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A NEW QUARK MASS MATRIX

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A new quark mass matrix

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Abstract

In the left-right symmetric three generation model with the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ we propose a new quark mass matrix. This matrix can be obtained naturally by invoking an additional U(1) gauge symmetry which distinguishes the different generations. Assuming that the Yukawa coupling constants are approximately equal one can derive relations between masses and mixing angles. Using the standard quark masses we find acceptable values for the angles. θ_3 is nearly independent of m_t whereas θ_2 varies within stringent bounds.

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Many authors have considered the quark mass matrix for three generations in the last years. Many of them have tried to predict the mass of the top quark m_t. However, many of these predictions are already ruled out by the experimental findings at PETRA [1].

In this note we avoid predicting another value for m_t. On the contrary we assume that to every generation belongs another Higgs multiplet and that the enormous differences in quark masses originate in vastly different vacuum expectation values of the Higgs fields whereas the Yukawa coupling constants are assumed to be approximately equal. In the left-right symmetric model with the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ this implies that by considering the mass matrices of the quarks one will not find any relations between the quark masses. One can, however, look for relations between masses and mixing angles.

That such relations should exist has been argued for quite convincingly by Harari [2]. Certain low energy quantities, such as the $K_S^o - K_L^o$ mass difference, are increasing functions of the masses of their intermediate quark lines. Now one would not like this mass difference to grow appreciably if the mass of the heaviest quark becomes large. This can only be avoided if the relevant mixing angle depends in a suitable way on the masses of the heaviest quarks.

In the left-right symmetric three generation model with the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ the left-handed quark fields

$$Q_{iL} = \begin{pmatrix} P_{iL} \\ N_{iL} \end{pmatrix} \qquad (1)$$

are in the (1/2, 0, 1/3) representation and the right-handed quark fields

$$Q_{iR} = \begin{pmatrix} P_{iR} \\ N_{iR} \end{pmatrix} \qquad (2)$$

are in the (0, 1/2, 1/3) representation of the gauge group. The index i labels the three generations. Higgs fields can only couple to such quark fields if they transform according to the (1/2, 1/2, 0) representation. Since we want to account for the enormous mass differences of quarks by vastly different vacuum expectation values of the Higgs fields and approximately equal Yukawa coupling constants we introduce three such multiplets ϕ_i, one for every generation. Then the most general invariant Yukawa interaction is

$$\mathcal{L}_y = \overline{Q_{iL}} \, (A^n_{ik} \phi_n + B^n_{ik} \tilde{\phi}_n) \, Q_{kR} + \text{H.c.} \qquad (3)$$

where $\hat{\phi} = \sigma_2 \phi^* \sigma_2$ and A^n_{ik} and B^n_{ik} are arbitrary complex coupling constants.

Invariance under parity reflection

$$Q_{iL} \leftrightarrow Q_{iR} \quad , \quad \phi_n \rightarrow \phi_n^+ \qquad (4)$$

requires A^n and B^n to be hermitian. One can eliminate the coupling of the quark fields to the $\tilde{\phi}_n$ by requiring invariance under

$$Q_{iL} \rightarrow Q_{iL} \, , \quad Q_{iR} \rightarrow -iQ_{iR} \, , \quad \phi_n \rightarrow i \, \phi_n \qquad (5)$$

Due to spontaneous symmetry breaking the neutral Higgs fields have non-zero vacuum expectation values

$$\langle \phi_n \rangle = \begin{pmatrix} v_n^P & 0 \\ 0 & v_n^N \end{pmatrix} \qquad (6)$$

which give rise to a mass term for the quark fields

$$\mathcal{L}_m = \overline{P_{iL}} \, M^P_{ik} \, P_{kR} + \overline{N_{iL}} \, M^N_{ik} \, N_{kR} + \text{H.c.} \qquad (7)$$

where

$$M^P_{ik} = A^n_{ik} \, v^P_n \quad , \quad M^N_{ik} = A^n_{ik} \, v^N_n \qquad (8)$$

