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Fluid/gravity model for the chiral magnetic effect

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We consider the STU model as a gravity dual of a strongly-coupled plasma with multiple anomalous $U(1)$ currents. In the bulk we add additional background gauge fields to include the effects of external electric and magnetic fields on the plasma. Reducing the number of chemical potentials in the STU model to two and interpreting them as quark and chiral chemical potential, we obtain a holographic description of the chiral magnetic and chiral vortical effects (CME and CVE) in relativistic heavy ion collisions. These effects formally appear as first-order transport coefficients in the electromagnetic current. We compute these coefficients from our model using fluid/gravity duality. We also find analogous effects in the axial current. Finally, we briefly discuss a variant of our model, in which the CME/CVE is realized in the late-time dynamics of an expanding plasma.

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I. INTRODUCTION

In the past two years, the chiral magnetic effect (CME) [1, 2] has received much attention in lattice QCD [3], hydrodynamics [4]-[7] and holographic models [8]-[14]. The CME states that, in the presence of a magnetic field $\mathbf{B}$, an electromagnetic current of the type

$$\mathbf{J} = C\mu_5 \mathbf{B}\,, \qquad C = \frac{N_c}{2\pi^2} \tag{1}$$

is generated in the background of topologically nontrivial gluon fields. This could possibly explain the charge asymmetry observed in heavy ion collisions at RHIC [15].

A hydrodynamic description of the CME has recently been found in [4] (see also [5]), using techniques developed in [6]. This model contains two $U(1)$ currents, an axial and a vector one, which are assumed to be conserved, at least in the absence of electric fields. This allows to introduce the corresponding chemical potentials, μ and μ_5, and the CME was shown to arise as a first-order transport coefficient, κ_B, in the constitutive equation for the electromagnetic current, $\Delta j^\mu = \kappa_B B^\mu$ with $\kappa_B = C\mu_5$. Since the CME is accompanied by a change in entropy, it is basically a non-equilibrium process.

There have also been several proposals for a holographic description of the CME. In the early works [8, 9], the axial anomaly was not realized in covariant form and the electromagnetic current was not strictly conserved. Aiming at restoring the conservation of the electromagnetic current, ref. [13] introduced the Bardeen counterterm into the action. However, as shown in [11, 13] for some standard AdS/QCD models, this typically leads to a vanishing electromagnetic current.

In [11, 12] the problem was traced back to the difficulty of introducing a chemical potential conjugated to a non-conserved charge. Ref. [11] therefore suggested a modification of the action in which the axial charge is conserved. This charge is however only gauge invariant when integrated over all space in *homogeneous* configurations [12], while the charge separation in heavy-ion collisions is clearly inhomogeneous. In contrast, ref. [12] introduced a chiral chemical potential dual to a gauge invariant current, despite it being anomalous. This required a singular bulk gauge field at the horizon, a phenomenon which seems to be generic in AdS black hole models of the CME.

In this letter, we propose a different approach which is based on the fluid/gravity correspondence [16] rather than a static (AdS/QCD) model. The fluid/gravity duality is more flexible since the hydrodynamic gradient expansion captures (small) deviations from equilibrium. This includes the CME and the change of the chiral charge density due to the anomaly $E \cdot B$ as first- and second-order effects, respectively. This allows us to introduce chemical potentials even for anomalous currents.

Our main goal is to construct a holographic dual of the hydrodynamic two-charge model of ref. [4]. We will start from the three-charge STU model [17] which we take as a prototype of an AdS black hole background with several $U(1)$ charges. We consider it as a *phenomenological* model of a strongly-coupled plasma with multiple chemical potentials, *i.e.* we prescind from the strict string theory interpretation of the three $U(1)$'s as R-charges inside the $SO(6)_R$ R-symmetry of $\mathcal{N} = 4$ super-Yang-Mills theory. This allows us to interpret one of them as an axial $U(1)$ charge and the other two as a single vector $U(1)$ charge. These charges are dual to μ and μ_5 required in the hydrodynamic description [4].

