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THE ON-SHELL QCD QUARK FORM FACTOR AND ITS DETERMINATION  
FROM TWO-PARTICLE CORRELATIONS IN  $e^+e^-$  ANNIHILATION

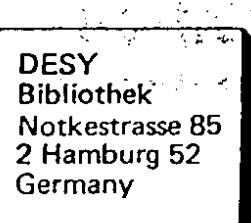
by

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THE ON-SHELL QCD QUARK FORM FACTOR AND ITS DETERMINATION

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$$d_s(q^2) = \frac{12\pi}{b \ln(q^2/\Lambda^2)}, \quad b = 33 - 2N_f \quad (1)$$

The acollinear 2-jet events reflect the deflection of the initially back-to-back quark-antiquark pair due to multiple small angle gluon emission : the highly off-shell quark (antiquark) radiates soft and collinear gluons until its invariant mass has reached  $\sqrt{q_0^2} \gg \Lambda$ .

The quark evolution from high invariant masses  $\sim \sqrt{q^2}$  down to  $\sqrt{q_0^2}$  has its clear manifestation in the well-known scaling violation of the fragmentation functions,  $D_F^h(x, q^2)$ , which is caused by the conversion of invariant mass into transverse momenta of radiated gluons, and therefore leads to a deviation from back-to-back direction by an acollinearity angle  $\theta \sim 2 p_t / \sqrt{q^2}$ , where  $p_t$  is the total radiated transverse momentum.

It has been pointed out by Dokshitzer, D'Yakonov and Troyan [1] that the deflection of the quark-antiquark pair from the original back-to-back direction can be described by an effective quark form factor which can be directly measured in a suitably energy-weighted angular correlation in  $e^+e^-$  annihilation into hadrons.

In this note we shall discuss the QCD prediction for the on-shell electromagnetic form factor, where special emphasis is put on the correct normalization over the whole kinematical range, and shall compare with recent data [2].

Let us start with the zeroth order result in QCD for the 2-particle inclusive cross-section ( $e^+e^- \rightarrow h_1 + h_2 + X$ ) for particles  $h_1$  and  $h_2$  belonging to opposite jets

$$\frac{dG_{\mu\mu}^{h_1 h_2}}{dx_1 dx_2 d\cos\theta} = \frac{3}{2} \sigma_{\mu\mu} \cdot$$

$$\sum_f Q_f^2 \left[ D_{of}^{h_1}(x_1) D_{of}^{h_2}(x_2) + D_{of}^{h_2}(x_1) D_{of}^{h_1}(x_2) \right] \delta(\cos\theta - 1) \quad (2)$$

Opposite side correlations in hadron jets produced in  $e^+e^-$  annihilation provide a possibility to measure directly the on-shell QCD quark form factor. Comparing recent FLUTO data with the leading log prediction yields  $0.54 \pm 0.1$  GeV for the QCD parameter  $\Lambda$ .

Abstract

where  $x_1 = E_1/\sqrt{q^2}$  are the fractional energies carried by particle  $h_1$  and  $h_2$ ,  $\pi - \theta$  is the angle between them ( $\theta = 0$  corresponds to back-to-back production), and  $G_{\mu\nu} = 4\pi\alpha'^2/3q^2$ . The physical meaning of the bare fragmentation functions implies the sum rule

$$\sum_h \int d\mathbf{x} \times D_h(\mathbf{x}) = 1 \quad (3)$$

Following DDT [1], we define an energy-weighted angular correlation ("acollinearity distribution")

$$\frac{dW}{d\cos\theta} = \frac{1}{G_{\mu\nu}} \sum_{h_1, h_2} \int d\mathbf{x}_1 d\mathbf{x}_2 \frac{d\sigma_{h_1, h_2}}{d\mathbf{x}_1 d\mathbf{x}_2 d\cos\theta} \quad (4)$$

with normalization

$$\int d\cos\theta \frac{dW}{d\cos\theta} = 1 \quad (5)$$

From eq. (2) one obtains the zeroth order QCD result

$$\frac{dW_0}{d\cos\theta} = \delta(\cos\theta - 1) \quad (6)$$

Eqs. (2) and (6) agree with the naive parton model, if one neglects the transverse momentum spread, i.e. assumes a transverse momentum distribution  $f(\vec{p}_T) = \delta^{(2)}(\vec{p}_T)$ . In a more realistic version of the parton model transverse momentum smearing is taken into account by a distribution with Gaussian momentum cut off

$$f_{\text{PARTON}}(\vec{p}_T) = \frac{1}{4\langle p_T \rangle^2} e^{-\frac{\pi}{4} p_T^2 / \langle p_T \rangle^2} \quad (7)$$

with a constant  $\langle p_T \rangle = 0.3$  GeV. The  $x_1, x_2$  dependence of the inclusive cross-section is still given by eq. (2), but the sharp angular correlation (6) gets smeared into

$$\left( \frac{dW}{d\cos\theta} \right)_{\text{PARTON}} = \frac{A}{2} e^{-A \sin^2(\theta/2)}, \quad A = \frac{\pi}{\langle \delta \rangle^2} \quad (8)$$

where the mean jet opening angle  $\langle \delta \rangle$  is defined by  $\langle \delta \rangle = \langle n \rangle \langle p_T \rangle / \sqrt{q^2}$ . Since no evidence has been found for a deviation from the 2-jet picture in  $e^+e^-$  annihilation for c.m. energies below 10 GeV, the parton prediction (8) should describe adequately the data at  $\sqrt{q^2} = 7.7$  and 9.4 GeV (see Figs. 1a, 1b).

At large acollinearity angles the angular correlation should reliably be given by the  $O(\alpha_s)$  contribution, i.e. by hard single gluon emission,  $e^+e^- \rightarrow q \bar{q} g$ , which can be exactly calculated in QCD [3]. For small

angles, however, the  $O(\alpha_s)$  contribution as well as all higher order contributions are infrared divergent due to both soft and collinear singularities. In order to obtain a physical sensible result, one therefore has to sum an infinite set of Feynman diagrams. Although the complete solution of this problem is not known, it has been shown by DDT [1] that a reliable summation can be done if the total radiated transverse momentum satisfies  $\Lambda^2 \ll p_T^2 \ll q^2$ . The latter region is characterized by large values of  $\ln(p_T^2/q^2)$  and requires summation of all contributions of the type  $(\alpha_s \ln^2(q/p_T^2))^n$ . Employing the so-called "planar" gauge, the dominant Feynman diagrams have a ladder-like structure (no interference diagrams), and can be summed to all orders. As a result one obtains the "DDT formula"

$$\begin{aligned} \frac{d\sigma_{h_1, h_2}}{d\mathbf{x}_1 d\mathbf{x}_2 d\cos\theta} &= \frac{3}{2} G_{\mu\nu} \sum_f Q_f^2 \frac{\partial}{\partial \cos\theta} \\ &\cdot \left[ D_f^{h_1}(x_1, p_T^2) D_f^{h_2}(x_2, p_T^2) + D_f^{h_2}(x_1, p_T^2) D_f^{h_1}(x_2, p_T^2) \right] T^2(q^2, p_T^2) \end{aligned} \quad (9)$$

