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Decentralized Coordination by Exchange of Primal Information

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We present a new approach for decentralized coordination in light of asymmetric information. The basic idea is to perform an iterative exchange of primal information, like proposals for purchase and supply quantities, in order to identify improvements over an initial, uncoordinated solution. Unlike most existing approaches, it will coordinate decisions of two or more decentralized parties, provided that the decision problems can be modeled and solved as linear or mixed-integer programs, while it does not require a central decision-making entity. We present a coordination scheme that identifies the system-wide optimum in a finite number of iterations, provided that all but one decentralized party report their cost changes incurred by new proposals and that the models of these parties can be formulated as linear programs, while the model of the remaining party may be of a mixed-integer type. In computational tests, we find that 20 iterations are sufficient to obtain considerable improvements over initial master plans in supply chains consisting of one buyer and one or multiple suppliers. Moreover, we show that the information exchange can be embedded into a strategy-proof mechanism, for which a lower bound on the efficiency can be derived.

1. Introduction

In decentralized organizations, decision makers are often misled due to double marginalization, a phenomenon that has already been recognized by Spengler (1950). Myopically optimizing their own profits, decentralized parties determine an allocation of interdependent or *central* resources that may be suboptimal for the whole system. Depending on the degree of this suboptimality, there is a need for the establishment of *coordination mechanisms*, which induce the decentralized parties to implement actions that result in system-wide gains. One such approach is revealing private information to enable a central entity

to determine an improved allocation of the central resources (see, e.g., Harris et al. 1982 and the example of Shirodkar and Kempf 2006).

However, decision makers frequently do not accept sharing of relevant data. Some private data (especially capacities and absolute cost figures) may be regarded sensitive because they constitute a strategic advantage for bargaining, which is lost after revelation. Hence, most of the recent studies assume that decisions are only made by decentralized parties and focus on the development of incentive schemes that align the decentralized objectives with the system-wide objective (e.g., Baldenius et al. 1999 and Chen et al. 2001). This paper also aims at improving decentralized decision-making without the involvement of a central entity. Additional assumptions are that data are deterministic, decentralized information is private, and that decision problems can be modeled as linear programs (LP) or mixed-integer programs (MIP). For this general setting, we propose a coordination mechanism that implies an iterative disclosure of primal, insensitive information guided by a *coordination scheme* (also abbreviated by “scheme” in the following), i.e., a set of rules that specify the information disclosed and the sequence of its exchange. With this additional primal information, the decentralized parties are able to generate and to identify an improvement (if one exists) over an initial solution. A new solution incurs a reallocation of the central resources (e.g., supply quantities). Hence, the mechanism must cope with two interdependent problems simultaneously: the generation and identification of an improved solution as well as the establishment of incentives for truthful information exchange.

The scheme presented here requires an iterative exchange of proposals regarding the use of the central resources and – for all but one decentralized party – the associated cost changes compared to the uncoordinated central resource allocation. The proposals are generated by mathematical programming models. The basic idea is to identify proposals that allow for considerable reductions in system-wide costs and at the same time address regions of the solution space that have not been examined in previous iterations. The system-wide optimum is found within a finite number of iterations if the optimization models of all but one decentralized party can be formulated as LP. In addition, we propose a modified version of this

scheme, which allows for discrete decisions in the models of the decentralized parties and shows improved convergence in general. Computational tests for the coordination of master plans in supply chains of one buyer and one or multiple suppliers result in considerable reductions of the system-wide costs after only 20 iterations.

Furthermore, in order to ensure truthful reporting of cost changes, we propose a strategy-proof, individually rational, and budget-balanced mechanism. This mechanism implies that one party receives the total surplus from coordination and pays previously fixed lump sums to the others. We assume that each cost-reporting party has prior, incomplete knowledge about the *marginal surplus*, i.e., the increase of the system-wide surplus that results from this party's participation in coordination. If the cost-reporting parties are risk-neutral and their prior knowledge is uniformly distributed, at least $3/4$ of the marginal surpluses can be realized on average.

In summary, the assumptions of our approach fit for a setting that has been identified by Stadtler (2007) in the context of collaborative supply chain planning as an existing gap in research: coordination of decentralized organizations of an arbitrary number of self-interested parties (e.g., a buyer-supplier or an n -tier supply chain) that hold and keep private information and rely on complex LP or MIP for their operational planning. This includes, for example, the alignment of different functional objectives in an intra-organizational supply chain (e.g., Karabuk and Wu 2002) as well as the coordination of material flows across legally separated companies in an inter-organizational supply chain with conflicting objectives (like a supply chain consisting of a manufacturer and a distributor, which is a fundamental issue that has not been addressed in the literature, e.g., Chen 2004).

The remainder of this paper is organized as follows: §2 reviews the relevant literature and points out some principal drawbacks of the exchange of dual information for coordination purposes. In §3 we describe the scheme and its convergence behavior. §4 presents a modified version of the scheme, which shows improved convergence and is applicable for multiple, decentralized MIP. The mechanism is outlined in §5.

In §6 we show how to customize the scheme to coordinate master plans in supply chains and provide the results of computational tests. §7 concludes the paper.

2. Literature Review

Coordination mechanisms in the form of contracts have been dealt with extensively in the literature (see Cachon 2003 and Tsay 1998 for detailed reviews). Frequent assumptions are demand uncertainty and symmetric information. Papers that consider asymmetric information include Schenk-Mathes (1995), Corbett and de Groote (2000), Cachon and Lariviere (2001), Corbett et al. (2004), Cachon and Zhang (2006), Özer and Wei (2006), and Taylor (2006). In contrast to the setting analyzed here, all of these papers focus on one-sided information asymmetry, rely on analytic decision models, and consider few uncertain or unknown parameters.

Other papers explicitly address the coordination of parties that employ mathematical programming models for decision-making. An elegant method is the application of classical decomposition techniques (see Dantzig and Wolfe 1960 and Benders 1962) or their modifications. This idea has a long tradition, beginning with Dantzig and Wolfe when they interpreted their decomposition procedure as decentralized decision-making. More recently, Karabuk and Wu (2002) addressed the intrafirm coordination of capacity planning in the semiconductor industry with demand and capacity uncertainty and applied an Augmented Lagrangian procedure. Arikapunam and Veeramani (2004) proposed a coordination scheme based on the L-shaped method. Ertogral and Wu (2000) and Walther, Schmid, and Spengler (2007) rely on subgradient procedures. All of these papers (implicitly) assume truthful exchange of information. Self-interested behavior (including potential incentives for distorted information exchange) has been taken into account by Kutanoglu and Wu (1999), Fan et al. (2003), and Guo et al. (2007). In line with Dantzig-Wolfe decomposition, they propose that the decentralized parties submit bids for the use of the central resources, while a central entity determines the market-clearing prices. The applicability of these mechanisms in decentralized environments, however, is limited due to the need for a trusted third party and a large number of

participants (e.g., in Fan et al. 2003, incentive compatibility can only be achieved if the number of participants approaches infinity).

Exchange of dual information is essential for approaches based on classical decomposition techniques. However, we argue that limiting the exchange to primal information is preferable for coordination purposes in our setting. If dual information is generated by decentralized models (e.g., Benders decomposition) and if these models contain integer variables, some difficulties arise. Early research has shown that for integer programming models, appropriate dual functions can be identified (Wolsey 1981). In contrast to dual prices of linear programming, these dual functions involve a large amount of data, which, depending on the algorithm used for their generation, may increase exponentially with the problem size. The disclosure of large amounts of data, however, may not be accepted by decision makers if they want to ensure that their private data will not be reconstructed by other parties (e.g., using inverse optimization, see Troutt et al. 2006). Apart from that, the computational effort for the identification of dual functions is considerable (e.g., Guzelsoy and Ralphs 2007). Alternatively, dual prices can be generated by a master problem and then be communicated to the decentralized parties (e.g., Dantzig-Wolfe decomposition). In this case, however, dual prices are not an effective guidance toward a system-wide improvement if duality gaps are large. Finally, we believe that the exchange of primal information has generally a better chance of acceptance in practice because decision makers understand clearly which information they actually disclose. In contrast, not all managers have an understanding of what dual information signifies.

Few papers have addressed the coordination of mathematical programming problems without the exchange of dual information. Schneeweiss and Zimmer (2004) proposed the use of hierarchical anticipation. In Fink (2005), a mediator generates proposals that decentralized parties accept or reject. Both approaches rely on strong assumptions about information availability (the possibility of estimating the follower's cost parameters, and the existence of a mediator knowing decentralized restrictions, respectively). In addition, the incentive compatibility of these approaches has not been shown. The papers most related to ours are those of Dudek and Stadtler (2005, 2007), which devise coordination schemes for interde-

pendent lot-sizing problems. Our research shares the idea of an iterative exchange of supply proposals with these papers, but differs on two important points. First, our scheme converges in a finite number of iterations for a general class of optimization problems, whereas Dudek and Stadtler only showed improvements by computational tests for their capacitated lot-sizing problems. Second, our approach is not limited to a team setting with frank disclosure of information.

