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QUARK CONFINEMENT AND GLUON RADIATION

by

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Abstract

Three problems of quark confinement by chromoelectric flux tubes are discussed in the case of e^+e^- annihilation. The first is the phenomenon, valid in classical electrodynamics, that there is almost no Coulomb-field between two charges which are accelerated up to $v/c \approx 1$. The second is the expectation that charges with their Coulomb-field trapped into a string do not radiate perturbatively. This may influence predictions for scaling violations for quark inclusive functions. Finally some properties of QCD-jets, which have evolved down to a mass Q_o, are shown to deviate from those of observed hadronic jets.

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I. Introduction

Quark pairs produced in e^+e^--annihilation are believed to be ultimately confined by the forces generated by a string of chromoelectric flux lines connecting the two charges [(1,2,3)]. On the other hand accelerated quarks are sources of gluon radiation fields which manifest themselves in gluon jets and in scaling violations of quark inclusive structure functions [(4)]. These two rôles of the colour field don't coexist without friction. Admittedly the problems are defined only on the rather intuitive level of ref. (1) which is based on analogies of QCD to classical electrodynamics (CED). This analogy may be useful at short distances, where the strong coupling constant α_s and with it the differences between QED and QCD due to the gluon self coupling are small.

Let me, after this warning, specify the classical framework and outline the problems. The flux tube model [(1,2,3)] assumes that the chromoelectric field lines between two quarks are compressed into a tube of fixed radius R_t and that for quark separation $r \ll R_t$ the field is close to the classical dipole field, yielding the classical Coulomb force. In section 2 we shall see that for $e^+e^- \to q\bar{q}$ at large energy there is essentially no electric Coulomb field. Almost all field is of radiative nature. This makes the transition from a CED-like field configuration to the string-like configuration difficult to imagine.

The second problem is likewise connected to the interplay of radiation and strings. If at a certain r (perhaps $r = R_t$) the Coulomb field configuration is modified as compared to the CED-like one, the perturbative emission of radiation will be strongly modified. The trivial reason is that Coulomb fields and radiation fields in the perturbative configuration are tied together. The radiation field itself is not divergence free. Now it is well known (5) that the emission of radiative quanta down to intrinsic masses Q_o requires the charges to run over a distance

$$r \lesssim R_W = W/Q_o^2 \tag{1}$$

with $W = \sqrt{q^2}$ = CM-energy. If we let the radiation end at $R_t \ll R_W$, much of the perturbative gluon radiation will be suppressed. The details and the consequences for quark and hadron inclusive structure functions will be discussed in sec. 3.

Probably in order to bypass the two previous difficulties, it was suggested (6) that gluon emission really proceeds to distances of order R_W and that confinement of colour charges occurs through appropriate combination of $q\bar{q}$-pairs, the individuals of which are produced perturbatively by the branching of gluon $\rightarrow q\bar{q}$. In section 4 we look into two aspects of this scenario, namely into the mass spectrum of the perturbatively generated jets and into the probability that there are no extra $q\bar{q}$-pairs around. Both points have difficulties because of the nonvanishing probability that no gluons with reasonable masses are emitted at all.

II. Fields for $r \ll R_t$

As stated before, I assume that the fields of QCD for small quark separation

$r(r \ll R_t$ = flux tube radius), have some similarity with those of CED. Before presenting those, let me depict in fig. 1 the field configuration of the flux tube model (1) for small and large r, as it may be correct for slow quarks ($v/c \ll 1$). Of importance is the constraint for the string tension

$$\text{energy/length} = \alpha_s/2R_t^2 \tag{2}$$

where α_s is the strong coupling constant. This is due to Gauss' law.

In CED, which does not know about pair creation, we can simulate the situation be letting a $q\bar{q}$ pair sit at $r = 0$ and accelerate it by electromagnetic fields within the quantum mechanical time interval *)

$$t \sim 1/W \tag{3}$$

up to velocity $\vec{\beta}$ with $|\vec{\beta}| \approx 1$ (see fig. 2). Then the field strengths in CED (g = charge) at the point x are (7)

$$\vec{E}(\vec{x},t)/g = \frac{\vec{n}-\vec{\beta}}{\gamma^2(1-\vec{\beta}\cdot\vec{n})^3R^2}\Big|_{ret} + \frac{\vec{n}\times\vec{n}\times\dot{\vec{\beta}}}{(1-\vec{\beta}\cdot\vec{n})^3\,R}\Big|_{ret} + \tag{4}$$