We proceed as follows. First, we show that the two-charge model of [4] can be considered as a special case of the hydrodynamic three-charge model of [6, 18]. Next, we reproduce the relevant magnetic conductivities in this three-charge model from the dual STU model (plus background gauge fields), using fluid/gravity duality [16]. We then reduce the model to two charges and recover the CME (*i.e.* κ_B of [4]) as well as other related effects. Finally, we present a time-dependent version of the STU model dual to a boost-invariant expanding plasma. –

Note added: After this work was completed, we learned of [27], in which similar transport coefficients are holographically computed from Kubo formulas.

II. CME AND CVE IN HYDRODYNAMICS

Hydrodynamics of a $U(1)^3$ plasma. The hydrodynamic regime of relativistic quantum gauge theories with triangle anomalies has been studied in [6, 18]. The anomaly coefficients are usually given by a totally symmetric rank-3 tensor C^{abc} and the hydrodynamic equations are

$$\partial_\mu T^{\mu\nu} = F^{a\nu\lambda} j^a_\lambda \,, \qquad \partial_\mu j^{a\mu} = C^{abc} E^b \cdot B^c \,, \quad (2)$$

where $E^{a\mu} = F^{a\mu\nu} u_\nu$, $B^{a\mu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F^a_{\alpha\beta}$ $(a = 1, 2, 3)$ are electric and magnetic fields, and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$ denotes the gauge field strengths. The stress-energy tensor $T^{\mu\nu}$ and $U(1)$ currents $j^{a\mu}$ are

$$\begin{aligned} T^{\mu\nu} &= (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu} \,, \\ j^{a\mu} &= \rho^a u^\mu + \nu^{a\mu} \,, \end{aligned} \quad (3)$$

where $\tau^{\mu\nu}$ and $\nu^{a\mu}$ denote higher-gradient corrections. ρ^a, ϵ and P denote the charge densities, energy density and pressure, respectively.

In the presence of E- and B-fields the first-order correction of the $U(1)$ currents is given by

$$\nu^{a\mu} = \xi^a_\omega \omega^\mu + \xi^{ab}_B B^{b\mu} + \ldots \,, \quad (4)$$

where $\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \partial_\lambda u_\rho$ is the vorticity. The ellipses indicate further terms involving electric fields. The conductivities ξ^a_ω and ξ^{ab}_B were first introduced in [6, 22] and are given by [18] (see also [6, 19])

$$\xi^a_\omega = C^{abc}\mu^b\mu^c - \frac{2}{3}\rho^a C^{bcd}\frac{\mu^b\mu^c\mu^d}{\epsilon + P} \,, \quad (5)$$

$$\xi^{ab}_B = C^{abc}\mu^c - \frac{1}{2}\rho^a C^{bcd}\frac{\mu^c\mu^d}{\epsilon + P} \,, \quad (6)$$

where μ^a are the three chemical potentials. These conductivities are specific for relativistic quantum field theories with quantum anomalies and do not appear in non-relativistic theories [6].

Magnetic and vortical Effects. For the hydrodynamical description of the chiral magnetic effect, we only need an axial and a vector chemical potential, μ_5 and μ. The three-charge model can be reduced to one with two charges by choosing the following identifications:

$$\begin{aligned} A^A_\mu &= A^1_\mu \,, & A^V_\mu &= A^2_\mu = A^3_\mu \,, \\ \mu_5 &= \mu^1 \,, & \mu &= \mu^2 = \mu^3 \,, \\ j^\mu_5 &= j^{1\mu} \,, & j^\mu &= j^{2\mu} + j^{3\mu} \,, \end{aligned} \quad (7)$$

and $C^{123} = C^{(123)} = \frac{C}{2}$. In the absence of axial gauge fields A^A_μ (which are not required), (2) simplifies to

$$\partial_\mu T^{\mu\nu} \simeq F^{V\nu\lambda} j_\lambda \,, \quad \partial_\mu j^\mu_5 = C E^\lambda B_\lambda \,, \quad \partial_\mu j^\mu \simeq 0 \,, \quad (8)$$

where $E^\mu \equiv F^{V\mu\nu} u_\nu$, $B^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F^V_{\alpha\beta}$. The symbol "$\simeq$" indicates that the equation only holds for $A^A_\mu = 0$.