3. Decision Problem and Coordination Scheme

Consider an optimization problem with a block-angular structure:

$$\begin{aligned}
 \min Z_C &= \sum_{i=1}^I c_i^T x_i \\
 \text{(C) s.t.} \quad &\sum_{i=1}^I A_i x_i \leq b_0 \\
 &B_i x_i \leq b_i \quad \forall i = 1, \dots, I \\
 &x_i \in X_i \quad \forall i = 1, \dots, I.
 \end{aligned} \tag{1}$$

$P = \{1, \dots, I\}$ is the set of decentralized parties i . $A_i \in \mathbb{R}^{n \times o_i}$ and $B_i \in \mathbb{R}^{m_i \times o_i}$ are rational matrices, $c_i \in \mathbb{R}^{o_i}$, $b_i \in \mathbb{R}^{m_i}$, and $b_0 \in \mathbb{R}^n$ are real vectors. The components of vectors $x_i \in \mathbb{R}^{o_i}$ are nonnegative decision variables that may be restricted to binary or integer values. After skipping the joint constraint (1), (C) decomposes into I decentralized problems (DP _{i}). We assume that (C) has a feasible solution. In case each party i implements the optimal solution of (DP _{i}) without regard to the actions of the others, this mostly will result in excess use of the central resource b_0 and infeasibility of the whole system. To overcome this problem, it is common practice to rely on simple rules that determine a feasible but often suboptimal solution. Examples include solving the decentralized problems in a given sequence (e.g., in a supply chain, often the upstream parties determine the supply quantities) or allocating the central resources according to a prespecified scheme. (Note that for implementing both these rules and the scheme proposed below, b_0 has to be publicly known. This is often a natural assumption; for the coordination of the material flow in

supply chains, $b_0=0$ holds because the quantities of the items delivered by the supplier have to be equal to the quantities received by the buyer.) In the following, we assume that an initial central resource allocation has been determined and devise a scheme that identifies the system-wide optimum if all but one decentralized party utilize LP for generating their decentralized plans.

The scheme specifies a sequential generation of proposals regarding the use of the central resources by parties $1, \dots, I-1$. The associated cost changes are reported *unilaterally*, i.e., parties $1, \dots, I-1$ report to party I both the cost changes of their own proposals compared to the initial solution as well as those resulting from a potential implementation of the proposals generated by I . Other decentralized data are kept private. In light of the almost complete information asymmetry, the key feature of the scheme is the search strategy for promising new proposals. We consider a proposal as promising if it shows a potential for system-wide cost savings and addresses regions of the solution space that have not been investigated in previous iterations of the scheme. Using model (CS1_{*i*}), each party $i=1, \dots, I-1$ can generate proposals with these characteristics.

$$\max Z_{CS1_i} = c_i^T (x_i^{st} - x_i) + k_i^T A_i (x_i^{st} - x_i) \quad (2)$$

$$(CS1_i) \quad \text{s.t.} \quad k_i^T A_i (x_i^e - x_i^{st}) \geq -c_i^T (x_i^e - x_i^{st}) \quad \forall e = 1, \dots, \bar{e}_i \quad (3)$$

$$k_i \leq M_i \quad (4)$$

$$B_i x_i \leq b_i \quad (5)$$

$$x_i \in X_i \quad (6)$$

$$k_i \geq 0. \quad (7)$$

This model aims at maximizing the potential savings of a solution x_i compared to a fixed starting solution x_i^{st} (note that the constant $c_i^T x_i^{st}$ is included for ease of exposition in (2)) and assigns penalties and bonuses for changes in the use of the central resources. Variables $k_i \in \mathbb{R}^n$ are endogenous prices for these changes. They are determined by constraints (3) such that $Z_{CS1_i} \leq 0$ with the repetition of a previous solu-

tion $x_i^e \in X_i^E$. X_i^E is the set of existing solutions for party i , including the initial solution and those identified in previous iterations of the scheme. The number of elements of X_i^E is denoted by \bar{e}_i . In order to avoid unboundedness, constraint (4) establishes an upper bound for k_i . M_i is a vector made up of big numbers that exceed marginal cost savings resulting from increases in central resource use (i.e., $M_i^T A_i (x_i^1 - x_i^2) > -c_i^T (x_i^1 - x_i^2)$ must hold for any two feasible solution vectors x_i^1 and x_i^2). Constraints (5) and (6) ensure the compliance of the decentralized restrictions of i . Finally, (7) is a non-negativity constraint.

After solving (CS1_i), the new supply proposal $\pi_i = A_i x_i$ together with $\chi_i = c_i^T (x_i - x_i^{init})$, the associated cost change compared to x_i^{init} , are communicated to party I . x_i^{init} is the initial solution and is determined based on the initial resource allocation π_i^{init} and a further model (CS-EVAL_i) described below.

With a fixed, feasible value for k_i , (CS1_i) corresponds to the subproblem of Dantzig-Wolfe decomposition. The fact that k_i is determined endogenously in (CS1_i) and not supplied by a master problem is the crucial point that allows us to limit the exchange to primal information. Lemma 1 makes this relationship explicit. It shows that the outcome of (CS1_i) is a feasible vertex solution to (DP_i), provided that the costs of the starting solution x_i^{st} do not exceed the costs of a convex combination of previous solutions with central resource use equal to or smaller than that of x_i^{st} . Define X_i^V as the set of vertex solutions to (DP_i).

All proofs have been relegated to the Appendix.

LEMMA 1. *Let (DP_i) only comprise continuous variables. Then:*

1. *There is a feasible solution to (CS1_i) if $c_i^T x_i^{st} \leq c_i^T \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e$ for all scalars $\lambda_{ie} \geq 0$ ($e=1, \dots, \bar{e}_i$)*

$$\text{with } A_i x_i^{st} \geq A_i \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e \text{ and } \sum_{e=1}^{\bar{e}_i} \lambda_{ie} = 1.$$

2. *For x_i^* , the optimal solution to (CS1_i), $x_i^* \in X_i^V$ holds.*

Party I runs a different model, (CS2 _{I}). This model minimizes both the costs of party I and a convex combination (determined by variables λ_{ie} through (10) and (11)) of the other parties' costs of previous proposals π_i^e (8), while securing that the parties' use of the central resources – including that of party I – does not exceed the availability b_0 (9).

$$\min Z_{CS2_I} = c_I^T x_I + \sum_{i=1}^{I-1} \sum_{e=1}^{\bar{e}_i} \lambda_{ie} \chi_i^e \quad (8)$$

$$(CS2_I) \quad \text{s.t.} \quad A_I x_I + \sum_{i=1}^{I-1} \sum_{e=1}^{\bar{e}_i} \lambda_{ie} \pi_i^e \leq b_0 \quad (9)$$

$$\sum_{e=1}^{\bar{e}_i} \lambda_{ie} = 1 \quad \forall i = 1, \dots, I-1 \quad (10)$$

$$\lambda_{ie} \geq 0 \quad \forall i = 1, \dots, I-1, e = 1, \dots, \bar{e}_i \quad (11)$$

(5), (6).

Note that in (5) and (6), i is set to I here. (CS2 _{I}) yields $\pi_i = \sum_{e=1}^{\bar{e}_i} \lambda_{ie} \pi_i^e$, the new proposals for the use of the central resources by parties $i = 1, \dots, I-1$, which are communicated to these parties then. (CS2 _{I}) is closely related to the master problem of Dantzig-Wolfe decomposition. However, the decentralized decisions for party I and the allocation of the central resources are determined simultaneously in (CS2 _{I}). Note that this transformation is a general means of making ideas from classical decomposition techniques applicable for decentralized coordination without a central entity.

Finally, we state the third model used by parties $i = 1, \dots, I-1$ to evaluate party I 's last proposals π_i .

$$\min c_i^T x_i$$

$$(CS-EVAL_i) \quad \text{s.t.} \quad A_i x_i \leq \pi_i$$

(5), (6).

Relevant outcomes of this model are $\chi_i = c_i^T (x_i - x_i^{init})$, the cost change for party i due to a potential implementation of π_i , which is communicated to I then, and the solution x_i , that is used as the starting solution for the next application of (CS1 _{i}). Below, we summarize the single steps of the scheme and state the main result of this section.

Algorithm 1: Coordination scheme

- *Initialization*: For all parties $i=1, \dots, I-1$: Solve (CS-EVAL _{i}) with $\pi_i = \pi_i^{init}$
- *Iterations*: Repeat
 - $\forall i=1, \dots, I-1$: Solve (CS1 _{i}) with x_i^{st} as the last outcome of (CS-EVAL _{i}) and communicate π_i and χ_i to I
 - I : Solve (CS2 _{i}) and communicate π_i separately to $i=1, \dots, I-1$
 - $\forall i=1, \dots, I-1$: Solve (CS-EVAL _{i}) with π_i as the last proposal by I and communicate χ_i to I

Until $Z_{CS1_i} \leq 0 \forall i=1, \dots, I-1$ and Z_{CS2_i} has not been improved compared to the last run of (CS2 _{i}).

THEOREM 1. Following the scheme, the optimal solution to (C) can be identified within a finite number of iterations if the decision problems of the cost-reporting parties can be formulated as LP.