+ analogous terms from the antiquark

and $$\vec{B}(\vec{x},t) = \vec{n}\times\vec{E}(\vec{x},t) \tag{5}$$

*) The detailed form of the acceleration curve has to be chosen such that the ensuing radiation spectrum agrees with that of QED (7).

with $$\vec{n} = (\vec{x} - \vec{y}(t_r))/|\vec{x} - \vec{y}(t_r)| , \tag{6}$$

where $\vec{y}(t)$ is the quark trajectory. The retarded time t_r is defined by the light cone condition

$$t_r < t, \qquad (\vec{x} - \vec{y}(t_r))^2 - (t-t_r)^2 = 0, \tag{7}$$

and $R = |\vec{x} - \vec{y}(t_r)|$. (8)

The velocity $\vec{\beta}$ and the acceleration $\dot{\vec{\beta}}$ are to be taken at time t_r.

Finally $$\gamma^2 = \frac{1}{1 - \vec{\beta}^2} \approx q^2/4m_o^2 , \tag{9}$$

where m_o is a quark mass.

The geometry is shown in fig. 3. The thin bubble of thickness $1/W$ I call the light shell. The first term in (4) is the Lorentz-boosted Coulomb field, the second term proportional to the acceleration is the radiation field. For $|\vec{x}| > t$ the fields are zero, as the terms coming from the two charges cancel. Now we see that except for $\vec{\beta}\cdot\vec{n} \approx 1$, which is around the intersection of the light shell with the jet axis, the Coulomb field vanishes asymptotically like $1/\gamma^2$, i.e. we have

$$\text{Field energy density} \sim (\frac{m_o}{W})^4 . \tag{10}$$

The field energy is concentrated on the light shell due to the form of $\dot{\vec{\beta}}(t)$.

Most importantly the energy is carried by the radiation field, and on the plane P orthogonal to $\vec{\beta}$ (see fig. 3) the electric flux integral is, in the limit $W \to \infty$, given by the radiation field alone.

We conclude that in CED for $W \to \infty$ there is no colour field energy to build up the string energy density, if one wants to preserve the energy flow of the gluon radiation. *) The origin of the string energy in QCD therefore cannot be seen in analogy to CED. Whether the gluon selfcoupling can change this situation, is not clear. Naively one could assume that the gluons, emitted because of the acceleration $\dot{\vec{\beta}}$ of the quark, represent themselves accelerated charges whose acceleration time coincides with that of the quarks. They produce then again bremsstrahlung which is also localized on the light shell. The effects of the gluon selfcoupling can also be estimated by a physical argument. In principle the gluons could feel a strong dispersion such that the gluon radiation field acquires a long "tail" into the interior of the light shell. This would mean that a significant fraction of the energy moves with much less than the speed of light. Since we know experimentally that gluon jets consist of pions with $\beta \approx 1$, such a picture is highly implausible. Thus I expect the field configuration of QCD at short distances not to differ significantly from that of CED, although formally the number of gluons emitted in an arbitrarily short time interval diverges [8] for $W \to \infty$ (see, however, sec. IV on the probability of having no gluons at all).

*) Ref. [1] contains conflicting statements. The rôle of the radiation field is not clarified in that paper.

III. String formation at fixed quark separation

The incompatibility of the radiation-like field configuration with the string configuration, emphasized in the last section, of course will have consequences on the perturbative gluon radiation, if we insist on string formation at a fixed quark separation, say R_t.

For a string we expect the Coulomb field of fig. 1b, i.e. the field lines "turn back" within a distance R_t. Since in the field configuration of eq. (4) the Coulomb field lines and the radiation field lines are continuously connected especially in the region S (fig. 3), the radiation field must change there considerably in the case of string creation. Especially it must be divergence free by itself, i.e. the field lines must be closed as shown in fig. 4. The detailed form of this modification is not known. As a definite example I shall consider what happens if the charges of the quarks are stopped or neutralized at the point $r = R_t$. The essential point is that also in this case the radiation field lines have to turn back as in fig. 4 contrary to eq. (4). We want to study the consequences of this stopping of the charges (not of the quarks) on the quark structure functions and ultimately on inclusive hadronic spectra and their scaling violations. The change in the gluon production amplitudes due to stopping the charge has been discussed *) in ref. [9]. If p and k are the external quark and gluon momenta respectively, the Feynman propagator of the internal quark in the matrix element has to be substituted by (see fig. 5a

