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Abstract

The method of asymptotic dynamics for large times developed by Kulish and Fadde'ev for QED is applied to QCD. We study the solution and calculate the on shell quark form factor in leading logarithmic order.

In a very translucent paper Kulish and Fadde'ev [1] have developed an approach which correctly describes the problem of the electromagnetic interaction at large distances. They define asymptotic states by means of an asymptotic Hamiltonian which takes masslessness of the photon and thus the long range of the Coulomb forces into account. With the help of these tools they define an S-matrix which is free of the infinite Coulomb phase problem and - following the earlier work of Chung [2], Greco and Rossi [3], and Kibble [4] on coherent states and infrared singularities - does not suffer from infrared singularities.

In this paper we sketch how the approach of Kulish and Fadde'ev can be carried over to QCD. Specifically, we discuss the on shell QCD form factor which has been studied in the work of Cornwall and Tiktopoulos [5] and Korthals Altes and de Rafael [6]. Cornwall and Tiktopoulos study the form factor by explicit calculation up to sixth order and conjectured renormalization group equations. They find exponentiation as Sudakov [7] in his classical QED calculation. Here we indicate how the leading logarithmic behaviour of the mass shell form factor can be derived from the asymptotic quark-gluon part of the QCD Hamiltonian.

In analogy to the argument of Kulish and Fadde'ev [1] the asymptotic limit $\tilde{H}_I$ for large times of the interaction of quarks and gluons in QCD can be formulated as

$$g\tilde{H}_I(t) = \int g\,\tilde{j}^a_\mu(x) A^\mu_a(x)\, d^3x \qquad (1)$$

with the gluon field $A^a_\mu(x)$ being coupled to the asymptotic quark current

$$\tilde{j}^a_\mu(x) = \rho^a \frac{p_\mu}{\omega} \delta^3\left(\frac{\vec{p}}{\omega}t - \vec{x}\right) \qquad (2)$$

of a color changing charge density

$$\rho^a = \sum_{i,k}\sum_r \left[a^+_i(\vec{p},r)\frac{\lambda^a_{ik}}{2} a_k(\vec{p},r) - b^+_i(\vec{p},r)\frac{\lambda^a_{ik}}{2} b_k(\vec{p},r)\right] \qquad (3)$$

moving with the velocity $\vec{v} = \vec{p}/\omega$. The current conserves separately quark and antiquark number and is diagonal in the momentum degree of freedom, in color space it is obviously a matrix. This then is a feature different from the electromagnetic case where as a consequence of the abelian character of the gauge group, the asymptotic current $\tilde{j}_\mu(x)$ is equivalent to a c-number. Since the color current $\tilde{j}^a_\mu(x)$ is an operator, the commutator of $\tilde{H}_I(t)$ with itself at two different times t', t" is not a c-number as in the case of an abelian gauge theory like QED.

With the help of the asymptotic Hamiltonian eq. (1) we transform the states $|\varsigma\rangle$ of the interaction picture into the asymptotic states

$$|\tilde{\varsigma}\rangle = V^+ |\varsigma\rangle . \qquad (4)$$

Here the unitary operator V(t) is the solution of the evolution equation

$$i\frac{dV}{dt} = g\tilde{H}_I V . \qquad (5)$$

Of course, since the commutator of $\tilde{H}_I$ with itself at different times is not a c-number, the T-product-solution of equ. (5) cannot be resolved into the simple structure of eq. (9) of ref. 1, which is of the form

$$V_{QED}(t) = \exp R(t) \cdot \exp i\phi(t) \tag{6}$$

where $\phi(t)$ represents the logarithmically divergent Coulomb phase and R describes the emission and absorption of photons. Instead we have to exploit the much more involved solution of W. Magnus [8], which also is of exponential type, however, with an infinite series in the exponent

$$V = \exp \Omega \tag{7}$$

where the exponent fulfills the equation

$$\Omega(t) = -ig \sum_{k=0}^{\infty} \beta_k \int_{t_0}^{t} \{ \tilde{H}_I(t'), \Omega^k(t') \} \, dt' . \tag{8}$$

Here the curly bracket stands for a repeated commutator of order k in Ω

$$\{ \tilde{H}_I, \Omega^k \} := [\cdots[\tilde{H}_I, \Omega]\,\Omega]\cdots]\,\Omega] \quad ,$$
$$\{ \tilde{H}_I, \Omega^0 \} := \tilde{H}_I \quad . \tag{9}$$

The coefficients β_k are related to the Bernoulli-numbers B_{2m} [9]

$$\beta_0 = 1 \; , \; \beta_1 = \tfrac{1}{2} \; , \; \beta_2 = \tfrac{1}{12} \; , \; \beta_{2m} = \frac{(-1)^{m-1}}{(2m)!} B_{2m} \; , \; m = 2, 3, \ldots$$

The implicit equation (8) can be solved with a power series expansion in g

$$\Omega(t) = \sum_{n=1}^{\infty} (-ig)^n \, \Delta_n(t) \quad . \tag{10}$$

Comparing the coefficients of equal powers in g after inserting eq. (10) into eq. (8) we have for the first few coefficients

$$\Delta_1(t) = \int_{t_0}^{t} \tilde{H}_I(t_1) dt_1 \quad , \quad \Delta_2(t) = \frac{1}{2} \int_{t_0}^{t} [\tilde{H}_I(t_2), \Delta_1(t_2)] \, dt_2 \quad ,$$
$$\Delta_3(t) = \frac{1}{2} \int_{t_0}^{t} [\tilde{H}_I(t_3), \Delta_2(t_3)] \, dt_3 + \frac{1}{12} \int_{t_0}^{t} [[\tilde{H}_I(t_2), \Delta_1(t_2)]\, \Delta_1(t_2)] \, dt_2 \; . \tag{11}$$

Actually the general expression for Δ_n can be given as a repeated commutator of $\tilde{H}_I$ and the coefficients Δ_m, $m < n$. A slightly different expansion has been given by Wilcox [10].