Theorem 1 is the first to show that coordination can be guaranteed by an exchange of only primal information. To achieve this, the interaction of a third party is not needed. Last but not least, finite convergence can even be achieved with discrete decisions for one decentralized party.

4. Extending the Applicability and Accelerating the Scheme

In this section, we propose some modifications to accelerate the convergence rate and to extend the scheme for settings with multiple parties running MIP. First, (CS1 _{i}) is modified to (CS1' _{i}).

$$\max Z_{CS1'_i} = c_i^T (x_i^{st} - x_i) - (k_i^T + \varepsilon) d_i \tag{12}$$

$$(CS1'_i) \quad \text{s.t.} \quad A_i x_i \leq A_i x_i^{st} + d_i \tag{13}$$

$$k_i^T A_i (x_i^e - x_i^{st}) \geq (-c_i^T (x_i^e - x_i^{st}))^+ \quad \forall e = 1, \dots, \bar{e}_i \quad (14)$$

$$d_i \geq 0 \quad (15)$$

(5)-(7).

We have introduced new variables $d_i \in \mathbb{R}^n$ depicting increments of the central resource use (13). As a consequence, only penalties, but no bonuses, are imputed in the second term of (12). Hence, (CS1'_i) will only generate proposals with cost savings of party i compared to x_i^{st} , which increases the probability for the identification of a system-wide improvement and thus tends to accelerate the convergence rate. The arbitrarily small penalty-cost factor ε helps to avoid exploring minor deviations that may occur if multiple optimal solutions to (CS1'_i) exist. The use of (14) instead of (3) assures feasible solutions for k_i , even if (DP_i) is an MIP. (For MIP, (3) may become infeasible since the condition stated in 1. of Lemma 1 is not necessarily fulfilled there.)

Second, party I runs (CS1''_I), a modification of (CS1'_i), in case (CS2_i) did not yield a new proposal.

$$\max Z_{CS1''_I} = c_I^T (x_I^{st} - x_I) - (k_I^T + \varepsilon) d_I$$

$$(CS1''_I) \quad \text{s.t.} \quad k_I^T A_I (x_I^e - x_I^{st}) \geq (-c_I^T (x_I^{st} - x_I^e))^+ \quad \forall i = 1, \dots, I-1, e = 1, \dots, \bar{e}_i \quad (16)$$

$$c_I^T x_I + \varepsilon \leq \sum_{i=1}^I c_i^T x_i^{best} \quad (17)$$

(5)-(7), (13), (15).

Again, i has to be replaced by I in (5)-(7), (13), and (15). Constraints (16) exploit the knowledge of party I about the cost changes previously reported by parties $1, \dots, I-1$. Especially for multiple cost-reporting parties, (16) permits a more accurate determination of the endogenous penalty costs than (14), since the cost changes of each single cost-reporting party can be considered. Constraint (17) limits the search to proposals with a potential for an improvement compared to x_i^{best} , the best solution found so far. Analogously to parties $i=1, \dots, I-1$, I evaluates the proposals of $i=1, \dots, I-1$ by (CS-EVAL_I) in each iteration.

Third, parties run models (CS1_i) and (CS1''_i) not only once, but successively with randomly chosen $x_i^{st} \in X_i^E$ until a new proposal with $Z_{CS1'_i} > 0$ or $Z_{CS1''_i} > 0$ has been identified or all previous proposals have been tried. That way, different regions of the solution space are searched, which helps mitigating difficulties due to the non-convexity of (DP_i) for MIP and thus constitutes a further advantage compared to the application of dual decomposition procedures in the setting considered by us.

Fourth, except for the first iteration, (CS2_i) is extended to (CS2'_i) by allowing deviations from the convex combination of the other parties' proposals at the expense of penalty costs (26).

$$\min c_I^T x_I + \sum_{i=1}^{I-1} \left(\sum_{e=1}^{\bar{e}_i} \lambda_{ie} \chi_i^e + d_i cp_i^T \right) \quad (25)$$

$$(CS2'_i) \quad \text{s.t.} \quad A_I x_I + \sum_{i=1}^{I-1} \left(\sum_{e=1}^{\bar{e}_i} \lambda_{ie} \pi_i^e - d_i \right) \leq b_0 \quad (26)$$

$$(5), (6), (10), (11), (13), (15).$$

As above, i has to be replaced by I in (5), (6), (13), and (15). To roughly anticipate the other parties' cost changes with deviations d_i , each element of vector $cp_i \in \mathbb{R}^n$, the unit penalty costs for changes in the central resource use, is set to $(c_i^T (x_i^{init} - x_i^{fst})) / dp_i$, with fst as a superindex denoting the first proposal generated by party i . dp_i is set to $\sum_{j=1}^n (A_{ij} (x_i^{fst} - x_i^{init}))^+$ if $c_i^T (x_i^{init} - x_i^{fst}) > 0$, and to $\sum_{j=1}^n (A_{ij} (x_i^{fst} - x_i^{init}))^-$ otherwise, with j as the dimensions of central resource use.

5. Coordination Mechanism

In this section, we present a mechanism for use in combination with the scheme proposed. The mechanism does not require the involvement of a third party and is strategy-proof, individually rational, and budget-balanced. A mechanism is individually rational and budget-balanced if parties will not incur any losses from participating and if the payments specified sum up to zero (see, e.g., Mas-Colell et al. 1995 for more detailed explanations). While strategy-proofness, which means that truth-telling is a weakly

dominant strategy for parties (if they are required to reveal their valuations), is favorable for practical interfirm coordination, the two latter properties appear to be indispensable (e.g., Chu and Shen 2006).

In the mechanism, we apply the following rule for sharing the *surplus from coordination*, i.e., the difference between the costs of the initial, uncoordinated solution and those of the solution determined by the scheme: if a coordinated solution is implemented, then the party I with knowledge about the other parties' cost changes (we abbreviate this party by IP="informed party" in the following) receives the total surplus from coordination and pays previously negotiated lump sums to parties $1, \dots, I-1$ (RP $_i$ ="reporting party i "; we omit the index i when we address a two-party setting). Note that the fact that only the surplus is shared implies that parties incurring cost increases by the implementation of coordinated proposals are compensated accordingly, which is necessary for the individual rationality of the mechanism.

The parties' actions within the mechanism can be divided into three steps: First, each RP $_i$ determines the lump sum L_i , which he receives in case of successful coordination and informs the IP accordingly. Second, parties exchange information along the lines of the scheme. The RP $_i$ decide in this step whether to report their cost changes truthfully. Third, the IP decides separately for each RP $_i$ whether to implement a coordinated solution and to pay L_i or to implement the initial solution. Of course, the IP will prefer the coordinated solution if and only if the marginal surplus S_i is greater than or equal to L_i . In case of multiple RP $_i$, the IP can model this decision by declaring variables $\lambda_{i,init}$ as binary in (CS2 $_i$) and (CS2 $_i'$) and setting χ_i^{init} to $-L_i$ in the objective functions of these models.

For our analysis of the mechanism, we introduce the following assumptions:

ASSUMPTION 1: The RP $_i$ have prior, incomplete knowledge about their marginal system-wide surpluses S_i .

This knowledge can be derived from the general experience of decision makers or be acquired by (Bayesian) learning if coordination is undertaken repeatedly (e.g., once a month). We define S_i as a random variable with the probability density function $f_i(S_i)$ over the interval $[a_i, b_i]$.

ASSUMPTION 2: In the second step of the mechanism, parties' expected marginal surpluses are maximized if parties implement the actions as specified by the scheme.

First, this assumption requires that the scheme underlying the mechanism is the most efficient among all known schemes based on the exchange of primal information, which holds, up to our knowledge, for the scheme presented here. (Note here that we do not recommend schemes requiring the exchange of dual information for use with this mechanism. Dual information communicated to the RP_i may enable them to estimate the cost changes of the IP in order to usurp additional shares of the marginal surpluses, such that strategy-proofness would get violated.) Second, we assume that an RP_i cannot strategically affect the marginal surpluses of other RP_i , since he cannot foresee the effects of his proposals on other RP_i due to the privateness of decentralized data.

ASSUMPTION 3: The information exchange required by the scheme does not violate individual rationality of parties.

Within supply chain management, an (iterative) exchange of demand forecasts is common practice, for example, in CPFR (e.g., Aviv 2001). Proposals about the central resource use (e.g., supply quantities) comprise comparable information. Hence, we argue that an exchange of a modest number of such proposals and the associated, aggregated cost-effects does not do harm to the party disclosing this information, as the exchange of production capacities might do (which practitioners often regard as sensitive information, see, e.g., Kersten 2003).

The mechanism is individually rational and budget-balanced. For evaluating strategy-proofness and efficiency, we model parties' actions by a multi-stage game. If a proposal of an RP_i is accepted by the IP, the RP_i 's gains comprise the lump sum and a potential markup l_i that is due to distorted cost reporting. Due to his information status (see Assumption 1), a rational RP_i will choose equal markups for all proposals. Hence, the decision problem of an RP_i in the second step of the mechanism is

$$\max_{l_i} \int_{L_i+l_i+s_i(l_i)}^{b_i} (L_i + l_i) f(S_i) dS_i, \text{ with } s_i(l_i) \text{ as the function that maps the expected reduction of } S_i \text{ with } l_i.$$

By Assumption 2, $s_i(l_i) \geq 0$ holds, which means that the performance of the scheme is highest with truthful reporting. The RP_i weakly prefers a lump-sum payment $L_i = \hat{L}_i + \hat{l}_i$ together with $l_i=0$ to any payment \hat{L}_i with any markup $\hat{l}_i > 0$. That is, truth-telling is a weakly dominant strategy for the RP_i , since he is allowed to determine L_i in the first step of the mechanism.