*) Actually in [9] the modification of radiation due to the quark deceleration has been studied. This is inconsistent, as we saw, if the decelerating force is due to the chromoelectric string: Once this is formed, the charges are essentially neutralized.

for kinematics)

$$\frac{1}{2p\cdot k} \rightarrow \frac{1}{2p\cdot k}\left(1 - \exp\left\{ i\, p\cdot k \frac{R_t}{W}\right\}\right) \tag{11}$$

The oscillations given by the exponential in (11) for large $p\cdot k$ were shown in [9] to depend on the details of the deceleration and are likely to disappear in realistic situations. The vanishing of the bracket for $p\cdot k \rightarrow 0$ and its slope with respect to $p\cdot k$ are, however, quite general. Eq. (11) is calculated neglecting the recoil to the quark due to gluon emission. The final quark momentum distribution can be found by solving the Bethe-Salpeter equation for the quark inclusive structure function. I represent the equation graphically in fig. 5b and refer to the details to ref. [10]. The important modification compared to the usual Bethe-Salpeter equation is that we imitate the suppression of small internal momentum transfers according to (11) by a fermion propagator

$$\frac{1}{2p\cdot k} \rightarrow \frac{1}{2(p\cdot k+\Lambda^2)} \tag{12}$$

with $\Lambda^2 = W/R_t$. (13)

The quark inclusive cross section $d\sigma/dx$ with

$$x = \frac{2p\cdot q}{q^2}\ , \tag{14}$$

obtained by solving the Bethe-Salpeter equation with external quark and gluon masses taken as zero has been folded with a fragmentation function

$$D(z) = \frac{1-z}{z}(1-1.3(z-z^3)) \tag{15}$$

to yield a realistic hadronic inclusive spectrum $d\sigma_h/dx$. In fig. 6 we show the ratio of the hadronic spectrum for W = 30 GeV to that at W = 100 GeV for the option (13) and for $\Lambda^2 = 1$ GeV^2, independent of q^2. Clearly the first choice removes about half of the scaling violations of the latter (the ratio 1 corresponds to exact scaling).

There is no (and probably will not be) experimental evidence for or against scale breaking of this order of magnitude. The above considerations show, however, that confinement and perturbative effects may not be as cleanly separable as it is assumed [(4)]. Although the space-time picture for deep inelastic scattering is more complicated than in the e^+e^--case, one may doubt that scaling violations predicted there will survive a string formation at a fixed distance without large modifications.

IV. Preconfinement?

Turning around the (wellknown) argument of the last section, unperturbed evolution of jets down to masses Q_o requires

$$R_t = W/Q_o^2 = R_W. \tag{16}$$

For Q_o = 1 GeV and W = 100 GeV we thus need a quark separation of 20 Fermi. Intuitively it seems hard to imagine that over such long distances there is a significant exchange of momentum between the two jets. A string which will start to be created at such distances will have Fourier-components $\sim Q_o^2/W$

which when added to the leading quark momentum, will produce a shift in squared mass of order Q_o^2. In the preconfinement scheme of ref. [(6)] colour is neutralized by recombination of colourless clusters of $q\bar{q}$ pairs, most of which belong to the same jet, as those have predominantly small relative mass and small spatial separations. These recombinations preserve the jet mass. Some mass will be generated by soft clusters which might bring a quark line from one jet to the other. I expect this effect to be small as it can go in both directions and will cancel out in first order. We thus simply should compare jet masses in QCD with experimental ones. Unfortunately, the latter ones are not yet available, since neutral particles and baryons are not completely detected. I have to rely on Monte-Carlo calculations which fit charged particle tracks. In fig. 7 I show the integrated QCD-mass spectrum *) coming from the program of ref. [(10)] with external quark and gluon masses of Q_o = 700 MeV for W = 30 GeV. I have chosen to present the integrated form

$$\frac{1}{\sigma_{tot}} \int_0^M dm \quad \frac{d\sigma_{jet}}{dm} \tag{17}$$

because there is a δ-function in $d\sigma_{jet}/dm$ at $m = Q_o$ corresponding to no gluon emission. The curve $MC^{\pm}$ comes from a Monte-Carlo program used by TASSO [(13)] and contains 3 and 4 jet events plus fragmentation of quarks and gluons à la Field and Feynman [(14)]. Only charged $\pi^{\pm}$ and K's are included.

