the Mellin transforms. We then present a comparison with the code ZWPROD [7, 8] and discuss possible applications and extensions within this framework.

2 Formalism

We consider the inclusive production of a single vector boson $V = W^+, W^-$ or Z^0 in hadron-hadron collision with a centre of mass energy s which subsequently decays into a lepton pair of an invariant mass Q^2. The decay of the vector boson is treated within the narrow width approximation which replaces the propagator by a delta function such that $Q^2 = M_V^2$. We consider massless quarks. The cross section for this process can be expressed as

$$\sigma^{h_1 h_2 \to V \to l_1 l_2}(s) = x\sigma^{V \to l_1 l_2} W^V(x, Q^2), \qquad x = Q^2/s, \tag{1}$$

where $\sigma^{V \to l_1 l_2}$ represents the kinematically independent part of the Born level subprocess $q\bar{q} \to V \to l_1 l_2$ (the point-like cross section) multiplied by the appropriate branching ratio. The exact form of the point like cross section can be found in Ref. [7], formulae (A.10) and (A.11). [1]

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[1] There is an extra factor of 2 in the denominator of the formula (A.11), corrected by [9]

The structure function $W^V(x, Q^2)$ is written as a convolution of two parton distribution functions $f_a(\mu_f^2)$ and $f_b(\mu_f^2)$ and a hard scattering cross section represented by coefficient functions Δ_{ab},

$$W^V(x, Q^2) = \sum_{a,b=q,\bar{q},g} C^V_{a,b}\Big[f_a(\mu_f^2) \otimes f_b(\mu_f^2) \otimes \Delta_{ab}(Q^2, \mu_f^2, \mu_r^2)\Big](x). \tag{2}$$

The perturbative coefficients are known up to NNLO [7, 8],

$$\Delta^{\mathrm{N}k\mathrm{LO}}_{ab}(x, Q^2, \mu_f^2, \mu_r^2) = \sum_{n=0}^{k} \frac{\alpha_s^k(\mu_r^2)}{4\pi} \Delta^{(k)}_{ab}(x, Q^2, \mu_f^2, \mu_r^2). \tag{3}$$

The factor $C^V_{a,b}$ in Eq.(2) contains information about couplings of vector bosons to partons a and b. For the detailed form of the Eq.(2) we refer the reader to the paper of Hamberg, Matsuura and van Neerven [7] whose notation we follow closely [2]. The convolution sign represents an integral

$$\begin{aligned}&(f_1 \otimes f_2 \otimes \cdots \otimes f_k)(x)\\ &= \int_0^1 \mathrm{d}\,x_1 \int_0^1 \mathrm{d}\,x_2 \ldots \int_0^1 \mathrm{d}\,x_k\, \delta(x - x_1 x_2 \ldots x_k) f_1(x_1) f_2(x_2) \ldots f(x_k).\end{aligned} \tag{4}$$

In principle one can perform the integrals in Eq.(2) directly however, the problem is much better addressed after transforming to Mellin space,

$$f(N) = \int_0^1 \mathrm{d}\,x\, x^{N-1} f(x), \tag{5}$$

This transformation turns the integrals in Eq.(2) into ordinary products such that the structure function reads

$$W^V(N, Q^2) = \sum_{a,b=q,\bar{q},g} C^V_{a,b} f_a(N, Q^2) f_b(N, Q^2) \Delta_{ab}(N, Q^2) \qquad \mu_f = \mu_r = Q^2, \tag{6}$$

and therefore it is possible to evaluate it in a fast and efficient way. The formula for the inverse Mellin transform defines how to recover the original momentum space result,

$$W^V(x, Q^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \mathrm{d}\,N\, x^{-N} W^V(N, Q^2), \tag{7}$$

where c represents a point on the real axis such that all poles N_i in the function $W(N, Q^2)$ lie to the left from c. Further on, we will refer to functions in Mellin space as N space functions and functions from momentum space as x space functions.

3 Implementation

The main ingredients of the calculation are the coefficient functions up to NNLO and the parton distribution functions in Mellin space in terms of a complex variable N. The condition $N \in \mathbb{C}$ is required for the numerical evaluation of the inversion formula (7). For this we adopted the technique implemented in QCD-PEGASUS [10]. The complex integral (7) is rewritten in terms of an integral over a real variable z

$$W(x, Q^2) = \frac{1}{\pi} \int_0^{\infty} \mathrm{d}\,z\, \mathrm{Im}[e^{i\varphi} x^{-c-ze^{i\varphi}} W(N, Q^2)] \qquad N = c + z\exp^{i\phi} \in \mathbb{C} \tag{8}$$

and evaluated using Gaussian quadratures. The parameter $\phi > \pi/2$ represents the angle with respect to the positive real axis. Since the rightmost pole of the structure function is $N_{\mathrm{max}} = 1$, we chose $c = 1.5$. These

[2] Several typos appearing in Ref. [7] have been pointed out in [9]

values as well as the maximum value of the integration variable z are flexible and can be modified by the user in the main program if desired. For a more detailed description of the shape of the integration contour we refer to the QCD-PEGASUS manual [10].