L_i^* , the optimal value for L_i , can be determined analytically if the cumulated density function of S_i has an increasing generalized failure rate, which holds for many of the commonly applied distributions (see Lariviere and Porteus 2001). Then L_i^* is the maximum of a_i and the solution to the first-order condition of the RP_i 's decision model: $\int_{L_i}^{b_i} f_i(S_i) dS_i - L_i f_i(L_i) = 0$. After some transformations, we get that $L_i^* = \max\{a_i, b_i/2\}$ if the RP_i is risk-neutral and his prior knowledge about S_i is uniformly distributed over the interval $[a_i, b_i]$. Provided that all RP_i choose $L_i^* = \max\{a_i, b_i/2\}$, $\int_{L_i^*}^{b_i} S_i f_i(S_i) dS_i / \int_{a_i}^{b_i} S_i f_i(S_i) dS_i = (3b_i^2/8(b_i - a_i)) / ((b_i + a_i)/2) \geq 3/4$ of the marginal surpluses and, hence, at least a share of $1 - \sum_{i \in P} E[S_i] / (4E[S^{sys}])$ of S^{sys} , the surplus for the whole system, can be realized. For a two-party setting, the sum of the marginal surpluses equals the surplus for the whole system. With three or more parties, the ratio between these surpluses depends on whether coordinated solutions of different RP_i are complementary and on the RP_i 's prior knowledge about that. Most frequent is the non-complementary case, which occurs, for example, if several RP_i are competing for one central resource (see the appendix for an example). Then, the system-wide surplus exceeds the sum of the marginal surpluses and more than $3/4 S^{sys}$ can be obtained. We summarize our main results in Theorem 2.

THEOREM 2. *The scheme proposed can be embedded into an individually rational, strategy-proof, and budget-balanced mechanism that does not require the involvement of a third party. If parties' prior knowledge about the marginal surpluses is uniformly distributed and the sum of the marginal surpluses is*

smaller than or equal to the system-wide surplus, at least 3/4 of the system-wide surplus can be realized on average.

Note that parties' roles (IP, RP_i) have to be determined before applying the mechanism. Considering a two-party supply chain, both roles have advantages. To obtain an indication about the allocation of the surplus realized by the mechanism, assume a uniform distribution for $f(S)$ with $a=0$, which means that solutions with quite small improvements exist, too. An easy calculation shows that 2/3 of the surplus realized are allocated to the RP then, which seems to favor the RP. However, there are reasons for the RP to claim smaller shares than anticipated by our model. First, the RP will do so if he is risk-averse. Second, experimental studies (e.g., Rapoport et al. 1998, Seale et al. 2001) indicate that parties (and especially the less informed ones, which holds for the RP here) tend to claim considerably smaller shares of the overall profit in sealed bid double auctions, a mechanism related to that proposed here. Third, settling for a smaller share strengthens long-term collaboration. This may induce the IP to put more effort into corresponding coordination activities and result in future increases of the marginal surplus. It is important to note that the overall efficiency increases sharply if the RP claims less. If, for example, the share of the RP is 1/2 instead of 2/3, the real decrease in the RP's profit from coordination will be only 1/9. The efficiency of the mechanism, in turn, will rise from 3/4 to 8/9.

Especially for supply chains with more than two parties, there is often a natural choice for the roles of IP and RP_i . For example, if an OEM (original equipment manufacturer) uses the mechanism for coordination with his suppliers, the OEM will serve as the IP and his suppliers as the RP_i . Finally, note that the choice of parties' roles only affects the relative shares of the surpluses allocated; irrespectively of this choice, each party will obtain nonzero benefits if a coordinated solution (i.e., a solution incurring a different use of the central resources by this party compared to the initial solution) is implemented (except for the very special case that the marginal system-wide surplus exactly matches the lump sum required), which further favors the applicability of the mechanism proposed.

6. Customization to Master Planning and Computational Tests

In order to evaluate the convergence behavior of the scheme, we consider a hypothetical application, the coordination of master plans among multiple units in a decentralized supply chain. Besides production quantities, decisions about inventory and overtime are essential for master planning (e.g., Billington et al. 1983). In addition, we consider backorders because they are frequently used in practical modeling and reveal inefficiencies of uncoordinated solutions. The formulation of the centralized master planning model considered is given in the appendix.

We consider a supply chain consisting of an OEM and multiple suppliers in a single-sourcing relationship. The interdependencies between parties in the centralized model are covered by the inventory balance constraints for the supplied items J^D .

$$I_{jt-1} + X_{jt} = \sum_{k \in S_j \cap J^B} r_{jk} X_{kt} + I_{jt} \quad \forall j \in J^D, t = 1, \dots, T. \quad (18)$$

I_{jt} is the inventory and X_{jt} is the production amount of item j in period t . r_{jk} is the number of units of item j required to produce one unit of the immediate successor item k , S_j is the set of immediate successors of item j in the bill of materials, and J^B the set of items produced by the buyer. We apply the following reformulation to (18):

$$IB_{jt-1} + XB_{jt} = \sum_{k \in S_j \cap J^B} r_{jk} X_{kt} + IB_{jt} \quad \forall j \in J^D, t = 1, \dots, T, \quad (19)$$

$$IS_{jt-1} + X_{jt} = XS_{jt} + IS_{jt} \quad \forall j \in J^D, t = 1, \dots, T, \quad (20)$$

$$I_{jt} = IS_{jt} + IB_{jt} \quad \forall j \in J^D, t = 1, \dots, T,$$

$$XB_{jt} \geq XS_{jt} \quad \forall j \in J^D, t = 1, \dots, T, \quad (21)$$

$$XB_{jt} \leq XS_{jt} \quad \forall j \in J^D, t = 1, \dots, T, \quad (22)$$

$$XB_{jt}, XS_{jt}, IB_{jt}, IS_{jt} \geq 0 \quad \forall j \in J^D, t = 1, \dots, T.$$

IB_{jt} and IS_{jt} are inventories at the buyer's and the suppliers' sites of an item $j \in J^D$ in period t . XB_{jt} is the amount of item j delivered to the buyer in period t , and XS_{jt} the amounts delivered by the suppliers. (19) and (20) are inventory balance constraints for the decentralized models. Constraints (21) and (22) correspond to the restriction of the central resource in (C) and link the decentralized models. With this decomposition, models (CS-EVAL_i), (CS1'_i), (CS1''_i), and (CS2'_i) can be derived directly.

To further speed up the convergence rate, a modification specific to supply chain planning is applied to (CS1'_i) and (CS1''_i). In order to avoid a penalization of complementary changes in the supply pattern, we replace constraints (21) and (22) by

$$\sum_{\tau=1}^t XB_{j\tau} \geq \sum_{\tau=1}^t XS_{j\tau} \quad \forall j \in J^D, t = 1, \dots, T, \quad (23)$$

$$\sum_{\tau=1}^t XB_{j\tau} \leq \sum_{\tau=1}^t XS_{j\tau} \quad \forall j \in J^D, t = 1, \dots, T.$$

(Note that a similar idea has been used by Dudek and Stadtler (2005) in their scheme.) Thereby, constraints (23) are only applied to penalize solutions that benefit from earlier supply.

In order to thoroughly evaluate the convergence behavior of the scheme, we aim at investigating test problems that are *hard to coordinate*. In principle, potential reasons for easy coordination may be large cost asymmetries among parties (then, proposals by the party with the greater costs may come very near to the system-wide optimum without further need for coordination) or among single items (which may allow the scheme to concentrate on the high-cost items and may have a similar effect like reducing the problem size). Both problems have been avoided here. In the master planning problem considered, the major cost components depending on central resource use are backorder costs for the buyer and overtime costs for the suppliers. Hence, to avoid easy coordination, we set the costs for an increase in capacity by overtime and the resulting decrease of backorder costs (for items produced on this resource or for their successor items) to be equal on average in our basic test set. Additionally, we conduct sensitivity analyses for different ratios between average overtime and backorder costs (1:5, 5:1). Define the cumulated capac-

ity requirements of an item j as $\bar{a}_j = \sum_{k \in R_j} r_{kj} \bar{a}_k + \sum_{m=1}^M a_{mj}$, with M as the number of resources and R_j as the set of immediate predecessors of j in the bill of materials. Backorder costs for one unit of item j in a period are set to $bl_j = \bar{a}_j K$, with K as a constant of an arbitrary value used for the correct transformation of the unit of \bar{a}_j . Overtime costs for one unit of capacity increase on resource m are set to $oc_m = \left(\sum_{j \in J_m} bl_j / a_{mj} \right) / |J_m|$, with J_m as the set of items produced on m . In order to determine unit costs for inventory holding, we assume that the values of end items equal their unit backorder costs and that the values of intermediate items equal their cumulated capacity requirements transformed as above. Further assuming monthly periods for master planning and capital costs of 10% p.a. for inventory holding, we obtain $h_j = \bar{a}_j K / (10T)$.