If the difference in quark masses is to be accounted for predominantly by the difference of the vacuum expectation values then one must have

$$|v_1^{P,N}| \ll |v_2^{P,N}| \ll |v_3^{P,N}| \qquad (9)$$

Both mass matrices obviously have the same form. We propose this form to be

$$M^{P,N} = \begin{pmatrix} 0 & 0 & \alpha v_1^{P,N} \\ 0 & \beta v_1^{P,N} & \gamma v_2^{P,N} \\ \alpha^* v_1^{P,N} & \gamma^* v_2^{P,N} & \delta v_3^{P,N} \end{pmatrix} \qquad (10)$$

where α , γ are complex and β , δ are real Yukawa coupling constants. These mass matrices have the following properties

a) they are not block diagonal, i.e. they imply the correct type of mixing,
b) in the limit $v_1^{P,N} = 0$ in each charge sector the lightest quark becomes massless, the others remain massive; there is still mixing between the massive quarks,
c) in the limit $v_1^{P,N} = v_2^{P,N} = 0$ in each charge sector the two lightest quarks become massless and the heaviest quark remains massive.

Every serious candidate for the mass matrix must obviously satisfy a). But also b) and c) are very sensible requirements to be imposed on the mass matrix as has been argued by Davidson and Wali [3] and by Harari [4].

To obtain the form (10) of the mass matrix naturally, i.e. as a consequence of the group structure and the representation content of the theory, we apply an idea due to Davidson and Wali [3]. One enlarges the gauge group by an additional U'(1) factor: $SU(2)_L \times SU(2)_R \times U(1) \times U'(1)$. The primary task of this additional gauge symmetry is to distinguish the different generations. Therefore if l_i, r_i, and h_i are the new quantum numbers of Q_{iL}, Q_{iR}, and ϕ_i respectively then one requires

$$l_i \neq l_j, \; r_i \neq r_j, \; h_i \neq h_j \quad \text{for } i \neq j \tag{11}$$

If we now choose

$$l_i = i, \; r_i = -i, \; h_i = i+3 \quad \text{for } i = 1,2,3 \tag{12}$$

then the proposed form (10) of the mass matrix results. This additional quantum number obviously is something like a generation number.

However, even if we do not adopt the special quantum number assignment (12) we can draw far reaching conclusions from (11) alone if we require the conditions a), b), and c) to be satisfied.

As has been pointed out by Davidson and Wali [3] (11) implies that the mass matrix has the properties

1) every matrix element is either zero or proportional to one v_i,
2) for a given index i, v_i can appear in any given row or column at most once.

The matrix must contain at least one element $\sim v_3$, because otherwise c) would not be valid. But on the other hand v_3 can appear only once. In fact if there were two elements proportional to v_3, then because of 2) they would not be in the same row nor in the same column. But then the matrix would have rank $\geqslant 2$ even in the limit $v_1 = v_2 = 0$. This would again contradict c. Due to invariance under parity reflection v_3 must appear on the diagonal. If necessary we renumber our fields such that $M_{33} \sim v_3$.

In the same way one can show that v_2 appears exactly twice. E.g. if it appeared once then because of parity reflection invariance it would have to appear on the diagonal. However, then in the limit $v_1 = 0$ there would be no mixing at all and b) would be violated. One can further derive that one can have only one of the two cases

$$\begin{pmatrix} & & \\ & & \sim v_2 \\ & \sim v_2 & \sim v_3 \end{pmatrix} \qquad \begin{pmatrix} & & \sim v_2 \\ & & \\ \sim v_2 & & \sim v_3 \end{pmatrix} \tag{13}$$

The second case can be reduced to the first by relabeling $Q_{1L} \leftrightarrow Q_{2L}$ and $Q_{1R} \leftrightarrow Q_{2R}$.

Now we come to v_1. It must appear in the first row exactly once. $M_{11} \sim v_1$ leads to a block diagonal matrix contradicting a). $M_{12} \sim v_1$ implies

$$M = \begin{pmatrix} 0 & \sim v_1 & 0 \\ \sim v_1 & 0 & \sim v_2 \\ 0 & \sim v_2 & \sim v_3 \end{pmatrix} \tag{14}$$

This type of mass matrix has been proposed by Fritzsch [5]. $M_{13} \sim v_1$ leads to (10).

Therefore (11) and a), b), and c) leave us with two possibilities. We will now investigate the relations between mixing angles and masses implied by (10) and then compare with the corresponding relations obtained from (14).