The coefficient functions in N space were published previously in Ref. [11], including corrections to the previous literature. The corresponding `FORTRAN` code is `DY.f` used together with `ANCONT` [12]. We performed the Mellin transforms starting from the x space expressions [13] using the `harmpol` package [14]. The results can be expressed mostly in terms of complex-valued simple harmonic sums [15, 16] and several more complicated ones which we approximated by using the minimax method[3] worked out in detail in [12] previously.[4] The absolute accuracy of our approximation is better than 10^{-9} over the whole kinematic range.

At the moment there are two options for the input parton distribution functions in N space. A toy input corresponds to the one used for the 2001/2 benchmark tables [18] and is used for comparisons with ZWPROD [7, 8] assuming no evolution of PDFs. The general form reads

$$xf_{i,\text{toy}}(x,\mu_0^2) = nx^a(1-x)^b, \qquad i = q,\bar{q},g, \qquad n,a,b \in \mathbb{R}, \tag{9}$$

which is in Mellin space represented by an Euler beta function

$$f_{i,\text{toy}}(N,\mu_0^2) = n\beta(a+N,b+1). \tag{10}$$

The second option for the PDF input is using the FORTRAN code QCD-PEGASUS [10] which can be linked to our program.

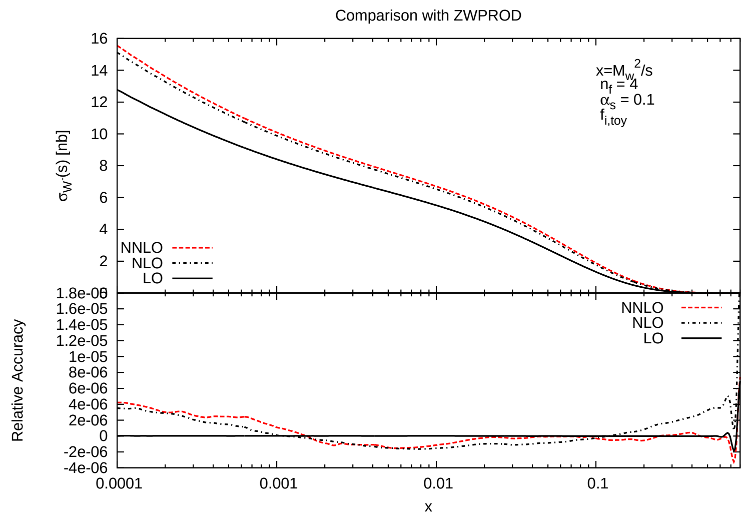

Fig. 1 Cross section for W^- production up to NNLO in the narrow width approximation using the toy parton distribution functions and a fixed value of the strong coupling constant. Upper part: The full cross section. Lower part: Relative accuracy with respect to the ZWPROD.

4 Results And Outlook

There are several programs on the market using the standard momentum space evaluation [19–21] which can provide a cross-check for our N space calculation. We performed comparisons of the full cross sections with a program ZWPROD written by the authors of the original calculation of the NNLO Drell-Yan

[3] We used the MINIMAX routine implemented in Maple
[4] Exact expressions were given in [17].

coefficient functions [7, 8]. The Fig. (1) shows a comparison for the W^- cross section using toy input for PDFs corresponding to the Eq.(10) with no evolution and a fixed value of the coupling constant $\alpha_s = 0.1$. The relative accuracy is better than 6×10^{-6} in the relevant kinematical range $x \in (10^{-4}, 0.8)$. As an intermediate check, we compared the Mellin inversion of N space coefficient functions against the x space expressions using a program of Gehrmann and Remiddi [22] for the numerical evaluation of harmonic polylogarithms. The framework presented here is suitable for a further implementation of those cross sections where N space coefficient functions are also available, like Higgs production and deep inelastic scattering (DIS) [11, 23–26]. The setup is well suited for merging the program with threshold resummation calculations which are typically performed in Mellin space (see e.g. [27]). For the extraction of PDFs from $W^{\pm}$ and Z production it would be desirable to have an access to the rapidity distributions in which case one will need to apply double Mellin transforms of two variables N_1 and N_2 however, this is a subject to further study. On the side of PDFs we aim for a direct interface to the LHAPDF grids [28] which will allow the user to freely choose any particular PDF set provided within this framework. Recent results [29] on N space input parametrizations also allow for more flexible input PDF parametrisations in QCD-PEGASUS. Further improvements with respect to the speed of the code are foreseen and together with an upgrade on the input PDFs this code can become a tool for PDF fits, where fast and accurate evaluations of cross sections are needed. The current version of the c++ code can be downloaded from `http://www-zeuthen.desy.de/~kpetra/sbp`.

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