Comparison of this expression with the zeroth order result (2) shows clearly two effects reflecting the quark evolution according to QCD: the bare fragmentation functions have been replaced by the  $q^2$  dependent scale breaking fragmentation functions,  $D_f^h(x) \rightarrow D_f^h(x, q^2)$ , and the fragmentation functions have been multiplied by the square of a QCD form factor  $T(q^2, p_T^2)$ . Since the  $q^2$  dependent fragmentation functions satisfy still the sum rule (3), one obtains immediately from eq. (9) the angular correlation for real gluon emission ( $p_T, \theta \neq 0$ )

$$\left( \frac{dW}{d\cos\theta} \right)_{\text{real}} = \frac{\partial}{\partial \cos\theta} T^2(q^2, p_T^2) \quad (10)$$

The uncalculable fragmentation functions have dropped out completely, and one is left with a direct measurement of the QCD form factor  $T$ . Eqs. (9), (10) have been derived in the region  $\Lambda^2 \ll p_T^2 \ll q^2$ . If one defines the form factor  $T$  in such a way that eq. (10) holds over the whole kinematical range  $0 \leq p_T^2 \leq q^2$ , the normalization (5) tells us immediately that  $w(p_T^2) = T^2(q^2, p_T^2)$  should be interpreted as the probability that the total radiated transverse momentum is less than  $p_T^2$  with  $0 \leq w(p_T^2) \leq w(q^2) = 1$  \*).

\*). For the zeroth order result (6) one obtains  $w_0 = 1$  for  $p_T > 0$  and  $w_0 = 0$  for  $p_T = 0$ , while the parton model (8) gives  $w_{\text{parton}} = (1 - \exp(-A p_T^2/q^2))$  with  $p_T^2 = q^2 \sin^2(\theta/2)$ .

In the region  $\Lambda^2 \ll p_\perp^2 \ll q^2$  DDT obtained ( $C_F = \frac{4}{3}$ ) :

$$\begin{aligned} T(q^2, p_\perp^2) &\simeq \int_1^2 dt \exp\left[-\frac{ds(p_\perp^2)}{2\pi} C_F \ln^2(p_\perp^2/q^2) \cdot \frac{1}{t^2}\right] \\ &= 1 - \frac{ds(p_\perp^2)}{4\pi} C_F \ln^2\left(\frac{p_\perp^2}{q^2}\right) + \frac{7}{6} \frac{1}{2!} \left[ \frac{ds(p_\perp^2)}{4\pi} C_F \ln^2\left(\frac{p_\perp^2}{q^2}\right) \right]^2 - \dots \end{aligned} \quad (11)$$

In view of the well-known exponentiation of the leading log contributions in QED, which has also been found in QCD [4], the non-exponential form (11) is somewhat surprising. One has to remember, however, that (11) has been derived after several approximations from a more complete expression replacing eq. (9) \*). Moreover, it is seen from the perturbation expansion given in (11) that the numerical deviation from the simple exponential

$$S(q^2, p_\perp^2) \equiv e^{-\frac{ds(p_\perp^2)}{4\pi} C_F \ln^2(p_\perp^2/q^2)} \quad (12)$$

is irrelevant for all physical applications \*\*), and it is therefore tempting to conjecture [5, 6] that the analytically correct angular correlation is given by eq. (10) with  $T$  being replaced by the exponential form factor  $S$ , eq. (12). It is interesting to note, that the form factor  $S$  is identical to the QCD electromagnetic form factor for an on-shell massless quark ( $p^2 = 0$ ) [4], if  $p_\perp$  is replaced by a small regularization mass  $\mu$  for the gluons :  $F_{\text{on-shell}}(q^2) = S(q^2, \mu^2)$ .

The appearance of the on-shell form factor in eq. (10) is not accidental but rather is required by the infrared finiteness of the integrated acollinearity distribution. In an integration over  $dw/d\cos\theta$  one has to add to eq. (10) the contributions from virtual gluon exchanges, which are infrared divergent in finite order in  $\alpha_s$ . But the sum of all virtual gluon corrections is nothing else than the total cross section for the exclusive channel  $e^+ e^- \rightarrow q \bar{q}$ , which by definition is given by the square of the on-shell quark electromagnetic form factor. Thus we have

$$\left(\frac{dw}{d\cos\theta}\right)_{\text{virtual}} = \delta(\cos\theta - 1) F_{\text{on-shell}}^2(q^2) \quad (13)$$

\* ) See the first paper cited in [1].

\*\*) E.g. at  $\sqrt{s} = 30$  GeV the difference between  $S$  and  $T$  is less than  $1\%$  for  $\theta > 20^\circ$  and less than  $10\%$  for  $10^\circ < \theta < 20^\circ$ .

Adding the real and virtual contributions and integrating we obtain \*)

$$\int_{-1}^{+1} d\cos\theta \left[ \left( \frac{dw}{d\cos\theta} \right)_{\text{real}} + \left( \frac{dw}{d\cos\theta} \right)_{\text{virtual}} \right] =$$

$$F_{\text{on-shell}}^2(q^2) + S^2(q^2, q^2) - S^2(q^2, \mu^2) = 1 \quad (14)$$

(since  $S(q^2, q^2) = 1$ ), and one observes that the infrared contributions from virtual exchanges have been exactly cancelled by the divergent contributions from real emissions. This proves that the simple exponential (12) is the correct QCD expression to be used in the acollinearity distribution (10) in the leading log region.

For a study of the  $\theta$  dependence of  $S$  one needs the correct relation between the total transverse momentum  $p_\perp$  and the acollinearity angle  $\theta$ . DDT used  $p_\perp^2 = q^2 \tan^2(\theta/2)$ , which restricts the kinematically allowed region to  $0 \leq \theta \leq \pi/2$  and leads to a vanishing acollinearity distribution at  $90^\circ$ . Since the experimentally accessible region is  $0 \leq \theta \leq \pi$ , this relation cannot be correct, and we shall use instead

$$p_\perp^2 = q^2 \sin^2(\theta/2) \quad (15)$$

Support for the correctness of this relation comes from two sources. The first is the naive parton model, where relation (15) is found to hold (this is the reason for the  $\sin^2(\theta/2)$  dependence in (8)). The second source is the exact  $O(\alpha_s)$  result for  $S$  [3, 7]

$$\begin{aligned} S(q^2, p_\perp^2) &= 1 - C_F \frac{ds}{4\pi} \left[ \ln^2(\sin^2(\theta/2)) + 3 \ln(\sin^2(\theta/2)) + 10.2 \right] \\ &\quad + O(p_\perp^2) \end{aligned} \quad (16)$$

where we have dropped terms which vanish in the limit of small acollinearity angles. A comparison of eq. (16) with eq. (12) leads immediately to the relation (15).

It remains to discuss the appearance of the running coupling  $\alpha_s(p_\perp^2)$  in the exponential form factor (12). Since the acollinearity distribution is characterized by two large invariants ( $p_\perp^2$  and  $q^2$ ), the determination of the correct argument requires a calculation beyond the leading logarithm approximation (LLA), i.e. a summation of single logs. Even though the final answer to this question is not yet known,

\*) Note, that in the integration the explicit form of the leading log result (12) is required only near  $p_\perp = 0$ , where use of the LLA is justified, if the real gluons have a fictitious mass  $\mu$ , yielding  $S(q^2, p_\perp^2/\mu^2)$  in eq. (10).

there are strong indications from the work of DDT and others [8], that  $\alpha_s(p_1^2)$  is the correct choice.