Let us also define

$$\begin{aligned} &\rho_5 = \rho^1 \,, \quad \rho = \rho^2 + \rho^3 \,, \\ &\kappa_\omega = \xi^2_\omega + \xi^3_\omega \,, \quad \kappa_B = \xi^{22}_B + \xi^{23}_B + \xi^{32}_B + \xi^{33}_B \,, \\ &\xi_\omega = \xi^1_\omega \,, \quad \xi_B = \xi^{12}_B + \xi^{13}_B \,. \end{aligned} \quad (9)$$

Then from (3)–(6) we get the constitutive equations

$$\begin{aligned} j^\mu &= \rho u^\mu + \kappa_\omega \omega^\mu + \kappa_B B^\mu \,, \\ j^\mu_5 &= \rho_5 u^\mu + \xi_\omega \omega^\mu + \xi_B B^\mu \,, \end{aligned} \quad (10)$$

with coefficients

$$\begin{aligned} &\kappa_\omega = 2C\mu\mu_5\left(1 - \frac{\mu\rho}{\epsilon + P}\right), \quad \kappa_B = C\mu_5\left(1 - \frac{\mu\rho}{\epsilon + P}\right), \\ &\xi_\omega = C\mu^2\left(1 - 2\frac{\mu_5\rho_5}{\epsilon + P}\right), \ \xi_B = C\mu\left(1 - \frac{\mu_5\rho_5}{\epsilon + P}\right), \end{aligned} \quad (11)$$

which to leading order are in agreement with [4].

The leading term in κ_B is nothing but the *chiral magnetic effect* (CME) [1, 2], $\kappa_B = C\mu_5$. There is a second effect given by the term $\kappa_\omega = 2C\mu\mu_5$ which has recently been termed *chiral vortical effect* (CVE) [21]. The CVE states that, if the liquid rotates with some angular velocity $\vec{\omega}$, an electromagnetic current is induced along $\vec{\omega}$. – There are analogous effects in the axial current j^μ_5. The leading term in ξ_B, $\xi_B = C\mu$, generates an axial current parallel to the magnetic field. There is also a vortical effect given by $\xi_\omega = C\mu^2$. This term describes chirality separation through rotation [7]. We may refer to these effects as quark magnetic (QME) and quark vortical effects (QVE) since ξ_ω/μ and ξ_B are proportional to the quark chemical potential μ (while κ_ω/μ and $\kappa_B \propto \mu_5$).

We may also shift all anomalies in (2) entirely into the current $j^{1\mu}$ $(= j^\mu_5)$ by adding Bardeen currents,

$$\begin{aligned} j'^\mu &\equiv j^\mu + j^\mu_B \,, \qquad j'^\mu_5 \equiv j^\mu_5 + j^\mu_{5,B} \,, \\ j^\mu_B &= c_B \varepsilon^{\mu\nu\lambda\rho}(A^V_\nu F^A_{\lambda\rho} - 2A^A_\nu F^V_{\lambda\rho}) \,, \\ j^\mu_{5,B} &= c_B \varepsilon^{\mu\nu\lambda\rho} A^V_\nu F^V_{\lambda\rho} \,, \end{aligned} \quad (12)$$

with $c_B = -C/2$ such that (2) becomes $(C' = 3C)$

$$\partial_\mu T^{\mu\nu} \simeq F^{V\nu\lambda} j'_\lambda \,, \quad \partial_\mu j'^\mu_5 = C' E^\lambda B_\lambda \,, \quad \partial_\mu j'^\mu = 0 \,.$$

This is formally identical to (8), leading again to (11).

III. FLUID/GRAVITY MODEL OF THE CME

Three-charge STU model with external fields. In the following we propose the three-charge STU-model [17] as a holographic dual gravity theory for the (chiral) magnetic and vortical effects in a relativistic fluid. We begin by showing that the first-order transport coefficients (5) and (6) of the $U(1)^3$ theory can be reproduced from the STU model [17]. Subsequently, we will reduce it to a two-charge model and recover the conductivities (11).