We evaluate 14 different test sets. The basic test set (BASE) comprises two parties running LP with $|J^D|=4$ (see the appendix for the structures) and $T=12$ as well as a unilateral exchange of cost changes from the supplier to the buyer. In line with most practical applications, we assume by default that the buyer establishes the initial, uncoordinated solution and additionally evaluate a setting (RAND) where the initial solution is determined randomly. We vary the number of periods ($T \in \{3, 24\}$), the problem structure ($|J^D|=10$), the model type (MIP), and the number of suppliers (two (2S) and five (5S)); to avoid distortions by incentive issues determined by the mechanism, we assumed lump sums of 0).

For each of these sets, 216 instances have been generated by combining different random values for demands, production coefficients, and capacities. For each instance, these random values have been drawn from normal distributions with base values of 1 (negative values have been replaced by 0 and all random values have been divided by their averages) and different combinations of coefficients of variation. Demand data differ by the coefficients of variation ($CV_d \in \{0.1, 0.3, 0.5\}$) and the inclusion of an additional seasonality component (generation based on a cosine oscillation with an amplitude of 0 (no seasonality) or 0.2). For production coefficients and period capacities, $CV_{coe} \in \{0.2, 0.5, 0.7, 1, 1.5, 2\}$

and $CV_{cap} \in \{0.001, 0.1, 0.2, 0.3, 0.4, 0.5\}$, respectively, have been chosen. The average capacity utilization has been set to 90%. For the model class MIP, overtime can only be taken in equally sized shifts of 1/8 of the average period capacity.

Solving (CS1_i) and its modifications is not trivial because of nonlinearities in the objective functions. Although there may be more elegant procedures (e.g., that of Baker and Lasdon 1985 for nonlinear programming problems with continuous variables only), a simple separable programming approach (see, e.g., Williams 1994) has been sufficient to successfully implement our computational study. For each dimension j of k_i , we transformed the nonlinear term $k_{ij}^T d_{ij}$ in the objective functions of (CS1'_i) and (CS1''_i) to $(1/2(k_{ij} + d_{ij}))^2 - (1/2(k_{ij} - d_{ij}))^2$, derived upper and lower bounds for the quadratic terms and linearized them with 10 nodes (in preliminary tests, more exact linearizations did not yield significantly better results). The lower bounds used for the linearization have been set to 0, the upper bounds for k_{ij} to the maximum cost changes of items per period in previous proposals, and the upper bounds for d_{ij} equal to dp_i .

Out of the 3024 instances generated, we obtained 2795 with a suboptimality of the uncoordinated solution greater than 0.005%, which we analyzed further. Optimization problems have been solved using Xpress-MP 2008A on a single thread of an Intel SMP with a clock speed of 3.20 GHz and 1.99 GB RAM. We applied a time limit of 600 sec. for (C)¹ and 10 sec. for the other models solved in the scheme. For the analysis of the results, we calculate the performance indicators $UNC = \left(\sum_{n=1}^N (c_{unc,n} - c_{opt,n}) / c_{opt,n} \right) / N$, the average gap between the costs of the uncoordinated solution $c_{unc,n}$, and the costs of the solution to (C) $c_{opt,n}$, and $CS = \left(\sum_{n=1}^N (c_{cor,n} - c_{opt,n}) / c_{opt,n} \right) / N$, the average gap between $c_{opt,n}$ and the costs after 20 iterations of the scheme $c_{cor,n}$.

¹ Using this time limit, all instances of (C) have been solved to optimality except for 34 instances for the MIP with $|J^D|=4$ and 103 instances for the MIP with $|J^D|=10$. There, the average remaining gaps have been 0.53% and 0.56%, respectively.

	BASE	$T=24$	$T=3$	$ \mathcal{J}^D =10$	2S	$ \mathcal{J}^D =10, 2S$	$ \mathcal{J}^D =10, 5S$
UNC	15.1%	22.5%	10.9%	28.2%	15.1%	28.2%	28.2%
CS	5.7%	8.1%	2.2%	9.2%	4.9%	9.7%	7.7%

Table 1: Basic computational results

We present our basic results in Table 1. The size of instances and the number of suppliers turned out as important drivers for the performance of the scheme. Not surprisingly, increases of both $|\mathcal{J}^D|$ and T led to (modest) decreases in performance. Although considerable improvements have been found for each test set, the results suggest that managers applying the mechanism benefit by limiting coordination activities on a small number of (key) items and periods (e.g., $T=3$), where near-optimal solutions could be obtained. Moreover, the solution performance of the scheme tends to increase with multiple suppliers. This might seem surprising at first glance, but can be explained by the option of the buyer for recombining a larger number of different proposals of the suppliers and the more detailed knowledge about their cost changes χ_i .

For all instances, more than 60% of UNC could be mitigated after 20 iterations. This modest number of iterations has several advantages. Decision makers are able to participate interactively in the coordination process and to manually control the cost-effects of the proposals exchanged. In addition, the duration of the coordination process is kept short and little information is disclosed, which keeps the risk low for a reconstruction of model parameters by other parties.

Table 2 provides the results of further sensitivity analyses. If an MIP is used instead of an LP, increases of UNC, but only minor decreases in the convergence rate have been observed, which underlines the versatility of the scheme. Moreover, using a randomly generated starting solution does not significantly affect the gap closure, which suggests that the performance of the scheme is not bound to the choice of the initial solution. Finally, we evaluated different ratios between average overtime and backorder costs. Since the initial solution is determined by the buyer here, increases of this ratio lead to increases of both UNC and CS, whereas decreases yield reversed effects. In both cases, however, the relative reduction of UNC

compared to the base setting has been greater, which underlines our suggestion that the coordination hardness is maximized for equally distributed costs among parties.

	MIP	$T=3$, MIP	$ \mathcal{J}^D =10$, MIP	$ \mathcal{J}^D =10$, 5S, MIP	RAND	5:1	1:5
UNC	21.8%	21.2%	35.1%	35.1%	114012.5%	66.4%	8.9%
CS	7.9%	4.3%	12.7%	10.8%	6.7%	18.7%	2.7%

Table 2: Sensitivities for different parameterizations

7. Conclusion

This paper proposes a mechanism for coordinating complex LP or MIP run by two or more self-interested parties holding private information. We have been able to show finite convergence of the underlying scheme in the case where all but one party report their cost changes and only include continuous variables in their optimization models. We have further provided a more effective modification of this scheme, which is also applicable with discrete decisions of two or more decentralized parties and yields considerable improvements over initial solutions after 20 iterations in computational tests. Moreover, we have designed a strategy-proof mechanism for evaluating the schemes and provided a lower bound on its efficiency. Characteristics of the mechanism – like the exchange of primal, insensitive information only, truth-telling as a weakly dominant strategy, few iterations, and no central entity needed – favor the practical applicability.

The empirical verification of the scheme has been limited in this paper to deterministic models for supply chains consisting of one buyer and one or several suppliers. Hence, further research possibilities include customizations and computational studies for different organizational structures like three-tier supply chains as well as for stochastic programming models with discrete probabilities, which can be rewritten as usual LP and, hence, constitute a further potential application of the scheme proposed.

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References

- Arikapuram, S.P., D., Veeramani. 2004. Distributed decision-making in supply chains and private e-marketplaces. *Prod. Oper. Management* **13** (1) 111-121.
- Aviv, Y. 2001. The effect of collaborative forecasting on supply chain performance. *Management Sci.* **47** (10) 1326-1343.
- Baker, T.E., L.S., Lasdon. 1985. Successive linear programming at Exxon. *Management Sci.* **31** (3) 264-274.
- Baldenius, T., S., Reichelstein, S., A., Sahay. 1999. Negotiated versus cost-based transfer pricing. *Rev. Acc. Stud.* **4** 67-91.
- Benders, J.F. 1962. Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik* **4** 238-252.
- Billington, P.J., J.O., McClain, L.J., Thomas. 1982. Mathematical programming approaches to capacity-constrained MRP systems: Review, formulation and problem reduction. *Management Sci.* **29** (10) 1126-1141.
- Cachon, G.P. 2003. Supply chain coordination with contracts. S. Graves, T. de Kok, eds. *The Handbook of Operations Research and Management Science: Supply Chain Management*. Kluwer, Amsterdam, The Netherlands.
- Cachon, G.P., F., Zhang. 2006. Procuring fast delivery: Sole sourcing with information asymmetry. *Management Sci.* **52** (6) 881-896.
- Cachon, G.P., M.A., Lariviere. 2001. Contracting to assure supply: How to share demand forecasts in a supply chain. *Management Sci.* **47** (5) 629-646.