Let us discuss the kinematic range, in which the LLA may be considered to be reasonable. Since the asymptotic freedom prediction (1) for  $\alpha_s(p_1^2)$  will be used, we obtain the limitation  $\theta \gg 2\Lambda/\sqrt{q^2}$ . However, the realistic restriction at small acollinearity angles comes from consideration of confinement effects, which we estimate using the parton model formula (8). We are then led to the condition  $\theta \gg \langle \delta \rangle = \langle n \rangle \langle p_1 \rangle / \sqrt{q^2} \approx 2 \langle \langle p_1 \rangle / \langle x \rangle \rangle / \sqrt{q^2}$ , i.e.  $\theta \gg 18^\circ$ ,  $7^\circ$  at  $\sqrt{q^2} = 13$ ,  $31.6$  GeV, respectively, if we use  $\langle p_1 \rangle = 0.3$  GeV,  $\langle x \rangle = 0.15$ .

Concerning large acollinearity angles one has to remember that single logs have been neglected in the LLA. In order to ensure the dominance of the double log terms, one therefore requires  $\ln(q^2/p_1^2) \gg \ln(q^2/p_1^2)$ , which yields the constraint  $\sin(\theta/2) \ll 1/\sqrt{q^2}$ , i.e.  $\theta \ll 75^\circ$ . While this angle is large enough to provide a useful comparison with experimental data, we think that this limit is not serious for the following reasons : i) in the exact  $O(\alpha_s)$  result, eq.(16), the single log term is very large, already at  $20^\circ$ , and opposite in sign compared to the leading log term, and seems therefore to invalidate the LLA. If one includes, however, the finite term, it is seen that the non-leading terms cancel almost completely \*), and that the double log term constitutes a good approximation. If this cancellation occurs also in higher orders, the leading log result would be much better than comparison between leading and non-leading terms would indicate;

ii) the QCD form factor (12) is correctly normalized over the whole kinematical range  $0 \leq \theta \leq \pi$  ( $S(q^2, q^2) = 1$ ). Therefore it is not unlikely that it constitutes a good approximation also at large acollinearity angles. Already at  $40^\circ$   $S^2$  has reached a value of about 0.75, and not much room is left for a variation between  $40^\circ$  and  $180^\circ$ .

The data at the lowest energies,  $\sqrt{q^2} = 7.7$  and  $9.4$  Gev, do not agree with the QCD prediction (Figs.1a and 1b), since at these energies the jet fragmentation is governed by hadronization and not by perturbation theory. Instead the data are well described by the parton formula (8).

At the higher energies, however, the data agree with QCD as well as could possibly be expected. In particular we observe that the maximum of  $dw/d\theta$  is correctly predicted by QCD. Since agreement is found over nearly the whole angular interval (i.e.  $10^\circ < \theta < 120^\circ$ ), this confirms our conjecture about the validity of the LLA. It is worthwhile to

\* ) Actually, the sum of the non-leading terms in eq.(16) has a zero at  $\theta = 21^\circ$ .

mention that the good overall agreement at large angles depends critically on the relation (15) and the choice  $\alpha_s(p_1^2)$ . If we would have used  $\alpha_s(q^2)$  instead of  $\alpha_s(p_1^2)$  in the form factor  $S$ , the result would have drastically changed. E.g. at  $31.6$  Gev the maximum of  $dw/d\theta$  would occur at  $\theta = 30^\circ$  (for  $\Lambda = 0.5$  Gev), which is certainly excluded by the data. To shift the maximum to the experimentally observed position, requires an unreasonable large value of  $\Lambda$ ,  $\Lambda = 2.5$  Gev.

Actually the data displayed in Figs. 1a - 1h show some fluctuations around the smooth theoretical curve, preventing us from a precise determination of the scale parameter  $\Lambda$  from a fit to  $dw/d\theta$ .

The most optimal observable for a quantitative QCD test is the square of the form factor  $S(\theta)$ , which can be obtained from the experimental data on  $dw/d\theta$  by a straightforward integration from zero angle up to  $\theta$ . Since the quantity  $w(\theta) = S^2(\theta)$  has the simple physical interpretation as the total probability that the acollinearity angle is less than  $\theta$ , it should be a smooth, monotonously increasing function of  $\theta$ , in which possible statistical fluctuations are washed out. If we remember that the measurement of  $dw/d\theta$  requires an energy-weighting (eq.(4)), it can be expected that  $S^2(\theta)$  provides a suitably inclusive observable for which a save QCD prediction can be made.

It is seen (Figs.2a - 2f) that theory and experiment are within experimental errors well compatible with each other for acollinearity angles outside the confinement region. The good agreement allows us to determine at each energy the only free parameter, the QCD scale  $\Lambda$ , from a best  $\chi^2$ -fit to the integrated data. Using a cut off at small angles to exclude hadronization effects, we obtain the  $\Lambda$  values shown in Fig.3. The fact that the  $\Lambda$  values obtained at different energies are well compatible with a common value,  $\Lambda = 0.54 \pm 0.1$  Gev, is a direct test of the asymptotic freedom behavior of the QCD quark-gluon coupling (1). Note that similar values for  $\Lambda$  have been found in other reactions [9].

Finally we test the  $q^2$  dependence of  $S^2(\theta)$  by keeping the angle fixed and well inside the proper leading log region, i.e.  $15^\circ < \theta < 35^\circ$ . This genuine QCD test is carried out in Fig.4 by plotting a particular combination  $F(S^2(\theta))$  of the integrated data against  $\ln q^2$  such that the data should lie on a straight line with slope one if the QCD prediction turns out to be correct.