The Lagrangian of the STU-model is given by [17]

$$\mathcal{L} = R - \frac{1}{2}G_{ab}F^a_{MN}F^{bMN} - G_{ab}\partial_M X^a \partial^M X^b \qquad (13)$$
$$+ \frac{1}{24}\sqrt{-g_5}\epsilon^{MNPQR}S_{abc}F^a_{MN}F^b_{PQ}A^c_R + 4\sum_{a=1}^{3}\frac{1}{X^a}\,,$$

where

$$G_{ab} = \frac{1}{2}\delta_{abc}(X^c)^{-2}, \qquad X^1X^2X^3 = 1\,. \qquad (14)$$

g_{MN}, X^a and A^a_M $(M, N = 0, 1, ..., 4,\ a, b, c = 1, 2, 3)$ denote the metric, three scalars and $U(1)$ gauge fields, respectively.

The boosted black brane solution corresponding to the three-charge STU model is given by [17]

$$ds^2 = -H^{-\frac{2}{3}}(r)f(r)u_\mu u_\nu dx^\mu dx^\nu - 2H^{-\frac{1}{6}}(r)u_\mu dx^\mu dr$$
$$+ r^2 H^{\frac{1}{3}}(r)\left(\eta_{\mu\nu} + u_\mu u_\nu\right)dx^\mu dx^\nu\,,$$
$$A^a = \left(A^a_0(r)\,u_\mu + \mathcal{A}^a_\mu\right)dx^\mu\,, \quad X^a = \frac{H^{\frac{1}{3}}(r)}{H_a(r)}\,, \qquad (15)$$

where $(\mu, \nu = 0, 1, 2, 3)$

$$f(r) = -\frac{m}{r^2} + r^2 H(r)\,, \quad H(r) = \prod_{a=1}^{3} H^a(r)\,,$$
$$H^a(r) = 1 + \frac{q^a}{r^2}\,, \qquad A^a_0(r) = \frac{\sqrt{mq^a}}{r^2 + q^a}\,, \qquad (16)$$

and u_μ is the four-velocity of the fluid with $u_\mu u^\mu = -1$. Following [6], we have formally introduced constant background gauge fields $\mathcal{A}^a_\mu$, which are necessary for the computation of the transport coefficients ξ^{ab}_B.

We now use the standard procedure [16] to holographically compute the transport coefficients ξ^a_ω and ξ^{ab}_B. We closely follow [20] which has already determined ξ^a_ω from the STU-model (but not ξ^{ab}_B relevant for the CME). Working in the frame $u_\mu = (-1, 0, 0, 0)$ (at $x^\mu = 0$), we slowly vary u_μ and $\mathcal{A}^a_\mu$ up to first order as

$$u_\mu = (-1, x^\nu\partial_\nu u_i), \qquad \mathcal{A}^a_\mu = (0, x^\nu\partial_\nu\mathcal{A}^a_i)\,. \qquad (17)$$

We may also vary m and q in this way, but it turns out that varying these parameters has no influence on the transport coefficients ξ^a_ω and ξ^{ab}_B.

As a consequence, the background (15) is no longer an exact solution of the equations of motion but receives higher-order corrections. The corrected metric and gauge fields can be rewritten in Fefferman-Graham coordinates and expanded near the boundary (at $z = 0$) as

$$ds^2 = \frac{1}{z^2}\left(g_{\mu\nu}(z, x)\,dx^\mu dx^\nu + dz^2\right)\,,$$
$$g_{\mu\nu}(z, x) = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}(x)\,z^2 + g^{(4)}_{\mu\nu}(x)\,z^4 + \ldots\,,$$
$$A^a_\mu(z, x) = A^{a(0)}_\mu(x) + A^{a(2)}_\mu(x)\,z^2 + \ldots\,. \qquad (18)$$