*) Similar spectra have been obtained before by E. Pietarinen (unpublished) using the approach of ref. [(11)]. The inconsistency of the QCD mass spectrum with the experimental one was pointed out to me by him. The large fraction P_o of events with mass Q_o is identical to the function $N_q(Q^2)$ in the quoted paper.

The curve MC^o also contains neutrals, but no gluons [15]. Baryons are included nowhere. The apparent disagreement is mainly due to the large fraction P_o of no-gluon events. It is 30 % of the total rate at 30 GeV (see ref. [11]). Apart from the quantitative difference between the mass distribution this fraction puts us in the puzzling situation of having no $\bar{q}q$-clusters around to achieve preconfinement (see also ref. [12] for the lack of clusters). Although by lowering the gluon mass we can also lower P_o, the basic problem does not disappear quickly thereby nor by increasing W.

V. Conclusions

Confinement of quarks by strings is a problematic concept. We have seen that in e^+e^--annihilation, as $W \to \infty$,

i) At small quark separation the space between the quarks, which should be filled by electric flux for the string, is rather empty,

ii) String formation at fixed R_t changes perturbative scaling predictions, which are believed to be rather fundamental,

iii) Perturbative QCD-mass distributions do not match experimental ones, if jets run "down to Q_o", and

iv) Preconfinement cannot work with probability one, since at finite energies often there are no gluons and, a fortiori, no $q\bar{q}$-clusters at finite energies.

The general feeling that confinement and short range effects are cleanly separated in hard processes does not seem to be true in the string picture, due to the double rôle of the gluon field. The essential ingredients in our analysis are Lorentz-invariance ($\rightarrow$ contraction of the Coulomb field) and four dimensions ($\rightarrow$ transverse radiation fields). Consequences for two dimensional models and lattice theories are obvious.

Acknowledgement

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Figure Captions

Fig. 1 Chromoelectric field configuration, possibly valid for static quarks:

a) separation $r \ll R_t$,

b) $r \gg R_t$.

Fig. 2 Velocity of quark in $q\bar{q}$ creation at large q^2.

Fig. 3 Kinematical definitions for field configuration corresponding to fig. 2 and formula (4).

P denotes surface to test flux of radiation field.

S denotes surface to test flux of radiation field after string formation.

Fig. 4 Separation of radiation field and Coulomb field after string formation (purely schematic!)

Fig. 5 a) Kinematics for gluon emission:

k = gluon momentum,

p = quark momentum.

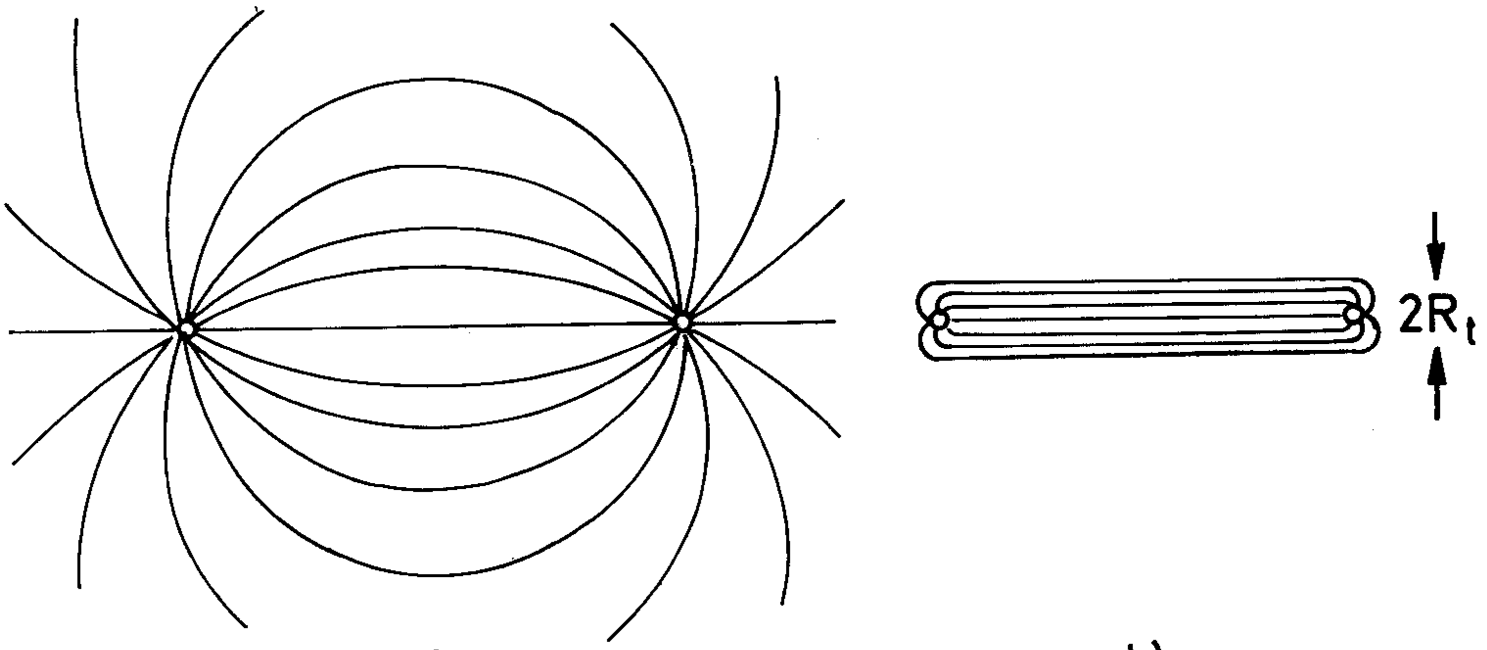
b) Bethe-Salpeter equation for multiple gluon emission.