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## Figure Captions

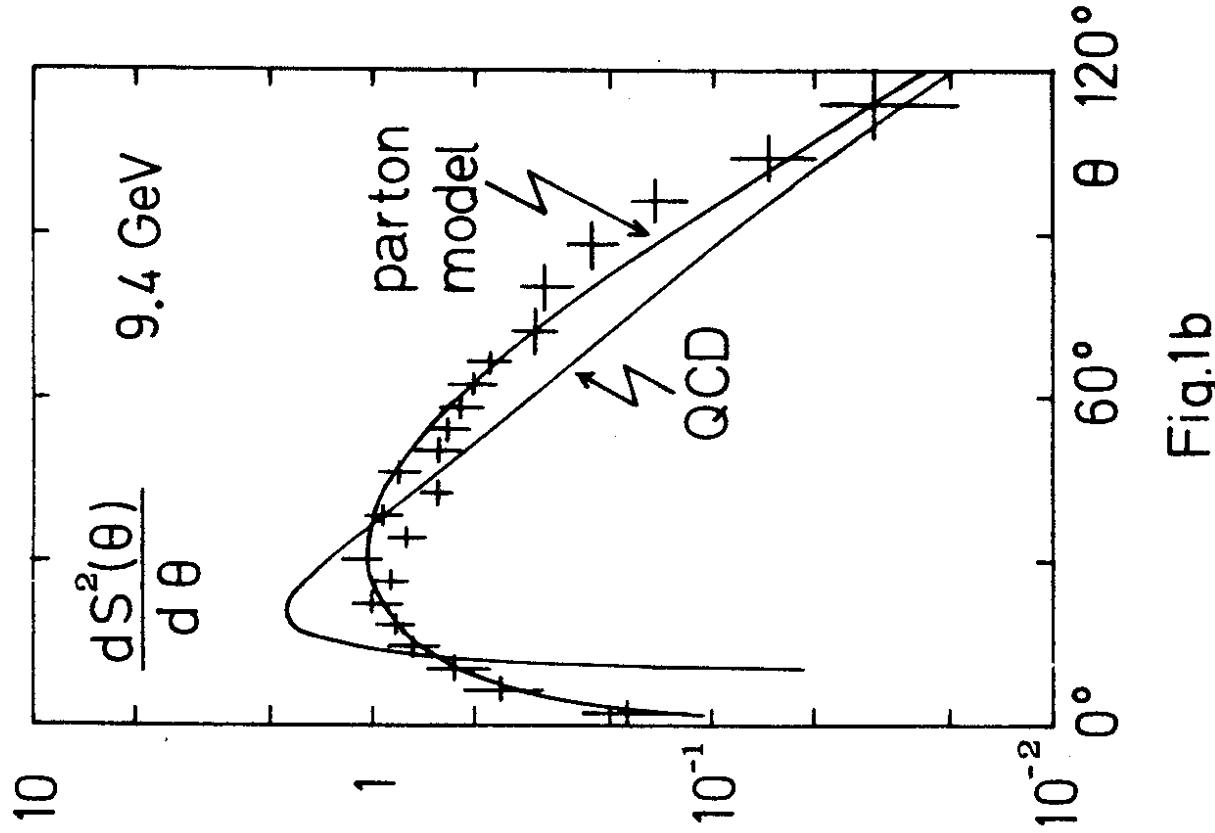
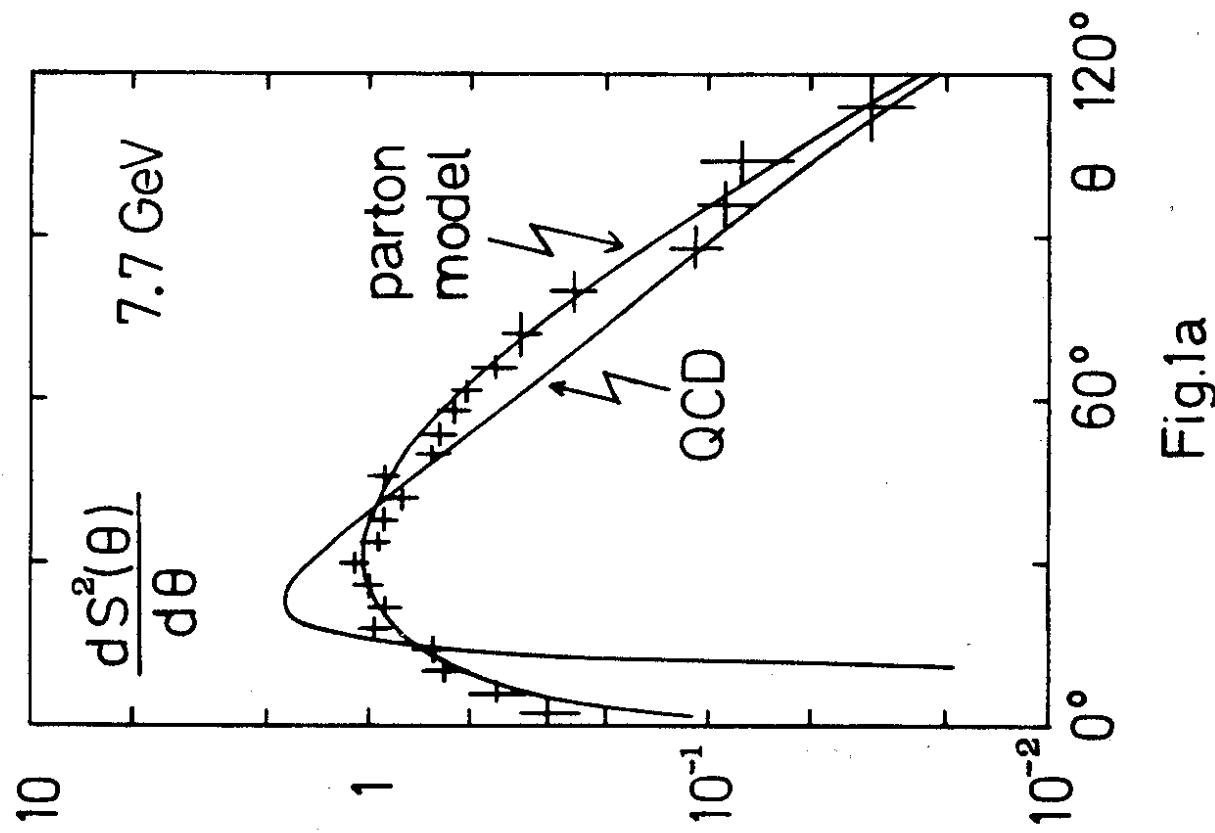
Fig.1 The acollinearity distribution  $dw/d\theta$  [2].

The QCD prediction is shown for  $\Lambda = 0.5$  GeV.  
In a) and b) the parton model result (8) is  
shown with  $\Lambda = 6.3$  .

Fig.2 The integrated acollinearity distribution  
and the QCD prediction (12) for  $\Lambda = 0.5$  GeV.

Fig.3 The QCD scale parameter  $\Lambda$  as obtained from  
best fits to the data points in Fig.2 using  
cut offs at  $\theta = 18^\circ$  at 13 GeV and  $\theta = 14^\circ$  at  
17, 22, 27.6, 30 and 31.6 GeV.

Fig.4 QCD test of the energy variation of the integrated  
acollinearity distribution at fixed angle.  
 $F$  is defined by eq.(12) and  $F(S^2(\theta)) = \ln q^2 - \ln \Lambda^2$ .  
The data are obtained from Fig.2 at 13, 17, 22 and 30 GeV.  
The point at 30 GeV represents the average over the data  
at 27.6, 30 and 31.6 GeV. Here we use  $\Lambda = 0.54$  GeV.