The first-order gradient corrections of the energy-momentum tensor and $U(1)$ currents (3) are then read off from [23, 24]

$$T_{\mu\nu} = \frac{g^{(4)}_{\mu\nu}(x)}{4\pi G_5} + c.t.\,, \quad j^\mu_a = \frac{\eta^{\mu\nu}A^{(2)}_{a\nu}(x)}{8\pi G_5} + \hat{j}^\mu_a\,, \qquad (19)$$
$$\hat{j}^\mu_a = -\frac{S_{abc}}{32\pi G_5}\epsilon^{\mu\nu\rho\sigma}A^{(0)}_{b\nu}(x)\partial_\rho A^{(0)}_{c\sigma}(x)\,, \qquad (20)$$

where *c.t.* denotes diagonal corrections to the energy-momentum tensor due to counterterms. The term $\hat{j}^\mu_a$ will be discussed below around (25).

The computation of the corrected metric and gauge fields is very similar to that in [20]. At zeroth order, we get the same expressions for the pressure P and charge densities ρ_a as in [20], $P \equiv m/16\pi G_5$ and $\rho_a \equiv \sqrt{mq_a}/8\pi G_5$, which may be combined to give

$$\frac{\sqrt{mq^a}}{2m} = \frac{\rho^a}{\epsilon + P} \qquad (\epsilon = 3P)\,. \qquad (21)$$

At first order, the transport coefficients ξ^a_ω and ξ^{ab}_B are read off from the near boundary behavior of A^a_μ via (19),

$$\xi^a_\omega = \frac{1}{16\pi G_5}\left(S^{abc}\mu^b\mu^c - \frac{\sqrt{mq^a}}{3m}S^{bcd}\mu^b\mu^c\mu^d\right)\,, \qquad (22)$$
$$\xi^{ab}_B = \frac{1}{16\pi G_5}\left(S^{abc}\mu^c - \frac{\sqrt{mq^a}}{4m}S^{bcd}\mu^c\mu^d\right)\,, \qquad (23)$$

with $\mu^a \equiv A^a_0(r_H) - A^a_0(\infty)$. Using a standard relation between the anomaly coefficients C_{abc} and the Chern-Simons parameters S_{abc}, $C_{abc} = S_{abc}/16\pi G_5$, as well as (21), we find that the holographically computed transport coefficients (22) and (23) coincide exactly with those found in hydrodynamics, (5) and (6).

Holographic magnetic and vortical effects. In order to obtain the holographic versions of the magnetic and vortical effects (11), we reduce the STU model to a two-charge model using the same identities as in hydrodynamics, (7) and (9). In particular, we define (vector and axial) gauge fields A^V_μ and A^A_μ, chemical potentials μ and μ_5, and currents j^μ and j^μ_5 as in (7) but now with $\mu^a \equiv A^a_0(r_H) - A^a_0(\infty)$, and A^a and j^μ_a as in (15) and (19), respectively. Moreover, we set $S_{abc} = S/2$ with $S = 16\pi G_5 C$ and keep S_{abc} general, as in [6].

Using also the identifications (9), but now for the holographically computed transport coefficients (22) and (23), we get

$$\kappa_\omega = 2C\mu\mu_5\left(1 - \mu\sqrt{\frac{q}{m}}\right)\,, \quad \kappa_B = C\mu_5\left(1 - \mu\sqrt{\frac{q}{m}}\right)\,,$$
$$\xi_\omega = C\mu^2\left(1 - \mu_5\sqrt{\frac{q_5}{m}}\right)\,, \quad \xi_B = C\mu\left(1 - \frac{\mu_5}{2}\sqrt{\frac{q_5}{m}}\right)\,,$$

in agreement with (11). This shows that the CME, CVE, *etc.* are realized in the STU-model, when appropriately reduced to a two-charge model.