In order to obtain relations between mixing angles and masses from (10) one must make some assumptions. First of all we assume that the Yukawa coupling

constants are real and approximately equal: $\alpha \sim \beta \sim \gamma \sim \delta$, and in our calculation we set them all equal to one coupling constant $\Gamma = \alpha = \beta = \gamma = \delta$. By redefining ϕ if necessary we have $\Gamma > 0$.
As a further approximation we assume that CP is conserved. This implies that the vacuum expectation values $v_i^{P,N}$ are real. Finally we assume that the parameters in the Higgs potential are chosen such that $v_i^{P,N} > 0$. Then the mass matrix (10) obtains the form

$$M^{P,N} = \begin{pmatrix} 0 & 0 & w_1^{P,N} \\ 0 & w_1^{P,N} & w_2^{P,N} \\ w_1^{P,N} & w_2^{P,N} & w_3^{P,N} \end{pmatrix} \tag{15}$$

with $w_i^{P,N} = \Gamma v_i^{P,N} > 0$.

Let us for a while suppress the index P,N. Expressing the w_i in terms of the eigenvalues λ_i of M we find

$$w_1 = -(\lambda_1 \lambda_2 \lambda_3)^{1/3} \tag{16}$$
$$w_3 = \lambda_1 + \lambda_2 + \lambda_3 - w_1 \tag{17}$$
$$w_2^2 = -\lambda_1\lambda_2 - \lambda_2\lambda_3 - \lambda_3\lambda_1 + w_1(\lambda_1 + \lambda_2 + \lambda_3 - 2w_1) \tag{18}$$

Obviously exactly one λ_i is negative. Because of $|\lambda_i| = m_i$ and the values of the quark masses one finds from (17) that $\lambda_3 > 0$. To decide whether λ_1 or λ_2 is negative we consider the limit $v_1 = 0$. According to property b) this should imply $m_1 = 0$ and hence $\lambda_1 = 0$. Then from (18) one finds $w_2^2 = -\lambda_2\lambda_3$. Hence we choose $\lambda_2 < 0$.

If U is the orthogonal matrix which diagonalizes M

$$UMU^{+} = \text{diag.}\ (\lambda_1, \lambda_2, \lambda_3) \tag{19}$$

then one has

$$UMU^{+}\ \text{diag.}\ (1,-1,1) = \text{diag.}\ (m_1, m_2, m_3) \tag{20}$$

and the Cabibbo matrix C is obviously given by

$$C = U^{P}(U^{N})^{+} \tag{21}$$

According to (19) the columns of U^{+} are the normalized eigenvectors of M. These eigenvectors $\vec{n}_i$ are easily calculated:

$$\vec{n}_i = N_i \left(w_1 (\lambda_i - w_1),\ w_2 \lambda_i,\ \lambda_i (\lambda_i - w_1) \right) \tag{22}$$

with

$$N_i = \left[w_1^2 (\lambda_i - w_1)^2 + w_2^2 \lambda_i^2 + \lambda_i^2 (\lambda_i - w_1)^2 \right]^{-\frac{1}{2}} \tag{23}$$

Hence for the Cabibbo matrix we find

$$C_{ik} = \vec{n}_i^{\,P} \cdot \vec{n}_k^{\,N} \tag{24}$$

We parameterize this matrix in the form

$$C = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 & c_1 c_2 s_3 + s_2 c_3 \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 & c_1 s_2 s_3 - c_2 c_3 \end{pmatrix} \tag{25}$$

where $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$.

Using $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$ we find for θ_1

$$\cos\theta_1 = 1 - \frac{1}{2}\left[\frac{m_u}{m_c} + \frac{m_d}{m_s} + \frac{m_u}{w_1^P} + \frac{m_d}{w_1^N} - 2\left(\frac{m_u m_d}{m_c m_s}\right)^{\frac{1}{2}} \left(1+\frac{m_c}{w_1^P}\right)^{\frac{1}{2}} \left(1+\frac{m_s}{w_1^N}\right)^{\frac{1}{2}} \right] \tag{26}$$

With Weinberg's current algebra estimate [6] of the quark masses

$$m_u = 4{,}2\ \text{MeV}, \quad m_d = 7{,}5\ \text{MeV}$$
$$m_c = 1150\ \text{MeV}, \quad m_s = 150\ \text{MeV} \tag{27}$$

and $m_b = 5$ GeV one finds that $\cos\theta_1$ does not depend strongly on m_t in the region 18 GeV $\leq m_t \leq \infty$. It is a decreasing function of m_t and one has

$$\cos\theta_1 = \begin{cases} 0{,}98 \text{ for } m_t = 18 \text{ GeV} \\ 0{,}97 \text{ for } m_t = \infty \end{cases} \tag{28}$$