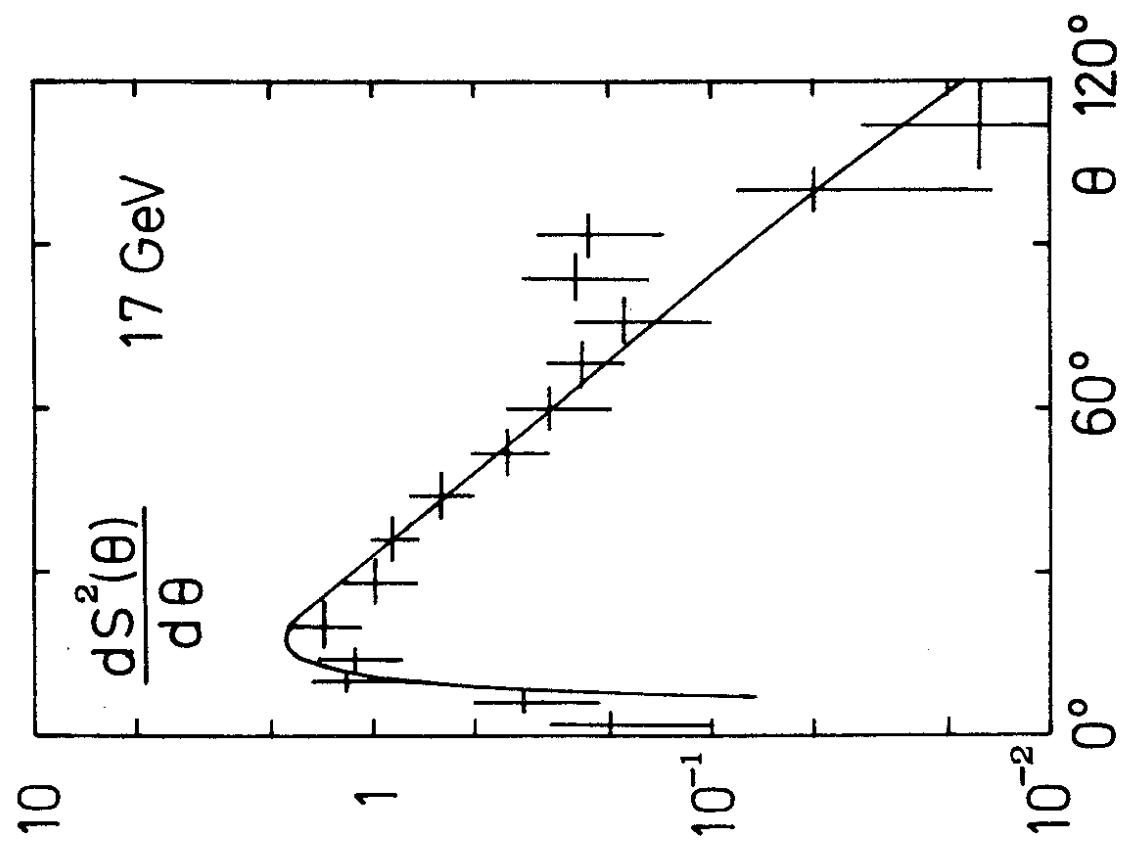


Fig.1d

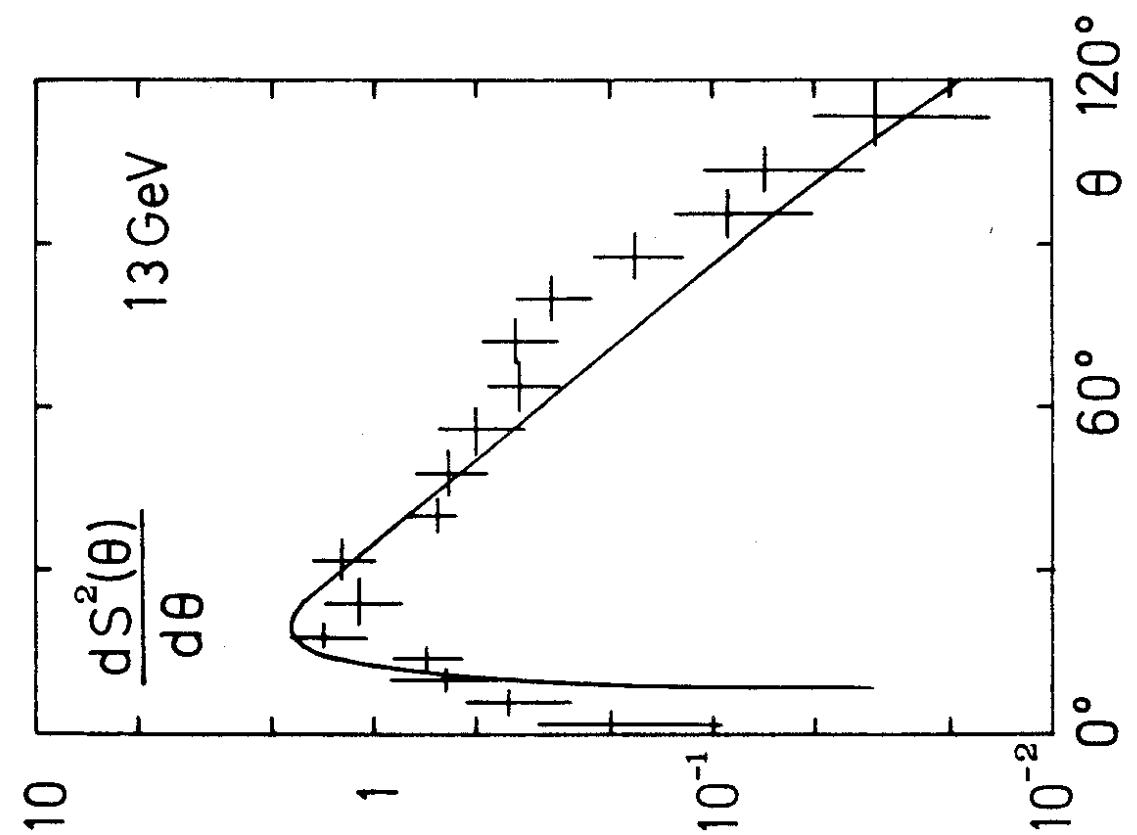


Fig.1c

