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ON NON-PERTURBATIVE EFFECTS IN THE JET PHYSICS*

by

M. Voloshin and V. Zakharov

Institute of Theoretical and Experimental Physics, Moscow
and
Deutsches Elektronen-Synchrotron DESY, Hamburg

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Institute of Theoretical and Experimental Physics, Moscow
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Abstract

We consider quark scattering off non-perturbative gluonic fields in the vacuum and its impact on the hadronic final state in the e^+e^--annihilation. A conservative estimate gives a non-perturbative $p_\perp \sim 1$ GeV. The modification of the final state survives even at an arbitrary high energy, where the corrections to the total cross-section die away. We also argue that the coupling constant extracted from the perturbative treatment of the jet phenomenology can well be different from that governing the correction to σ_{tot}.

The structure of the physical vacuum state in QCD is quite different from what is suggested by perturbation theory. In particular, the quark fields develop a nonvanishing vacuum expectation value $\langle \bar{q}q \rangle \neq 0$ which is responsible for the spontaneous breaking of chiral symmetry [1]. Note that $\langle \bar{q}q \rangle$ vanishes in the chiral limit in perturbation theory so that we are dealing with a nonperturbative effect. More recently, it has been suggested [2] that the gluonic field also develops vacuum-to-vacuum matrix elements which go beyond the perturbation theory. In particular

$$\langle 0 | : G^a_{\mu\nu} G^a_{\mu\nu} : | 0 \rangle \neq 0 \qquad (1)$$

where $G^a_{\mu\nu}$ is the gluon field strength tensor and the N-ordering refers to the perturbation theory.

The only well-understood nonperturbative fluctuation which contributes to (1) is provided by the instanton solution [3]. However, the instanton calculus does not allow to construct a real theory of the vacuum state.

The vacuum expectation value (1) enters the so called QCD sum rules [2] which look like

$$\int ds\, \sigma(e^+e^- \to X)\, s\, \exp(-s/M^2) = = const \cdot M^2 \left[1 + \frac{\alpha_s(M)}{\pi} + c \frac{\langle \alpha_s G^2 \rangle}{M^4} + \dots \right] \qquad (2)$$

and which can be derived by considering vacuum polarization in the Euclidean region alone (the bridge to the observable cross section is provided by the general dispersion relations). The evidence in favor of the nonvanishing $\langle \alpha_s G^2 \rangle$ is ample in the charmonium and bottonium physics [2,4].

Given the effect of the nonperturbative gluon fluctuations on the integrals like (2) one may wonder how the cross section itself is affected. It turns out that asymptotically the correction to the total cross section vanishes along with the (current) quark mass [2]:

$$\frac{\delta\sigma}{\sigma} \underset{s\to\infty}{\sim} \frac{m_q^2}{s^6} \langle \alpha_s G^2 \rangle .$$

Moreover, the power corrections become important only at a low mass scale of order $M^2 \sim m_\rho^2$ where the integrals over the cross section are dominated by resonances.

The vanishing of the correction to the total cross section does not necessarily imply, however, that the effect of the vacuum fields on the final state is also negligible. Indeed, the perturbative correction to the total cross section [5], $\alpha_s(s)/\pi$, also vanishes at high energy so that the total cross section of e^+e^- annihilation approaches the bare quark one. On the other hand, gluon emission can never be neglected as far as the final state in e^+e^- annihilation is concerned since it gives rise to terms proportional to $\alpha_s \ln^2 \frac{s}{\mu^2}$ where μ is the intrinsic infrared cutoff. Thus, the effect on the final state becomes especially important at high energies where the change in the total cross section is negligible (the large inelastic cross section $\sigma(e^+e^- \to q\bar{q}g)$ is cancelled by the same large negative correction to the cross section of "elastic" annihilation $e^+e^- \to q\bar{q}$).

Nowadays, with jet physics attracting a great deal of attention, it is only natural to ask whether something similar happens to the non-perturbative corrections and whether they can be important in jet physics at high energy

despite the absence of any effect on the total cross section. We try to answer this question in the present note.