Fig. 6 Ratio of hadronic inclusive cross sections at W = 100 GeV and W = 30 GeV:

$$S(x) = \frac{d\sigma_h/dx \ (W=30\,\mathrm{GeV})}{d\sigma_h/dx \ (W=100\,\mathrm{GeV})}$$

for Λ^2 as given by eq. (13) and for $\Lambda^2 = 1\ \mathrm{GeV}^2$. $R_t = 10\ \mathrm{GeV}^{-1}$.

Fig. 7 Integrated jet mass spectrum for QCD (Q_o = 700 MeV, W = 30 GeV) and for realistic Monte Carlo simulations [13,15] (see text). $MC^{\pm}$ refers to charged particles only, MC^{o} includes neutrals.



a) b)

Fig.1

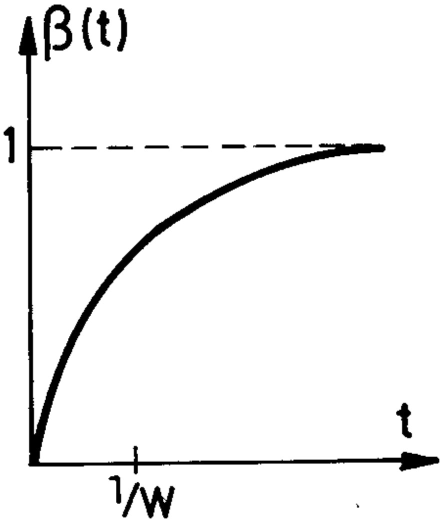