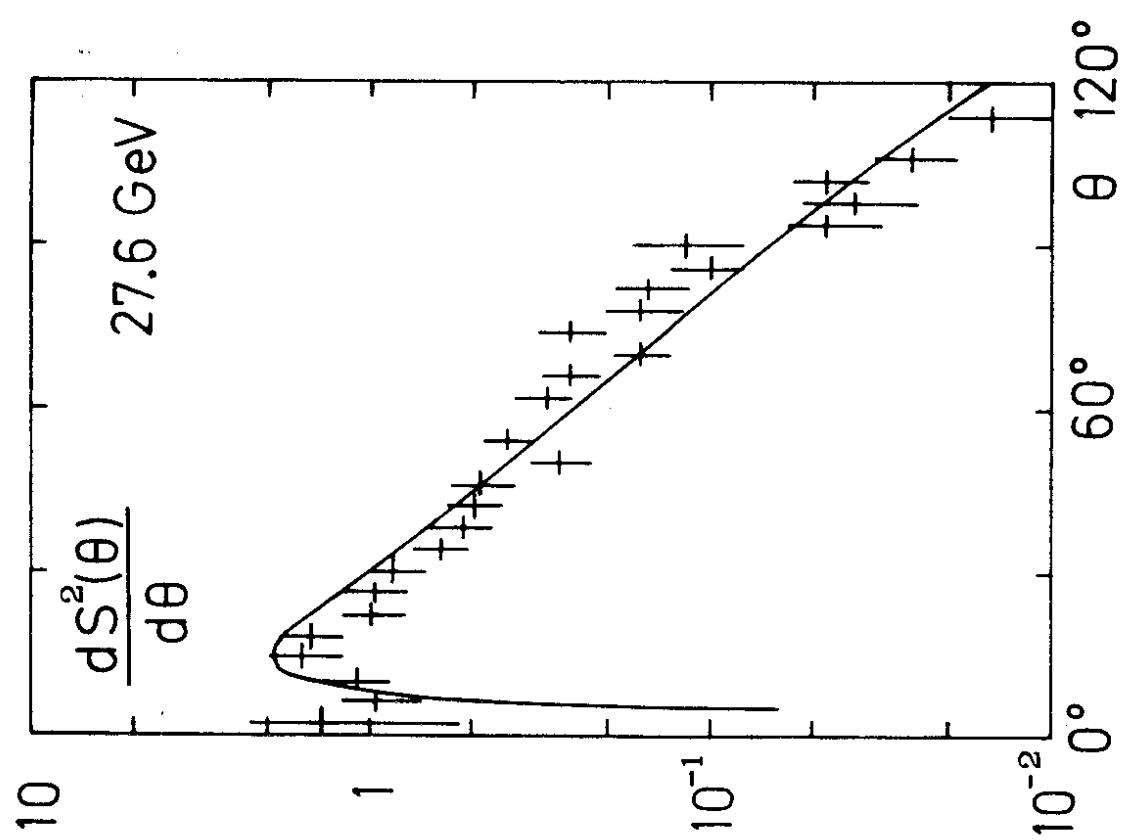


Fig.1f

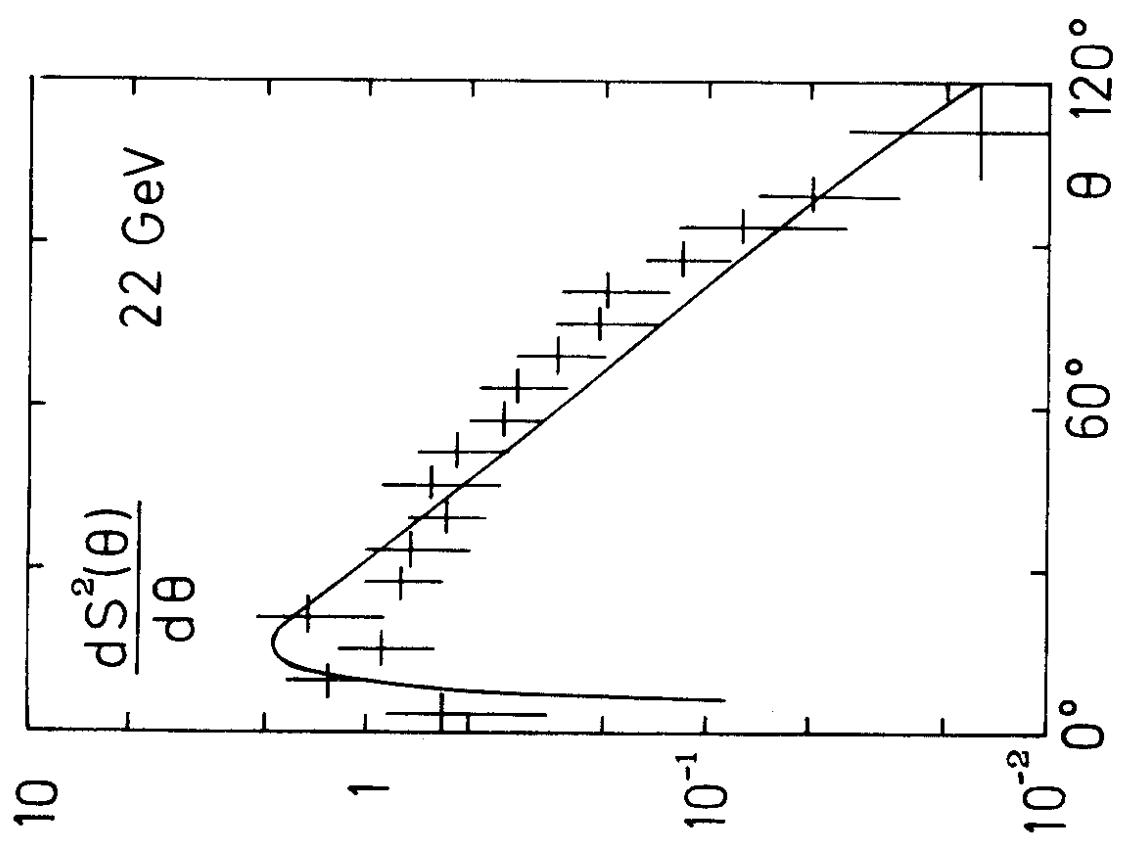


Fig.1e