Unfortunately, an analysis of the final state is always a very delicate problem in QCD, since one cannot deal directly with large distance physics. One usually makes some cavalier assumptions concerning effects of confinement (or lack of it), which in the case of perturbation theory turn to be justified, at least a posteriori. For nonperturbative fields the problem is even harder since little can be said about exact field configurations which are responsible for, say, the nonvanishing vacuum expectation value (1). Even such basic problems as introduction of the vacuum fields in a manifestly Lorentz-invariant way are not straightforward to solve. For example, leaning on the instanton gas with an infrared cutoff easily violates general properties of QCD [2].

Under the circumstances, we try mostly simple estimates. They seem to be safe enough to demonstrate that the nonperturbative corrections to the final state persist at very high energies. Several important questions remain unanswered, however. The only reason to present these estimates which are vague in many points is our belief that introduction into theoretical consideration of a new process -- scattering off the vacuum gluonic fields -- will eventually pay off.

2. Simple-minded estimates

Consider first the simplest picture which visualizes the vacuum field as an ordinary external field affecting the trajectory of a quark injected into the vacuum by the electromagnetic current. Then the quark travelling through a vacuum fluctuation of dimension ρ acquires a transverse momentum of order

$$p_\perp^2 \simeq \frac{4}{3}\pi \alpha_s H^2 \rho^2 \qquad (p_\perp^2 << p_\parallel^2) \tag{3}$$

where α_s is the strong interaction coupling constant, H is the (color) magnetic field and we neglect the electric field for the moment.

Thus, we need to estimate the magnetic field H and the fluctuation size ρ . The former estimate is straightforward. Indeed, by virtue of Lorentz-invariance

$$\langle \alpha_s H^2 \rangle = \frac{1}{4} \langle \alpha_s G^2 \rangle \simeq 0.01\ GeV^4 \tag{4}$$

where we used the numerical value found in the analysis of Ref. 2. As for the estimate of ρ it is more ambiguous. There are essentially two ways to estimate ρ. The first one is to elaborate some particular model of the vacuum fluctuations and the second is to look more carefully into the QCD phenomenology.

Unfortunately, at present one understands only the instanton solution which most probably has little bearing on the real vacuum fields. Still, one can try the dilute gas approximation [6] with an infrared cutoff to get the feeling of how large ρ is. One such an estimate is published [7]:

$$\rho_c \simeq (0.2\ GeV)^{-1} \tag{5}$$

It arises from the fit of the known value of $\langle \alpha_s G^2 \rangle$

A more reliable way to find ρ is provided by the QCD sum rules. The physical

picture behind them is that quarks do not disturb vacuum fields and the sum rules are expected to be valid over the space-time intervals that are confined to a single vacuum fluctuation. On the other hand, phenomenologically the sum rules for bottonium [4] work for time intervals of order $(0.1\ GeV)^{-1}$. Of course there could be some geometrical factors but the estimate obtained is rather close to what instanton calculations indicate. Substituting then the numbers we get quite a sizable momentum

$$(p_\perp^2)_{quark} \simeq 1\ GeV^2 \tag{6}$$

The same approach applied to a gluon yields

$$(p_\perp^2)_{gluon} \simeq \frac{9}{4} (p_\perp^2)_{quark} \tag{7}$$

where the difference between quarks and gluons is common to all the jet physics calculations [8-10].

Note that so far we referred actually to the stimates of ρ in Euclidean space-time where short interval $\Delta t^2 + \Delta x^2 \simeq \rho^2$ means short space and time distances. In the real Minkowski space there could be some cancellations between Δt^2 and Δx^2. These would only increase Δx .

Incorporation of Lorentz-Invariance

The consideration of the previous section gives a feeling of how important the scattering off the vacuum fields could be numerically but fails to account explicitly for an important property of the vacuum state, that is its Lorentz-invariance. Here we try to fill the gap.

Consider again the final state physics in e^+e^- annihilation into hadrons. It is helpful to keep in mind that the ordinary perturbative bremsstrahlung of gluons by quarks can be translated into the language of quark scattering off the vacuum fluctuations. In perturbation theory the latter are essentially the zero-point fluctuations of the harmonic oscillator and the momentum transferred from the quark to the vacuum appears as an excitation of the gluonic vacuum, that is, the plane-wave gluon.