- Chen, Z.-L. 2004. Integrated production and distribution operations: Taxonomy, models, and review. D. Simchi-Levi, S.D. Wu, Z.M. Shen, eds. *The Handbook of Operations Research and Management Science: Modeling in the E-Business Era*. Kluwer, Amsterdam, The Netherlands.
- Chen, F., A., Federgruen, Y.-S., Zheng. 2001. Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management Sci.* **47** (5) 693-708.
- Chu, L.Y., Z.-J.M., Shen. 2006. Agent competition double-auction mechanism. *Management Sci.* **52** (8) 1215-1222.
- Corbett, C.J., X., de Groote. 2000. A supplier's optimal quantity discount policy under asymmetric information. *Management Sci.* **46** (3) 444-450.
- Corbett, C.J., D., Zhou, C.S., Tang. 2004. Designing supply contracts: Contract type and information asymmetry. *Management Sci.* **50** (4) 550-559.
- Dantzig, G.B., P., Wolfe. 1960. Decomposition principle for linear programs. *Oper. Res.* **8** (1) 101-111.
- Dudek, G., H., Stadler. 2005. Negotiation-based collaborative planning between supply chains partners. *Eur. J. Oper. Res.* **163** (3) 668-687.
- Dudek, G., H., Stadler. 2007. Negotiation-based collaborative planning in divergent two-tier supply chains. *Internat. J. Product. Res.* **45** (2) 465-484.
- Ertogral, K., S.D., Wu. 2000. Auction-theoretic coordination of production planning in the supply chain. *IIE Trans.* **32** (10) 931-940.
- Fan, M., J., Stallaert, A.B., Whinston. 2003. Decentralized mechanism design for supply chain organizations using an auction market. *Inf. Sys. Res.* **14** (1) 1-22.
- Fink, A. 2005. Supply chain coordination by means of automated negotiations between autonomous agents. B. Chaib-draa, J. Mueller eds. *Multiagent based Supply Chain Management*. Springer, Berlin.
- Guo, Z., G.J., Koehler, A.B., Whinston. 2007. A market-based optimization algorithm for distributed systems. *Management Sci.* **53** (8) 1345-1358.

- Guzelsoy, M., T.K., Ralphs. 2007. Duality for mixed-integer linear programs. *Internat. J. Oper. Res.* **4** (3) 118-137.
- Harris, M., C.H., Kriebel, A., Raviv. Asymmetric information, incentives and intrafirm resource allocation. *Management Sci.* **28** (6) 604-620.
- Karabuk, S., S.D., Wu. 2002. Decentralizing semiconductor capacity planning via internal market coordination. *IIE Trans.* **34** (9) 743-759.
- Kersten, W. 2003. Internetgesteuerte Beschaffung komplexer Produktionsmaterialien - das Ende der Wertschöpfungspartnerschaft in der Automobilindustrie?. H. Wildemann, ed. *Moderne Produktionskonzepte für Güter- und Dienstleistungsproduktionen (in German)*, Munich.
- Kutanoglu, W., S.D., Wu. 1998. On combinatorial auction and Lagrangean relaxation for distributed resource scheduling. *IIE Trans.* **31** (9) 813-826.
- Lariviere, M.A., E.L., Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing Service Oper. Management* **3** (4) 293-306.
- Mas-Colell, A., Whinston, M.D., Green, J.R. 1995. *Microeconomic Theory*. Oxford University Press, New York.
- Özer, Ö., W., Wei. 2006. Strategic commitments for an optimal capacity decision under asymmetric forecast information. *Management Sci.* **52** (8) 1238-1257.
- Rapoport A., T.E., Daniel, D.A., Seale. 1998. Reinforcement-based adaptive learning in asymmetric two-person bargaining with incomplete information. *Exper. Econ.* **1** 221-253.
- Schenk-Mathes, H.Y. 1995. The design of supply contracts as a problem of delegation. *Eur. J. Oper. Res.* **86** (1) 176-187.
- Schneeweiss, C., K., Zimmer. 2004. Hierarchical coordination mechanism within the supply chain. *Eur. J. Oper. Res.* **153** (3) 687-703.
- Seale, D.A., E.D., Terry, A., Rapoport. 2001. The information advantage in two-person bargaining with incomplete information. *J. Econ. Behav. Organ.* **44** (2) 177-200.

- Shirodkar, S., K., Kempf. 2006. Supply chain collaboration through shared capacity models. *Interfaces* **36** (5) 420-432.
- Spengler, J.J. 1950. Vertical integration and antitrust policy. *J. Political Econom.* **58** (4) 347-352.
- Stadtler, H. 2007. A framework for collaborative planning. *OR Spectrum*, online version available, DOI: 10.1007/s00291-007-0104-5.
- Taylor, T.A. 2006. Sale timing in a supply chain: When to sell to the retailer. *Manufacturing Service Oper. Management* **8** (1) 23-42.
- Troutt, M. J., W.-K. Pang, S.-H. Hou. 2006. Behavioral estimation of mathematical programming objective function coefficients. *Management Sci.* **52** (3) 422-434.
- Tsay, A.A., S., Nahmias, N., Agrawal. 1998. Modeling supply chain contracts: A review. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative models for supply chain management*, Kluwer, Boston, MA.
- Walther, G., E., Schmid, T., Spengler. 2007. Negotiation based coordination in product recovery networks. *Internat. J. of Prod. Econ.* **111** (2) 334-350.
- Williams, H.P. 1994. *Model building in mathematical programming*. Wiley, Chichester, England.
- Wolsey, L.A. 1981. Integer programming duality: Price functions and sensitivity analysis. *Math. Program.* **20** (1) 173-195.

Appendix

Proofs

PROOF OF LEMMA 1. 1. We begin with a proof of Lemma 2, a technical result, which is needed for proving this lemma and Theorem 1.

LEMMA 2. Consider (CS1_i) with x_i fixed to an arbitrary value x_i^f . For k_i^{f*} , the optimal solution to this

problem, there are scalars $\lambda_{ie} \geq 0$ ($e=0, \dots, \bar{e}_i$) with $(k_i^{f*})^T A_i (x_i^{st} - x_i^f) = M_i^T \lambda_{i0} - \sum_{e=1}^{\bar{e}_i} c_i (x_i^{st} - x_i^e)^T \lambda_{ie}$ and

$$\lambda_{i0} + \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T A_i^T \lambda_{ie} \geq (x_i^{st} - x_i^f)^T A_i^T.$$

PROOF OF LEMMA 2. Write (CS1_f) for (CS1_i) with x_i fixed to x_i^f . (CS1_f) can be reduced to

$$\max_{k_i} \left\{ k_i^T A_i (x_i^{st} - x_i^f) \mid k_i^T A_i (x_i^{st} - x_i^e) \leq -c_i^T (x_i^{st} - x_i^e) \quad \forall e = 1, \dots, \bar{e}_i, k_i \leq M_i, k_i \geq 0 \right\}.$$

(CS1_f-D), the dual problem to (CS1_f), is

$$\min_{\lambda_i} \left\{ M_i^T \lambda_{i0} - \sum_{e=1}^{\bar{e}_i} c_i (x_i^{st} - x_i^e)^T \lambda_{ie} \mid \lambda_{i0} + \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T A_i^T \lambda_{ie} \geq (x_i^{st} - x_i^f)^T A_i^T, \lambda_{ie} \geq 0 \quad \forall e = 0, \dots, \bar{e}_i \right\}.$$

According to duality theory, the objective function values of the optimal solutions to (CS1_f) and (CS1_f-D) are equal, which proves this lemma. \square

We now proceed with the proof of the first statement of Lemma 1. Since there are feasible solutions to (C) and thus to (DP_i), it suffices to show that constraints (3), (4), and (7) do not render (CS1_i) infeasible. Since these constraints do not depend on x_i , (CS1_i) is feasible, provided the feasibility of (CS1_f) for at least one x_i^f . To demonstrate the latter, we prove the feasibility and the boundedness of the dual problem (CS1_f-D), which has been stated in the proof of Lemma 2. (CS1_f-D) has a feasible solution since λ_{i0} can take any value greater than zero. The optimal solution to (CS1_f-D) is bounded, e.g., if

$$\sum_{e=1}^{\bar{e}_i} c_i^T (x_i^{st} - x_i^e) \lambda_{ie} \leq 0 \quad \forall \lambda_{ie} \geq 0 \quad (e=0, \dots, \bar{e}_i), \quad \lambda_{i0} + \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T A_i^T \lambda_{ie} \geq (x_i^{st} - x_i^f)^T A_i^T. \quad (24)$$

Due to the assumptions of this lemma (1.),

$$\sum_{e=1}^{\bar{e}_i} c_i^T (x_i^{st} - x_i^e) \lambda_{ie} \leq 0 \quad \forall \lambda_{ie} \geq 0, \quad A_i \sum_{e=1}^{\bar{e}_i} \lambda_{ie} (x_i^{st} - x_i^e) \geq 0, \quad \sum_{e=1}^{\bar{e}_i} \lambda_{ie} = 1. \quad (25)$$

The condition $\sum_{e=1}^{\bar{e}_i} \lambda_{ie} = 1$ can be omitted in (25) since (25) implies $\sum_{e=1}^{\bar{e}_i} c_i^T (x_i^{st} - x_i^e) \lambda_{ie} \alpha \leq 0$ and

$A_i \sum_{e=1}^{\bar{e}_i} \lambda_{ie} (x_i^{st} - x_i^e) \alpha \geq 0$ for any scalar $\alpha \geq 0$ with $\sum_{e=1}^{\bar{e}_i} \alpha \lambda_{ie} = 1$. Together with (24), we get that (CS1f_i-D)

is bounded if $\lambda_{i0} \geq (x_i^{st} - x_i^f)^T A_i^T \forall \lambda_{i0} \geq 0$. This inequality is fulfilled for, e.g., $x_i^f = x_i^{st}$. Hence, (CS1f_i-D) has a feasible, bounded solution and, according to duality theory, (CS1f_i) and thus (CS1_i).