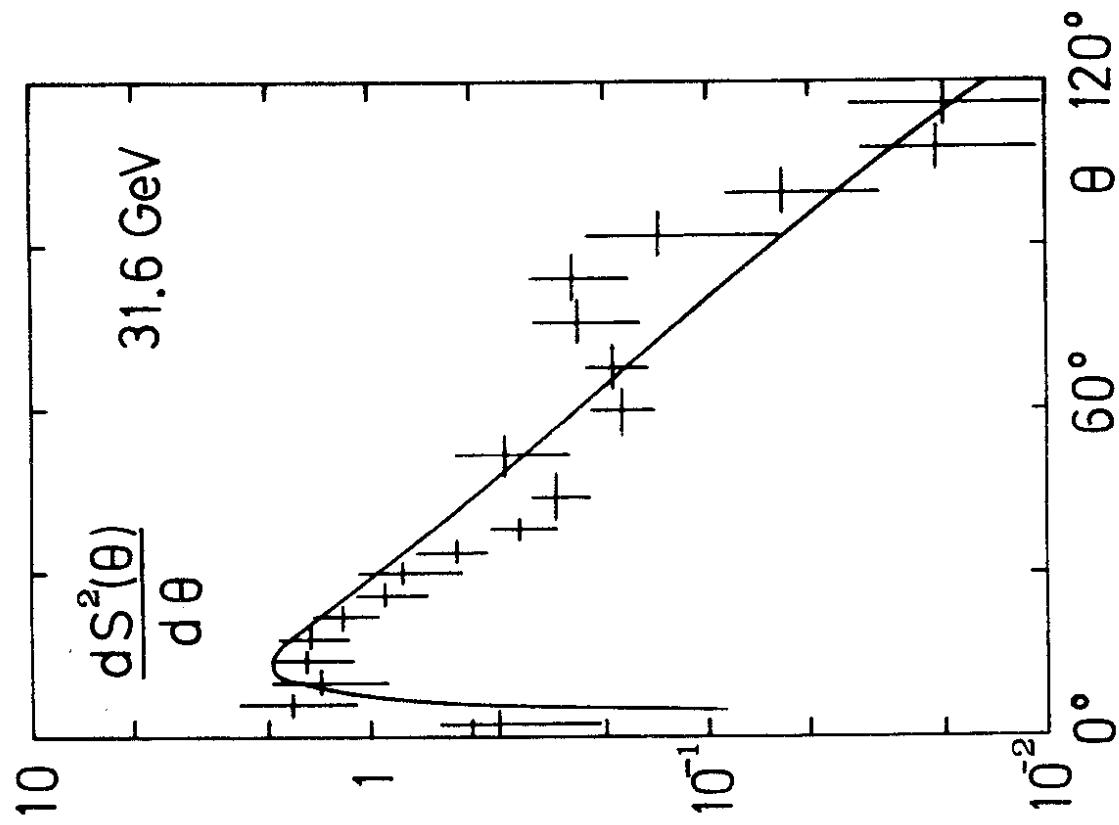


Fig.1h

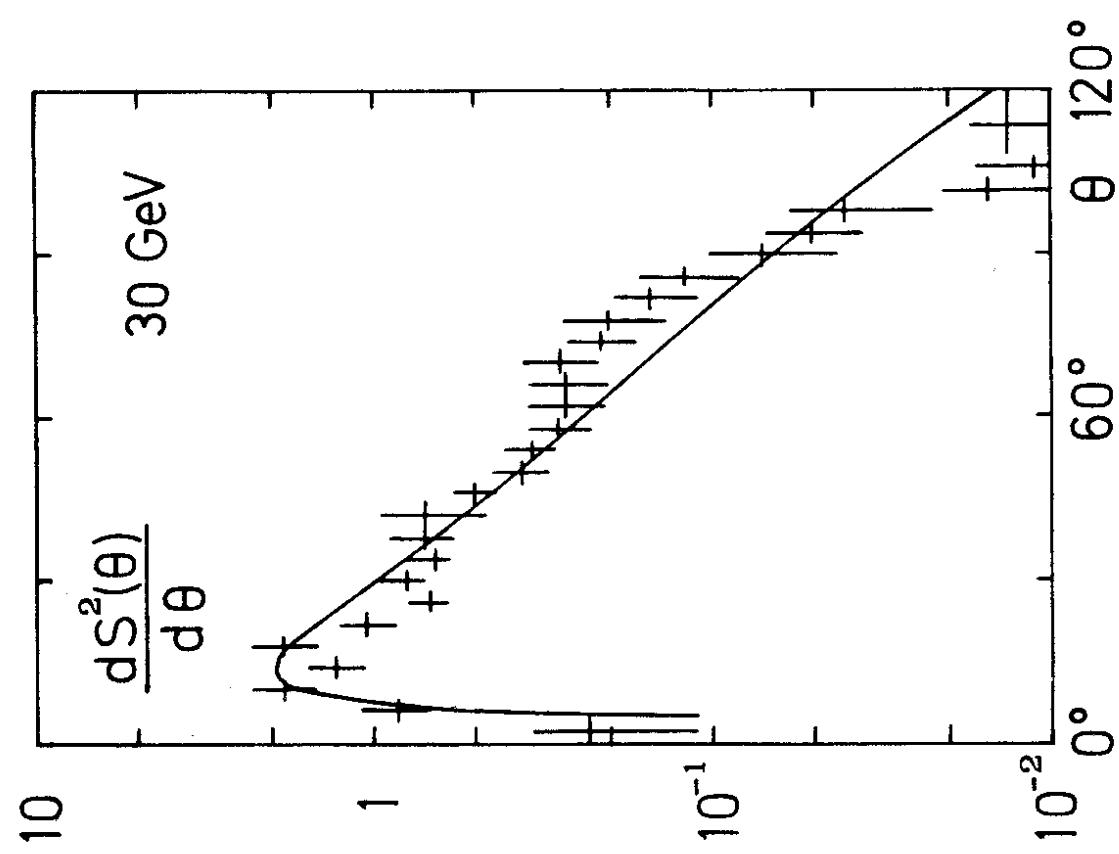


Fig.1g

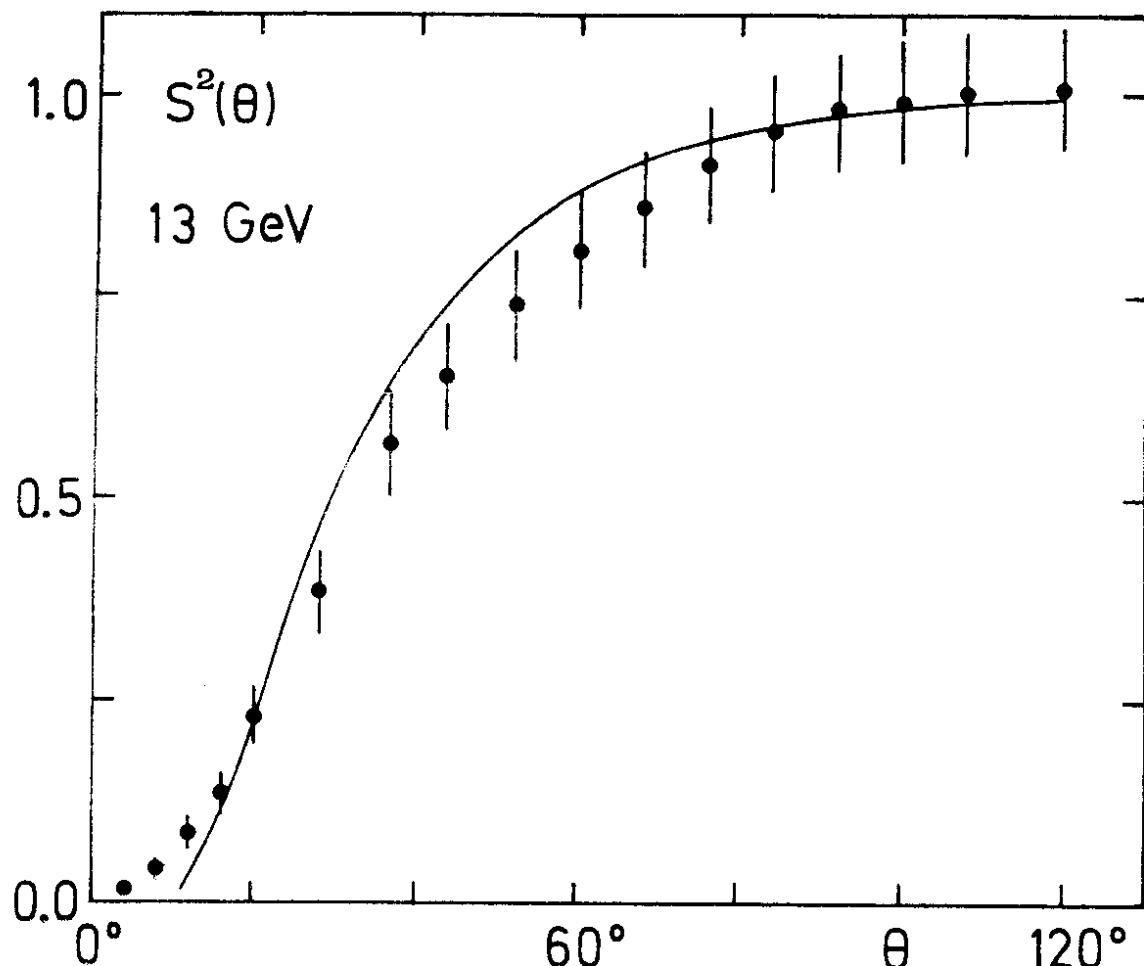