This is in agreement with the value given by Shrock and Wang [7]

$$\cos\theta_1 = 0{,}9737 \pm 0{,}0025 \tag{29}$$

Again using $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$ we find for C_{13}

$$C_{13} = -\frac{w_1^N}{m_b} - \frac{m_u}{w_1^P} + \left(\frac{m_u m_d m_s}{m_c}\right)^{\frac{1}{2}} \frac{1}{w_1^N} \left(1 + \frac{m_s}{w_1^N}\right)^{\frac{1}{2}} \left(1 - \frac{m_d}{w_1^N}\right)^{\frac{1}{2}} \left(1 - \frac{m_d m_s}{(w_1^N)^2}\right)^{\frac{1}{2}} \left(1 + \frac{m_c}{w_1^P}\right)^{\frac{1}{2}} \quad (30)$$

Using the same mass values as above one finds that C_{13} is a decreasing function of m_t (18 GeV $\le m_t \le \infty$) varying between

$$C_{13} = \begin{cases} -0{,}017 \text{ for } m_t = 18 \text{ GeV} \\ -0{,}021 \text{ for } m_t = \infty \end{cases} \quad (31)$$

Therefore C_{13} depends only slightly on m_t. This is as it should be because C_{13} determines the decay $b \to u$ and should therefore not depend strongly on m_t. This is different for the mass matrix (14). As has been calculated by Li [8] this matrix leads to

$$s_3 = - \frac{\left(\frac{m_s}{m_b}\right)^{\frac{1}{2}} - \left(\frac{m_c}{m_t}\right)^{\frac{1}{2}}}{1 - \left(\frac{m_c m_d}{m_u m_s}\right)^{\frac{1}{2}}} \quad (32)$$

Obviously s_3 and hence $C_{13} = -s_1 s_3$ depend critically on m_t. For $m_t \sim 38$ GeV one even has $C_{13} = 0$. Our value for C_{13} leads to

$$\sin\theta_3 \sim 0{,}1 \quad (33)$$

This value for $\sin\theta_3$ is within the bounds given by Ellis, Gaillard, and Nanopoulos [9] and by Shrock and Wang [7].

For C_{31} we obtain

$$C_{31} = N_1^N (w_1^N)^2 N_3^P m_t^2 \left(1 - \frac{m_u m_c}{(w_1^P)^2}\right) \left(T_1 + T_2\right) \quad (34)$$

where

$$T_1 = - \left(1 - \frac{m_d}{w_1^N}\right)\left(\frac{m_u m_c}{(w_1^P)^2} + \frac{m_d}{w_1^N}\right) \quad (35)$$

$$T_2 = \frac{w_3^N}{(w_1^N)^2} m_d \frac{(m_u m_c)^{\frac{1}{2}}}{w_1^P} \left(1 + \frac{1}{w_1^P} \frac{m_c - m_u}{1 - \frac{m_u m_c}{(w_1^P)^2}}\right)^{\frac{1}{2}} \quad (36)$$

For the quark mass values used above one finds

$$C_{31} = \begin{cases} 0{,}023 \text{ for } m_t = 18 \text{ GeV} \\ -0{,}040 \text{ for } m_t = \infty \end{cases} \quad (37)$$

That means that C_{31} depends critically on m_t. This, however, is sensible since C_{31} determines the decay $t \to d$. In the region 18 GeV $\le m_t \le \infty$ T_1 is an increasing and T_2 is a decreasing function of m_t. Since T_1 varies between -0,064 and -0,040 and T_2 varies between 0,088 and zero one has

$$-0{,}064 \le T_1 + T_2 \le 0{,}048 \quad (38)$$

Since the factor which multiplies $T_1 + T_2$ in (34) is approximately equal to unity one has

$$|C_{31}| \leq 0{,}07 \qquad (39)$$

Since $C_{31} > 0$ for $m_t \sim 20$ GeV one has $\sin \theta_2 > 0$ for $m_t \sim 20$ GeV. This implies

$$|C_{23}| >> |C_{13}| \quad \text{for } m_t \sim 20 \text{ GeV} \qquad (40)$$

which means that b decays predominantly into c if $m_t \sim 20$ GeV.

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