2. Since we only consider the properties of x_i^* , we can omit (3), (4), and (7). Then, (CS1_i) reduces to

$$\begin{aligned} & \max c_i^T (x_i^{st} - x_i) + k_i^T A_i (x_i^{st} - x_i) \\ & \text{s.t.} \quad (5), (6). \end{aligned}$$

Each solution x_i is a vertex of the polytope defined by inequalities (5) and (6), and hence, a vertex solution to (DP₁)². □

PROOF OF THEOREM 1. The proof of this theorem is structured into five parts. In (a), we prove that (CS1_i) always yields a feasible solution. This allows us to derive a lower bound on the costs of the solutions $x_i \in X_i$ that have not been identified after the termination of the scheme (b). Based on that, we can characterize a subset of the solutions to (C) for which the system-wide costs are greater than or equal to the costs of $x^l = (x_1^l, \dots, x_l^l)$, the system-wide solution resulting from the last run of (CS2₁) (c). Since this subset comprises an r -neighborhood of x^l , the optimality of x^l follows (d). Last (e), we show that the convergence is finite.

(a) By Lemma 1, (CS1_i) yields a feasible solution if

$$c_i^T x_i^{st} \leq c_i^T \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e \quad \forall \lambda_{ie} \geq 0 \quad (e=1, \dots, \bar{e}_i), \quad A_i x_i^{st} \geq A_i \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e, \quad \sum_{e=1}^{\bar{e}_i} \lambda_{ie} = 1. \quad (26)$$

² Note that (DP₁) and (CS1_i) may have several optimal solutions including non-vertex solutions. Here we assume that (CS1_i) is solved using an algorithm that limits its search to vertex solutions (e.g., a primal simplex).

We prove by contradiction that (26) is fulfilled for all starting solutions used in the scheme. For this purpose, we assume the existence of $\lambda_{ie} \geq 0$ with $c_i^T x_i^{st} > c_i^T \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e$, $A_i x_i^{st} \geq A_i \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e$ and $\sum_{e=1}^{\bar{e}_i} \lambda_{ie} = 1$.

Since (DP_i) are LP, the solution $x_i' = \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e$ is feasible. Since starting solutions x_i^{st} have been optimal for the last run of (CS-EVAL_i), $c_i^T x_i^{st} \leq c_i^T x_i$ holds for all feasible x_i with $A_i x_i^{st} \geq A_i x_i$ and, hence, also for $x_i = x_i'$. Hence, $c_i^T x_i^{st} \leq c_i^T x_i' = c_i^T \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e$ and a contradiction results.

(b) As a consequence, with the termination of the scheme, $Z_{CS1_i} \leq 0$ holds and, hence, $c_i^T (x_i^{st} - x_i) + k_i^T A_i (x_i^{st} - x_i) \leq 0$ for any x_i . With $x_i^f = x_i$, we get by Lemma 2 that there are scalars $\lambda_{ie} \geq 0$ ($e=0, \dots, \bar{e}_i$) with

$$c_i^T x_i \geq c_i^T x_i^{st} + \lambda_{i0} M_i^T - c_i \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T \lambda_{ie}, \quad \lambda_{i0} + \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T A_i^T \lambda_{ie} \geq (x_i^{st} - x_i)^T A_i^T. \quad (27)$$

Next, we show by contradiction the existence of scalars $\mu_{ie} \geq 0$ ($e=1, \dots, \bar{e}_i$) with

$$c_i^T x_i \geq c_i^T x_i^{st} - c_i \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T \mu_{ie}, \quad \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T A_i^T \mu_{ie} \geq (x_i^{st} - x_i)^T A_i^T. \quad (28)$$

Assume the contrary, i.e.,

$$c_i^T x_i < c_i^T x_i^{st} - c_i \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T \lambda'_{ie} \quad \forall \lambda'_{ie} \geq 0 \quad (e=1, \dots, \bar{e}_i), \quad \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T A_i^T \lambda'_{ie} \geq (x_i^{st} - x_i)^T A_i^T. \quad (29)$$

Together with (27), we get that there are $\lambda_{ie} \geq 0$ with

$$\lambda_{i0} M_i^T < -c_i \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T (\lambda'_{ie} - \lambda_{ie}) \quad \forall \lambda'_{ie} \geq 0, \quad \lambda_{i0} \geq \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T A_i^T (\lambda'_{ie} - \lambda_{ie}). \quad (30)$$

Recall the definition of M_i , $M_i^T A_i (x_i^1 - x_i^2) > -c_i^T (x_i^1 - x_i^2) \quad \forall x_i^1, x_i^2 \in X_i$. Substituting

$x_i^1 = \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T \lambda'_{ie}$ and $x_i^2 = \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T \lambda_{ie}$ in (30), we obtain $\lambda_{i0} \geq A_i (x_i^1 - x_i^2)^T$. Multiplying this

inequality by M_i^T and replacing x_i^1, x_i^2 again yields $\lambda_{i0} M_i^T \geq M_i^T A_i (x_i^1 - x_i^2) > -c_i^T \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e) (\lambda'_{ie} - \lambda_{ie})$,

which contradicts (30).

(c) Define X_i^S as the subset of the solutions to (CS1_i) for which $\mu_{ie} \geq 0$ ($e=1, \dots, \bar{e}_i$) exist that fulfill (28)

and $\sum_{e=1}^{\bar{e}_i} \mu_{ie} \leq 1$. Hence, $\forall x_i \in X_i^S$, there are $\mu_{ie} \geq 0$ with $c_i^T x_i \geq c_i^T x_i^{st} - c_i \sum_{e=1}^{\bar{e}_i} ((x_i^{st} - x_i^e)^T \mu_{ie} + (x_i^{st} - x_i^e)^T \mu'_{ie})$,

$\sum_{e=1}^{\bar{e}_i} ((x_i^{st} - x_i^e)^T A_i^T \mu_{ie} + (x_i^{st} - x_i^e)^T A_i^T \mu'_{ie}) \geq (x_i^{st} - x_i)^T A_i^T$, and $\sum_{e=1}^{\bar{e}_i} \mu_{ie} \leq 1$ for all scalars $\mu'_{ie} \geq 0$

($e=1, \dots, \bar{e}_i$). Set $\mu'_{ig} = 1 - \sum_{e=1}^{\bar{e}_i} \mu_{ie}$ for $g = st$, $\mu'_{ig} = 0$ otherwise, and define scalars $\lambda_{ie} = \mu_{ie} + \mu'_{ie}$

($e=1, \dots, \bar{e}_i$). Since $x_i^{st} \in X_i^E$, $c_i^T x_i \geq c_i^T x_i^{st} - c_i \sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T \lambda_{ie}$, $\sum_{e=1}^{\bar{e}_i} (x_i^{st} - x_i^e)^T A_i^T \lambda_{ie} \geq (x_i^{st} - x_i)^T A_i^T$,

and $\sum_{e=1}^{\bar{e}_i} \lambda_{ie} = 1 \quad \forall x_i \in X_i^S$. Hence, $\forall x_i \in X_i^S$, there are $\lambda_{ie} \geq 0$ with

$$c_i^T x_i \geq c_i^T \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e, \quad x_i^T A_i^T \geq \sum_{e=1}^{\bar{e}_i} \lambda_{ie} (x_i^e)^T A_i^T, \quad \sum_{e=1}^{\bar{e}_i} \lambda_{ie} = 1. \quad (31)$$

Denote x_I^l, λ_{ie}^l ($i=1, \dots, I-1, e=1, \dots, \bar{e}_i$) as the last solution to (CS2_i) before the termination of the scheme.