Fig.2

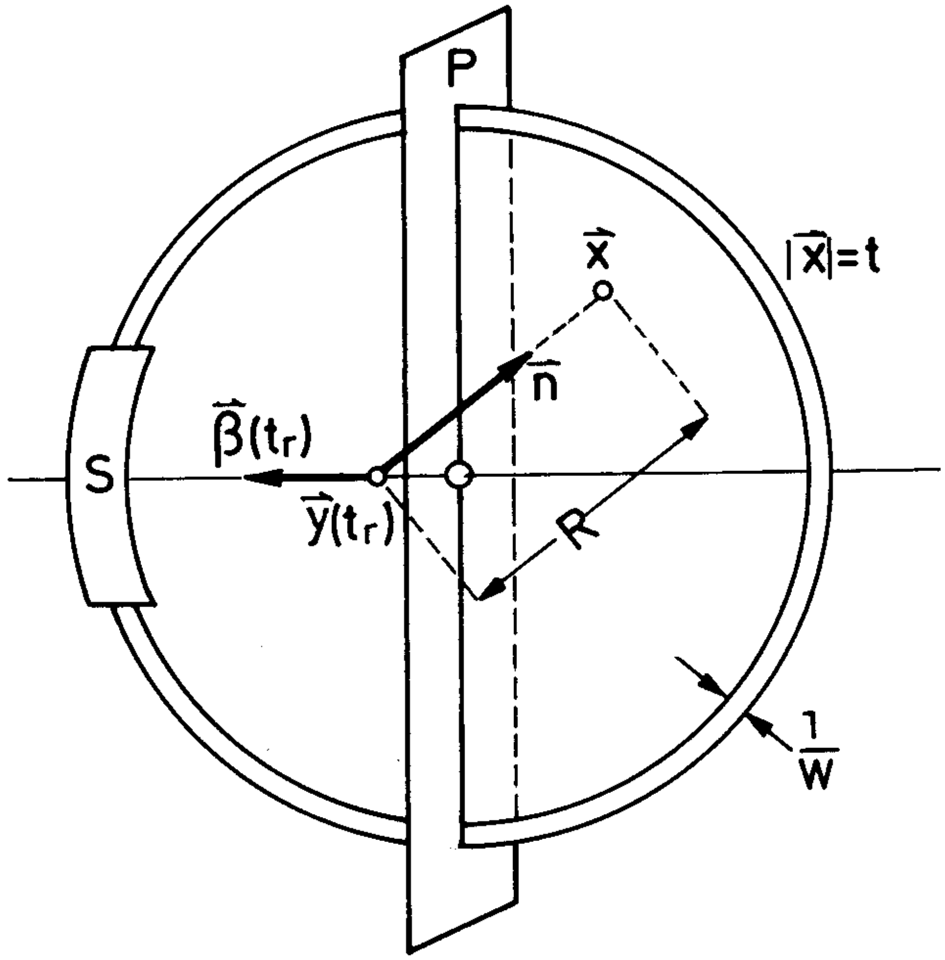


Fig.3

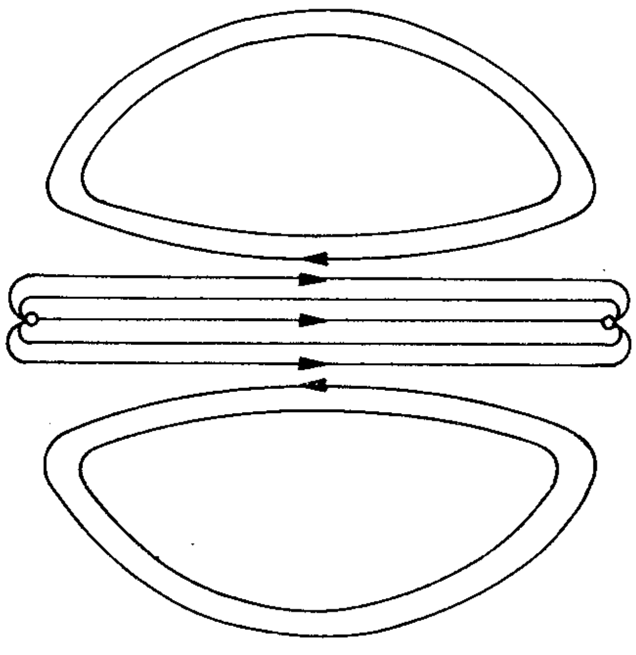


Fig.4

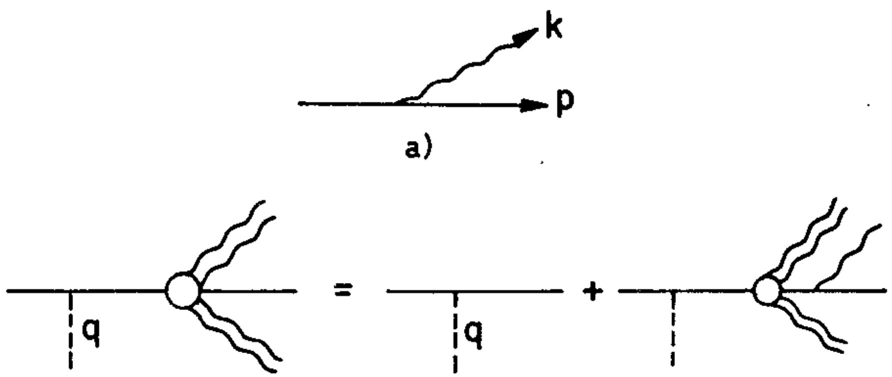


Fig.5

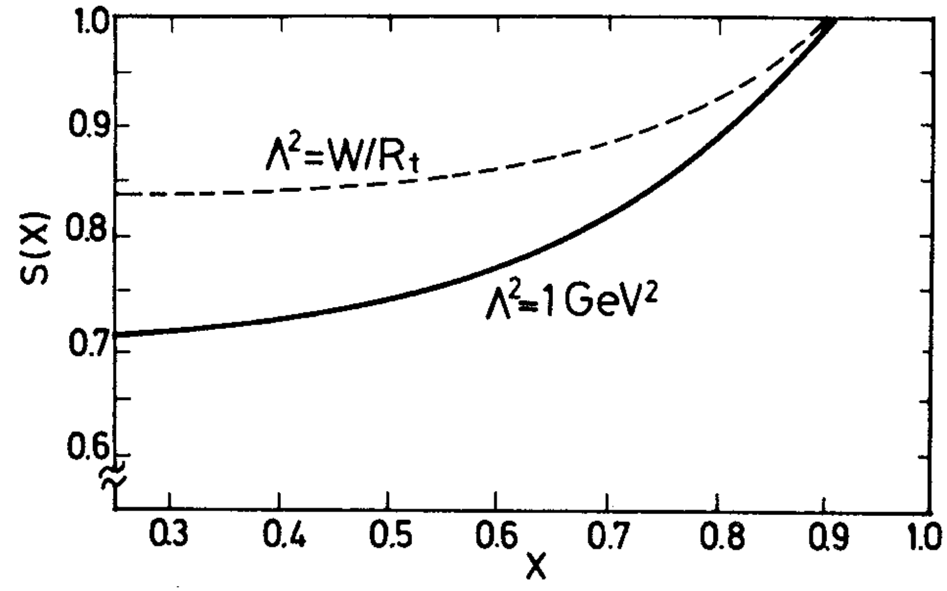


Fig.6

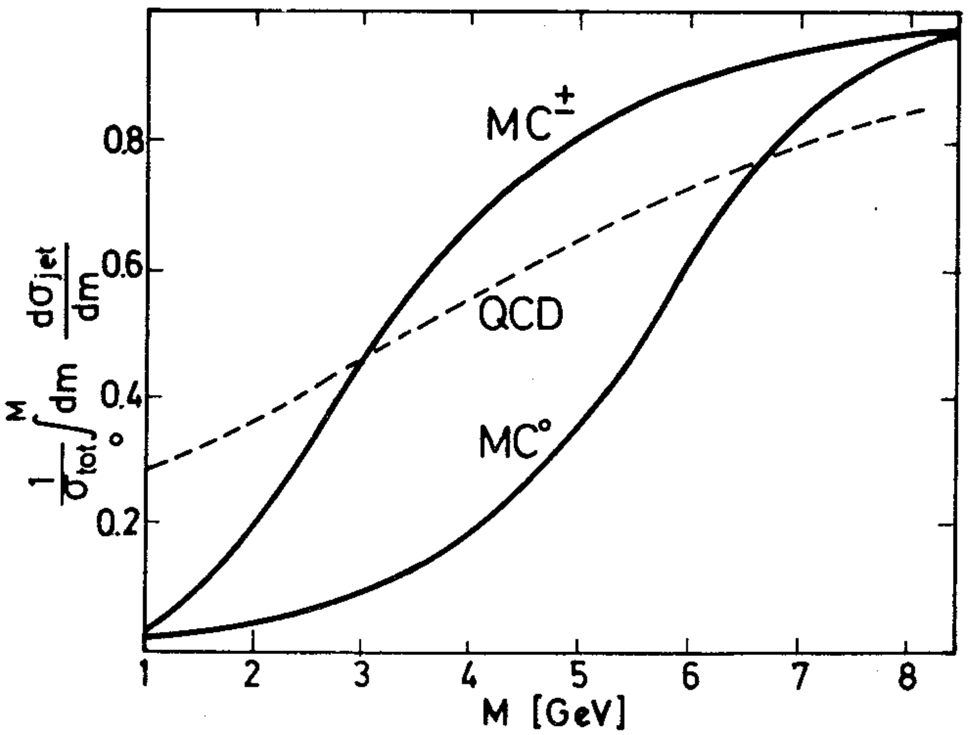


Fig.7