There is one problem which perturbation theory helps to recognize. Usually it does not make much sense to consider interaction of a single quark, without keeping in mind that there should be another one, because of color charge conservation. The calculations simplify in the case of the gluon bremsstrahlung in the forward direction. There in the so-called double log approximation the results of the quantum theory essentially coincide with the classical radiation theory. This fact was used recently to elaborate tests of the perturbative QCD [9,11]. Here we will also consider the case of the quark momentum much larger than the momentum transferred to the gluonic field.

In this kinematical region the amplitude of the interaction with the gluon field can be written as

$$T = T_0 \left(\frac{p_\mu A_\mu(k)}{pk} - \frac{p'_\mu A_\mu(k)}{p'k} \right) \qquad (8)$$

where T_0 is the amplitude of the bare process, $e^+e^- \to q\bar{q}$, p and p' are the momenta of the quarks and $A(k)$ is the Fourier transform of the gluonic 4-potential. To evaluate the cross section associated with the amplitude (8) one needs to know the density matrix:

$$\rho_{\mu\nu}(k) = \langle 0 | A_\mu(k) A_\nu(-k) | 0 \rangle . \qquad (9)$$

In a Lorentz-invariant gauge the density matrix is given, in general, by two invariant functions:

$$\rho_{\mu\nu}(k) = -F(k^2)\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + G(k^2)\frac{k_\mu k_\nu}{k^2} \qquad (10)$$

The gauge dependent longitudinal term $G(k^2)$ gives no contribution to the physical cross section while the function $F(k^2)$ is gauge independent provided that the gauge fixing condition preserves Lorentz-covariance.

In perturbation theory

$$(VT)^{-1} F(k^2) = \frac{4}{3} g^2 \, 2\pi \, \delta(k^2)\, \theta(k_0) \qquad (11)$$

where VT is the normalization 4-volume, $\theta(k_0)$ is the step function and g is the QCD coupling constant. Since we assume that fluctuations of the physical vacuum are far from being simple zero-point harmonic modes we replace eq. (11) by the more general one:

$$(VT)^{-1} F(k^2) = \frac{4}{3} g^2 f(k^2)\, \theta(k_0) \qquad (12)$$

Thus, because of Lorentz-invariance the nonperturbative effects can be described by a single function of the kinematical variable k^2. Little can be said specifically about the magnitude and shape of $f(k^2)$ at present. Most probably, it is a bell-like, with the maximum at $k^2 \simeq 0$.

The correction to the cross section is then given by

$$d\sigma = d\sigma_0 \, \frac{4}{3} g^2 \frac{s}{(pk)(p'k)} \, f(k^2) \, \frac{dk^2}{2\pi} \, \theta(k_0) \frac{d^3k}{(2\pi)^3 2k_0}$$

where $d\sigma_0$ is the bare cross section and $s = 2pp'$ (we consider massless quarks so that $p^2 = p'^2 = 0$).

In the bremsstrahlung region

$$\frac{s}{(pk)(p'k)} \simeq (k_\perp^2 + k^2)^{-1}$$

and

$$d\sigma = d\sigma_0 \times \left\{ \frac{4}{3} \frac{\alpha_s}{4\pi} \theta(k_0) \frac{dk_0}{k_0} \frac{dk_\perp^2}{k_\perp^2 + k^2} f(k^2) \frac{dk^2}{2\pi} \right\} \quad (13)$$

which reduces to the well known perturbative result once $f(k^2) = 2\pi\,\delta(k^2)$. It is clear that the function $f(k^2)$ simulates a sort of the Källen-Lehmann-type distribution over nonvanishing gluon masses [+]. As was already mentioned,

+) We would not claim any analytical properties for $f(k^2)$, however.

our present knowledge of $f(k^2)$ is confined to the fact that at sufficiently large k^2 it vanishes so that k^2 is bounded.