Comments. To get an anomaly-free three-point function for j^μ, we also need to add the Bardeen currents (12). As in hydrodynamics, this does not change the structure of the transport coefficients. Note however that, together with (20), the Bardeen term gives rise to additional contributions of the type

$$\begin{aligned}\Delta j^\mu &= \hat{j}^{2\mu} + \hat{j}^{3\mu} + j^\mu_B \\ &\supset \varepsilon^{\mu\nu\rho\sigma}(\mathcal{A}^A_\nu(x)\mathcal{F}^V_{\rho\sigma}(x) - \mathcal{A}^V_\nu(x)\mathcal{F}^A_{\rho\sigma}(x)) \,. \end{aligned} \quad (24)$$

If we choose $\mathcal{A}^A_\nu = \alpha^A u_\nu$ (at $x = 0$) with some constant $\alpha^A \neq 0$, we get terms of the type $\mathcal{A}^A_0 B^\mu$ which are forbidden by electromagnetic gauge invariance [11]. As in [11, 12], we are therefore forced to switch off the axial background gauge field $\mathcal{A}^A_\mu$ completely, $\alpha^A = 0$. This corresponds to a non-vanishing gauge field at the horizon, as in [12]. – There is also an additional term in j^μ_5,

$$\Delta j^\mu_5 = \hat{j}^\mu_5 + j^\mu_{5,B} \propto \epsilon^{\mu\nu\rho\sigma}\mathcal{A}^V_\nu(x)\mathcal{F}^V_{\rho\sigma}(x) \,, \quad (25)$$

which reflects the effects of the chiral anomaly $E \cdot B$ on the charge density $\rho_5 \equiv j^0_5$ at second order.

To summarize the hydrodynamic effects, we note that the CME, CVE, *etc.* are first-order effects away from equilibrium, while the changes of the chiral charge density due to the anomaly appear at second order in j^5_μ.

Holographic time-dependent model for the CME. As a final remark, we present a time-dependent version of the STU-model. It is well-known that the hydrodynamic gradient expansion of a fluid is also realized in the *late-time* evolution of a boost-invariant expanding plasma *à la* [25]. In the dual time-dependent gravity background the gradient expansion appears as an expansion in time.

Recently, a time-dependent Reissner-Nordström-type solution was found in [26] which describes the late-time evolution of an expanding $\mathcal{N} = 4$ super Yang-Mills plasma with a *single* chemical potential. Similarly, we now construct a late-time solution from the boosted black brane solution (15) (dual to *three* chemical potentials). Proceeding as in [26], we assume the late-time behavior

$$m = \tilde{\tau}^{-4/3} m_0 \,, \qquad q^a = \tilde{\tau}^{-2/3} q^a_0 \quad (26)$$

for the parameters m and q^a and find the zeroth-order solution ($v = \tilde{\tau}^{1/3} r$)

$$\begin{aligned} ds^2 &= -H^{-\frac{2}{3}}(v) f(v) d\tilde{\tau}^2 + 2H^{-\frac{1}{6}}(v) d\tilde{\tau} dr \\ &\quad + H^{\frac{1}{3}}(v)\left((1 + r\tilde{\tau})^2 dy^2 + r^2 dx^2_\perp\right) \,, \end{aligned} \quad (27)$$

$$A^a = -A^a_0(v) d\tilde{\tau} + \mathcal{A}^a_\mu dx^\mu \,, \quad X^a = \frac{H^{\frac{1}{3}}(v)}{H_a(v)} \,,$$

with

$$f(v) = r^2\left(-\frac{m_0}{v^4} + H(v)\right) \,, \quad H(v) = \prod_{a=1}^{3} H^a(v) \,,$$

$$H^a(v) = 1 + \frac{q^a_0}{v^2} \,, \qquad A^a_0(v) = \frac{1}{\tilde{\tau}^{1/3}} \frac{\sqrt{m_0 q^a_0}}{v^2 + q^a_0} \,. \quad (28)$$

This background is a good approximation of the full time-dependent solution at large $\tilde{\tau}$ (as we have explicitly checked using computer algebra for $\mathcal{A}^1_\mu = 0$, $\mathcal{A}^{2,3}_\mu = (0, -x_2 B, 0, 0)$). At smaller $\tilde{\tau}$, it receives subleading corrections in $\tilde{\tau}^{-2/3}$ corresponding to higher-order gradient corrections. It has been shown many times that first-order transport coefficients appear in the first correction in $\tilde{\tau}^{-2/3}$. It therefore follows from the above discussion that the conductivities ξ^{ab}_B, relevant for the CME, appear in the first-order correction to the solution (28).

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