Then,

$$c_I^T x_I^l + \sum_{i=1}^{I-1} \sum_{e=1}^{\bar{e}_i} \lambda_{ie}^l \mathcal{X}_i^e \leq c_I^T x_I + \sum_{i=1}^{I-1} \sum_{e=1}^{\bar{e}_i} \lambda_{ie} \mathcal{X}_i^e \quad \forall x_I \in X_I, \lambda_{ie} \geq 0, i=1, \dots, I-1, b_0 \geq A_I x_I + \sum_{i=1}^{I-1} \sum_{e=1}^{\bar{e}_i} \lambda_{ie} \pi_i^e. \quad (32)$$

Let x_i^l be the optimal solution to (CS-EVAL_{*i*}) subject to $\pi_i = \sum_{e=1}^{\bar{e}_i} \lambda_{ie}^l A_i x_i^e$. Replacing $\chi_i^e = c_i^T (x_i^e - x_i^{init})$

and $\pi_i^e = A_i x_i^e$ in (32), we get with (26) (setting $x_i^{st} = x_i^l$ there) that $\sum_{i=1}^I c_i^T x_i^l \leq c_I^T x_I + \sum_{i=1}^{I-1} c_i^T \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e$

$\forall \lambda_{ie} \geq 0$ ($i=1, \dots, I-1, e=1, \dots, \bar{e}_i$) and all $x_I \in X_I$ with $b_0 \geq A_I x_I + \sum_{i=1}^{I-1} A_i \sum_{e=1}^{\bar{e}_i} \lambda_{ie} x_i^e$. Since, for all $i=1, \dots, I-$

1, x_i^l has been used as the starting solution in (CS1_{*i*}) before terminating the scheme, we get together with

(31) that

$$\sum_{i=1}^I c_i^T x_i^l \leq \sum_{i=1}^I c_i^T x_i \quad \forall x_I \in X_I, x_i \in X_i^S, i=1, \dots, I-1, b_0 \geq \sum_{i=1}^I A_i x_i. \quad (33)$$

(d) For any $x_i \notin X_i^S$ characterized by (28) with $x_i^{st} = x_i^l$ and $\sum_{e=1}^{\bar{e}_i} \mu_{ie} = \bar{\mu}_i > 1$, there is a $x_i^\kappa \in X_i^S$ with

$x_i^\kappa = x_i^l + (x_i - x_i^l)\kappa, \kappa = 1/\bar{\mu}_i$. Hence, the r -neighborhood $B_r(x_i^l)$ of x_i^l defined with an Euclidean metric

and the radius $r > 0$ chosen such small that $r \leq \|x_i^\kappa - x_i^l\|^2 \forall x_i^\kappa$ comprises solely $x_i \in X_i^S$. Since x_I can be

chosen freely among X_I in (33), there is a system-wide feasible solution $x^l = (x_1^l, \dots, x_I^l)$ for which

$\sum_{i=1}^I c_i^T x_i^l \leq \sum_{i=1}^I c_i^T x_i^B \quad \forall x_i^B \in B_r(x_i^l), i=1, \dots, I$. Hence, on each edge outgoing from x^l , there is a solution

$x_i = (x_1, \dots, x_I)$ with $\sum_{i=1}^I c_i^T x_i^l \leq \sum_{i=1}^I c_i^T x_i$. Since no further improvements are possible when moving

from x_i^l into the direction of any of its outgoing edges, x^l is the optimal solution to (C).

(e) By Lemma 1, only vertices and, hence, a finite number of different solutions are identified by (CS1_{*i*}).

Then, (CS2_{*i*}) is run a finite number of times, too. \square

Example for the Calculation of the Marginal System-wide Surplus (with Non-Complementary Solutions)

Consider two different RP_i A, B that are competing for the use of one central resource which is expandable by the IP at maximum by 3[CU] (CU=capacity unit) with costs of 1[\$/CU]. Moreover, let A have offered within the scheme two proposals A1, A2 with overuses of 1[CU] and 2[CU] of this resource by A compared to the initial solution and resulting cost savings of 2\$ and 5\$, respectively. B1, B2, the proposals generated by B, show overuses of 1[CU] and 3[CU] of this resource by B and savings of 2\$ and 5\$, respectively. The best solution for the whole system is the simultaneous implementation of A2 and B1 resulting in a system-wide surplus of 5-2+2-1=4\$. Marginal surpluses are 4-(5-3)=2\$ for A (only B2 is implemented if A does not participate) and 4-(5-2)=1\$ for B. Hence, the sum of marginal surpluses is less than the surplus realized by the scheme.

Centralized Master Planning Model

$$\begin{aligned}
 \min \quad & \sum_{j=1}^J \sum_{t=1}^T h_j I_{jt} + \sum_{m=1}^M \sum_{t=1}^T k o_{mt} o c_m O_{mt} + \sum_{j=1}^{J^E} \sum_{t=1}^T b l_j B L_{jt} \\
 \text{s.t.} \quad & B L_{jt} + I_{jt-1} + X_{jt} = d_{jt} + B L_{jt-1} + I_{jt} \quad \forall j = 1, \dots, J^E, t = 1, \dots, T \\
 & I_{jt-1} + X_{jt} = \sum_{k \in S_j} r_{jk} X_{kt} + I_{jt} \quad \forall j = J^E + 1, \dots, J, t = 1, \dots, T \\
 & \sum_{j=1}^J a_{mj} X_{jt} \leq k_{mt} + k o_{mt} O_{mt} \quad \forall m = 1, \dots, M, t = 1, \dots, T \\
 & B L_{jT} = 0 \quad \forall j = 1, \dots, J^E \\
 & B L_{j0} = 0 \quad \forall j = 1, \dots, J^E \\
 & I_{j0} = 0 \quad \forall j = 1, \dots, J \\
 & B L_{jt} \geq 0 \quad \forall j = 1, \dots, J^E, t = 0, \dots, T
 \end{aligned}$$

$$I_{jt} \geq 0 \quad \forall j = 1, \dots, J, t = 0, \dots, T$$

$$O_{mt} \in \mathbb{R}_0^+ \text{ (for LP: } O_{mt} \geq 0) \quad \forall m = 1, \dots, M, t = 1, \dots, T$$

$$X_{jt} \geq 0 \quad \forall j = 1, \dots, J, t = 1, \dots, T$$

Indices and index sets:

j, k items, $j=1, \dots, J$; $1, \dots, J^E$ is the subset of items sold to external customers

m resources (e.g. personnel, machines, ...), $m=1, \dots, M$

t periods, $t=1, \dots, T$

S_j set of immediate successors of item j in the bill of materials.

Data:

a_{mj} capacity needed on resource m for one unit of item j

bl_j backorder costs for one unit of item j in a period

d_{jt} demand for item j in period t

h_j holding costs for one unit of item j in a period

k_{mt} available capacity of resource m in period t

ko_{mt} additional capacity of resource m with one unit of overtime in period t (e.g., one shift)

oc_m costs for one unit of overtime on resource m in a period

r_{jk} number of units of item j required to produce one unit of the immediate successor item k .

Variables:

BL_{jt} amount of backorders of item j in period t

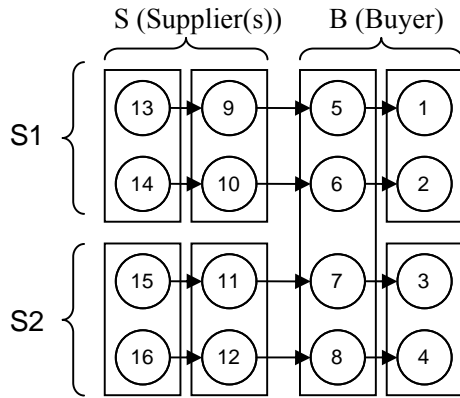
I_{jt} inventory of item j at the end of period t

O_{mt} amount of overtime on resource m in period t

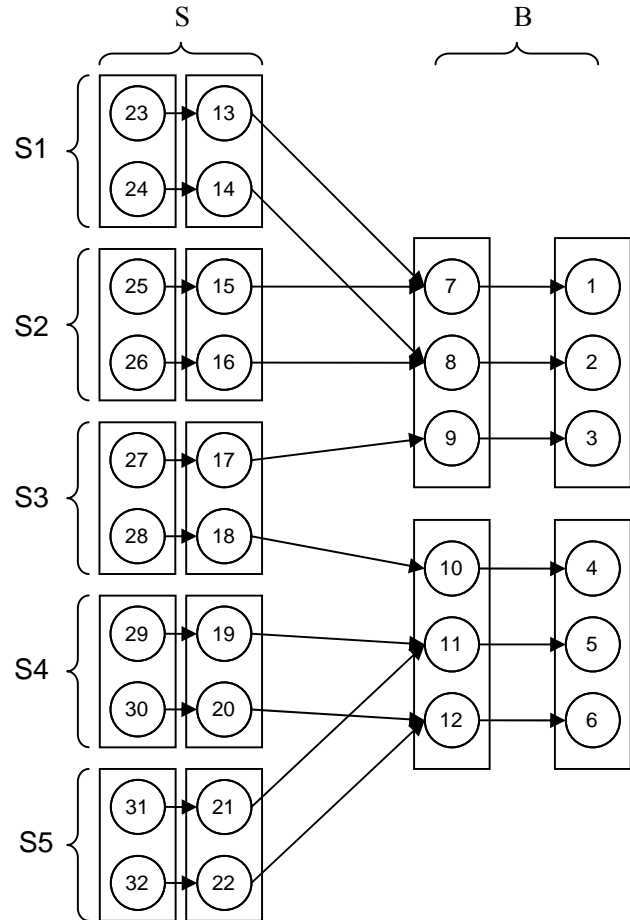
X_{jt} production amount of item j in period t .

Structure of Test Instances

$|J^D|=4, J=16, M=7$



$|J^D|=10, J=32, M=14$



Explanations:

- \textcircled{j} items j
- $\boxed{\text{○ ○}}$ set of items that are produced on a specific resource
- 2S separate production of items by S1; S2 ($|J^D|=4$), S1,S2; S3-S5 ($|J^D|=10$)
- 5S S1; S2; S3; S4; S5 ($|J^D|=10$)