What is most interesting, is that some integrals of $f(k^2)$ may be directly measurable. Indeed, consider $k_\perp^2 \gg (k^2_{\text{characteristic}})$. Then the integration over dk^2 results in a mere renormalization of the coupling constant by the factor

$$I = \int f(k^2)\, dk^2/2\pi \quad (14)$$

In perturbation theory the integral is unity and α_s is the standard perturbation theory coupling constant. An independent measurement of α_s can be provided by, say, the total cross section where all the corrections discussed now cancel out (see the Introduction). With the nonperturbative effects included, the effective α_s, measured by analysing the final state, can be different.

In perturbation theory the integral (14) reduces to the renormalization factor Z_3 which is eventually absorbed into the coupling constant renormalization. This occurs because of normalization of the gluon wave-function to the one-quantum state. Beyond perturbation theory the exitation of the gluonic vacuum is realized in physical particles so that it does not seem, at least at present, necessary for the same normalization to hold. As far as the Euclidean space calculation of the asymptotic correction to σ_{tot} is concerned, the non-perturbative contribution dies away rapidly (see the Introduction) and ordinary perturbative pattern sets in.

Phenomenological Implications and Discussion

There are several points which seem to deserve further comments.

1. We obtained a rather large value of $p_\perp^2$ (see eqs. (6) and (7)) due to the quark or gluon interaction with the vacuum fields. To realize such p one needs to have quarks and gluons of much higher energy. This condition certainly is not met for gluons at present energies. Thus, it is natural to expect some preasymptotic growth of $p_\perp$, which could be partly responsible for the observed broadening of the jets (for a review see Refs. 10,12)).

Scattering off the vacuum field could be partly responsible for the intrinsic quark $p_\perp$ introduced phenomenologically. Note, however, that the nonperturbative effects are described in terms of k^2, not $k_\perp^2$ as is assumed phenomenologically (for extensive phenomenological study see Ref. 13).

2. Another way to determine the strong interaction coupling constant from jet physics is to study what is called the clean three-jet events which are less dependent on the details of the quark or gluon fragmentation functions. [+)] As is argued in the previous section, due to nonperturbative effects, the coupling constant governing the three jet events could be different from that entering the total cross section. On the other hand, the nonperturbative effects in the total cross section are proportional to $\langle G^2 \rangle$ and vanish at large energy. The large nonperturbative correction to the three-jet cross-section is cancelled in the total cross-section rate by the corresponding

+) We would like to acknowledge very useful discussions of this point with G. Wolf and Sau Lan Wu.

correction to the two jet cross-section.

3. There is a recent suggestion by Parisi and Petronzio [14)] to introduce a gluon mass of order 0.8 GeV into low energy QCD phenomenology. The function $f(k^2)$ introduced above could partly imitate the effect of the gluon mass. In this respect our approach is close to that of Ref. 14. We differ drastically, however, as far as corrections to the total cross section are concerned. In particular, there is a theorem [15)] that there is no correction of order μ^2/s to the total cross section of the e^+e^- annihilation. The gluon mass, as introduced in Ref. 14, would induce such corrections. Thus, our proposal is rather opposite to that of Ref. 14. We find large correction to the final state which is cancelled in the total cross section, while Parisi and Petronzio look for a large correction to the total cross section and leave QCD corrections at large momentum transfer intact.

4. $B_{\mu\mu}(\Upsilon)$; we also note that the two cases could be differentiated experimentally by measuring the total hadronic width of the Υ-resonance. According to our point of view the genuine coupling constant is small which leads to the prediction [16)]

$$B_{\mu\mu}(\Upsilon) = (4.0 \pm 0.5)\% \quad .$$

On the other hand, from Ref. 14 one can deduce

$$B_{\mu\mu}(\Upsilon) \simeq 1.5\% \qquad \text{(nonvanishing gluon mass)} \quad .$$

Conclusions

Unlike the total cross-section which is determined by short distances at large energies, the final state is sensitive to the gluon field integrated along the particle trajectory. As a result, the correction to the final state is different in nature from that studied for the total cross section. Numerically, the nonperturbative corrections turn out to be large. An intriguing possibility is that they persist even at very large $p_\perp$, resulting in a multiplicative renormalization of the coupling constant in the double log (or forward bremsstrahlung) region.

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