Of course, the above expansion is of easy usage only if low orders dominate the result in some way. Actually the calculations up to fourth order in the exponent show for the color singlet matrix elements $\langle q\bar{q} \,|\, \Delta_n(t) | q\bar{q} \rangle$, n = 1,2,3,4 that asymptotically, i.e. for large times only the first term Δ_1 is leading, if not all gluon operators are contracted. The terms without remaining gluon operators contribute to the generalization of the Coulomb phase $\phi(t)$, eq. (6), of QED to the phase of QCD. In contrast to QED, where the Coulomb phase is determined by a single logarithm, in QCD it is given by an infinite sum of powers $c_A^{n-1} \left(\frac{g^2}{4\pi} c_F \ln |t|/t_0 \right)^n , n \geq 1$, where c_A is the eigenvalue of the quadratic Casimir operator of the gluons. Of course, in QCD a deviation from the Coulomb phase of QED is to be expected if quarks are confined. As long as we consider processes with the production of gluons, we drop the phase and thus have only to take the first order contribution Δ_1 in the exponent of Magnus' solution into account. Then V(t) is given by

$$V(t) = \exp\left\{-i\int_{t_0}^{t} g\,\tilde{H}_I(t')\,dt'\right\} \tag{12}$$

transforming every state in the Fock space into a coherent state, i.e. a superposition of states with growing numbers of gluons.

We now turn to the calculation of the on shell form factor of QCD of a colour singlet state of quark and antiquark in the asymptotic approximation. The asymptotic electromagnetic current $\tilde{J}_\mu(x)$ is given by

$$\tilde{J}_\mu(x) = V^+(t)\, J_\mu(x)\, V(t) \tag{13}$$

where $J_\mu(x)$ is the electromagnetic current of the point-like quarks. The colour singlet form factor is then determined by

$$\lim_{t\to\infty} \langle 0|\,\tilde{J}_\mu\,|\sum_b q_b\bar{q}_b\rangle = \langle 0|\,J_\mu\,|\sum_b q_b\bar{q}_b\rangle \cdot F(s) \tag{14}$$

where $s = (q_b + \bar{q}_b)^2$ and

$$F(s) = \lim_{t\to\infty} \langle 0|\exp\left\{-ig\int_{t_0}^{t} \tilde{j}^a_{2\mu}(x')\,A^\mu_a(x')\,d^4x'\right\}|0\rangle . \tag{15}$$

Here $\tilde{j}^a_{2\mu}(x)$ is the asymptotic color current of quark and antiquark.

For the calculation of the form factor normal ordering of the gluon creation and annihilation operators has to be carried out. This is done with the help of the Zassenhaus formula [11]

$$\langle 0|\exp\left\{-ig\int_{t_0}^{t}\tilde{j}^a_{2\mu}A^{\mu(+)}_a d^4x' - ig\int_{t_0}^{t}\tilde{j}^a_{2\mu}A^{\mu(-)}_a d^4x'\right\}|0\rangle$$

$$= \exp -\frac{g^2}{2}\int_{t_0}^{t}\int_{t_0}^{t}\tilde{j}^a_{2\mu}(x')\, D^{\mu\nu}_+(x'-x'')\,\tilde{j}^a_{2\nu}(x'')\,d^4x'\,d^4x'' \; . \tag{18}$$

Again terms originating from higher commutators do not contribute in the color singlet case. An actual calculation of the exponent in Feynman gauge yields the result for the form factor in leading logarithmic order

$$F(s) = e^{-G(s)} \tag{19}$$

with $(\alpha = g^2/4\pi)$

$$G(s) = \frac{\alpha}{4\pi}\, c_F\, \ln^2\frac{s}{m^2} \tag{20}$$

where c_F is the eigenvalue of the quadratic Casimir operator of the quarks, m is the quark mass.

This result has been derived in perturbation theory in ref. 5 and 6. There are a few points we would like to emphasize:

i) the exponentiation, usually a result which has to be proved by detailed discussions of graphs of arbitrary order, is in this approach built into the solution of the asymptotic dynamics,

ii) the asymptotic Hamiltonian inducing a formal unitary transformation into the coherent states governs the approximations to leading logarithmic order,

iii) the gluon self coupling which has not been taken into account up to now, can easily be incorporated into an asymptotic treatment thereby restoring gauge invariance. For the calculation of the quark form factor the triple and quartic gluon vertices needed not be considered, since they do not contribute to the leading logarithms in Feynman gauge [5].

Of course, the proposed method has a wide range of applications. In addition we have extended the approximation of the asymptotic current as to incorporate the hard collinear gluons in high energy processes. This approximation of the current reproduces the correct Altarelli-Parisi probability function. It enables us to calculate the Sterman-Weinberg cross section [12] for jet production in e^+e^- annihilation, which has also been studied by Curci, Greco and Srivastava [13] with the help of coherent states. The rather extensive details of this improved approximation of the asymptotic current will be published elsewhere.

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