Fig.2a

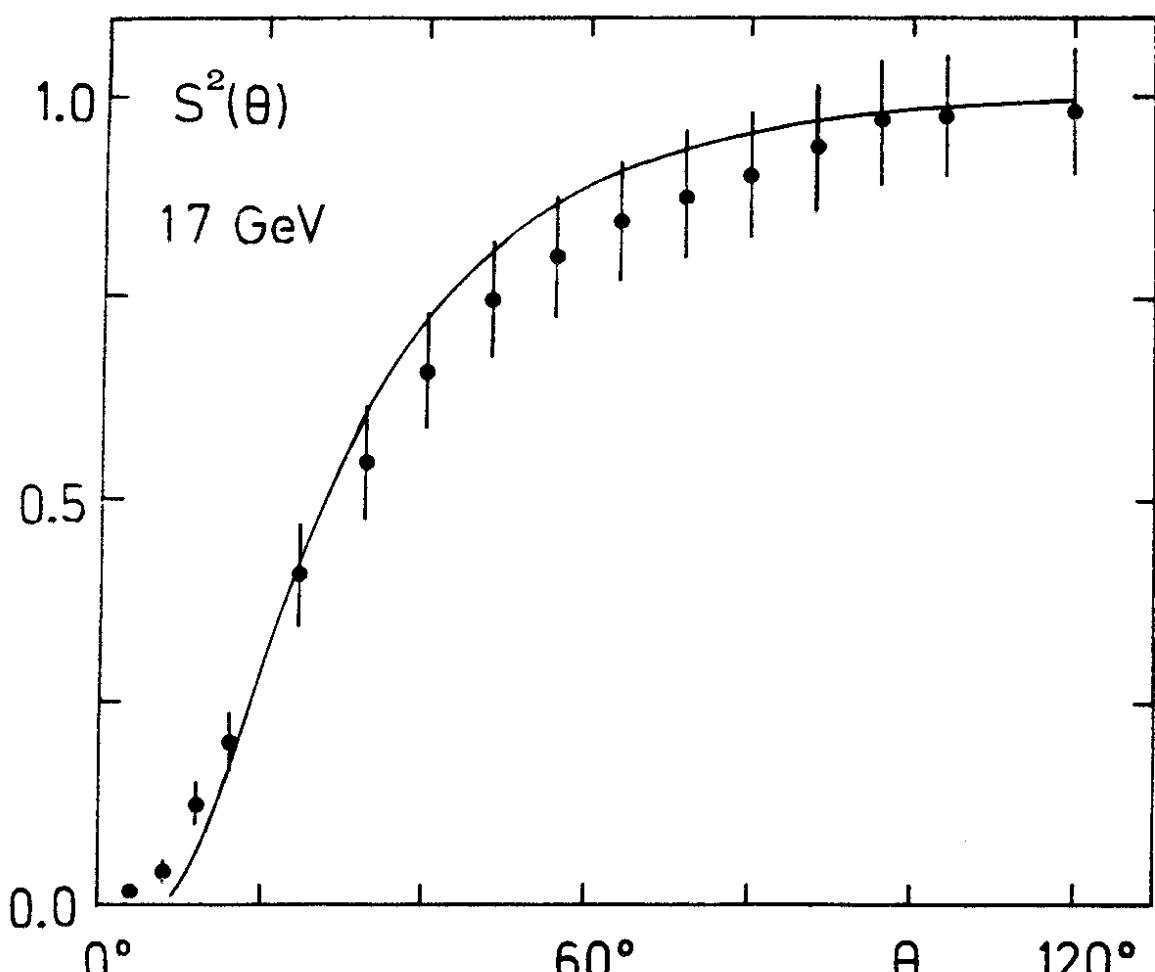
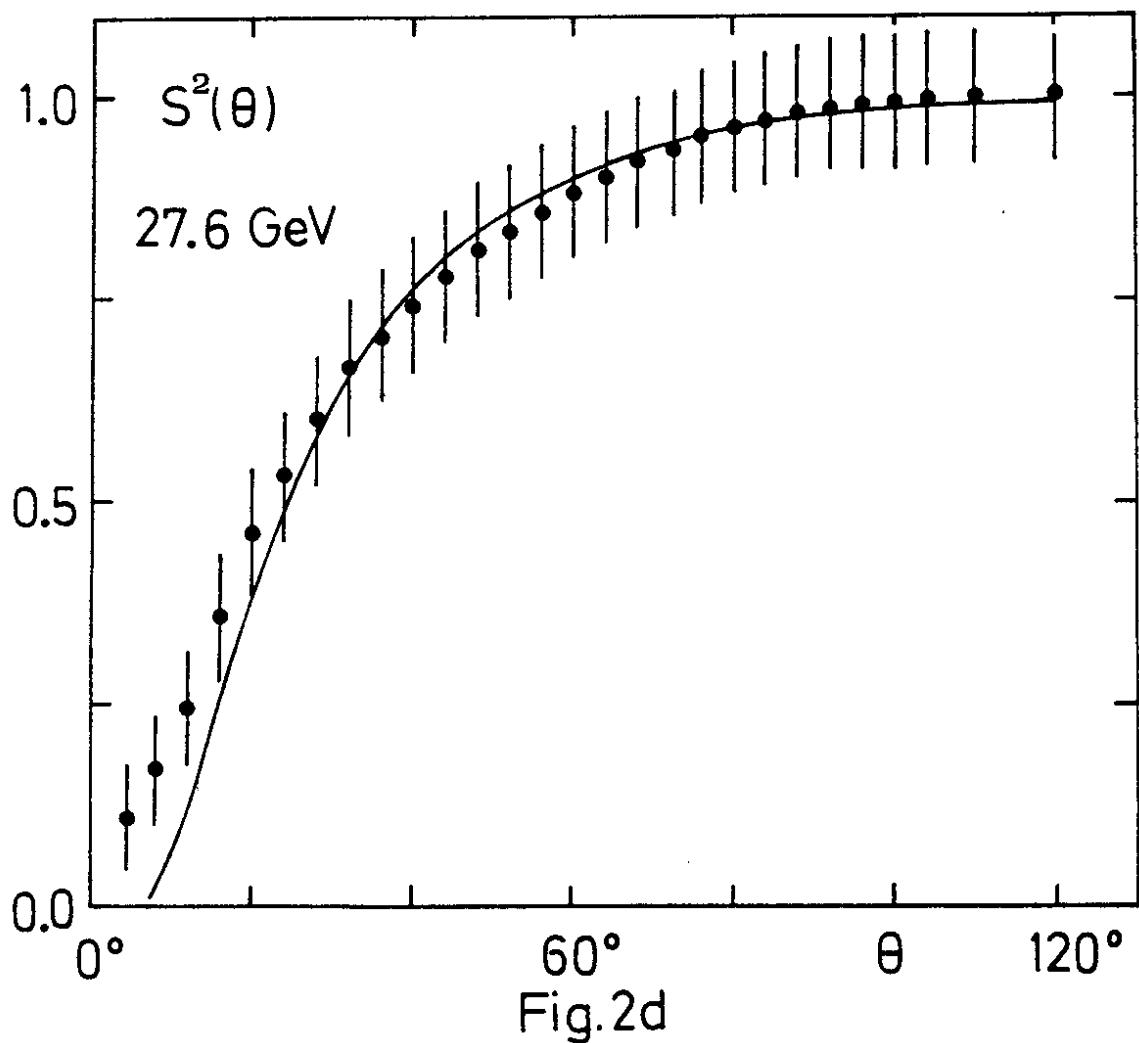
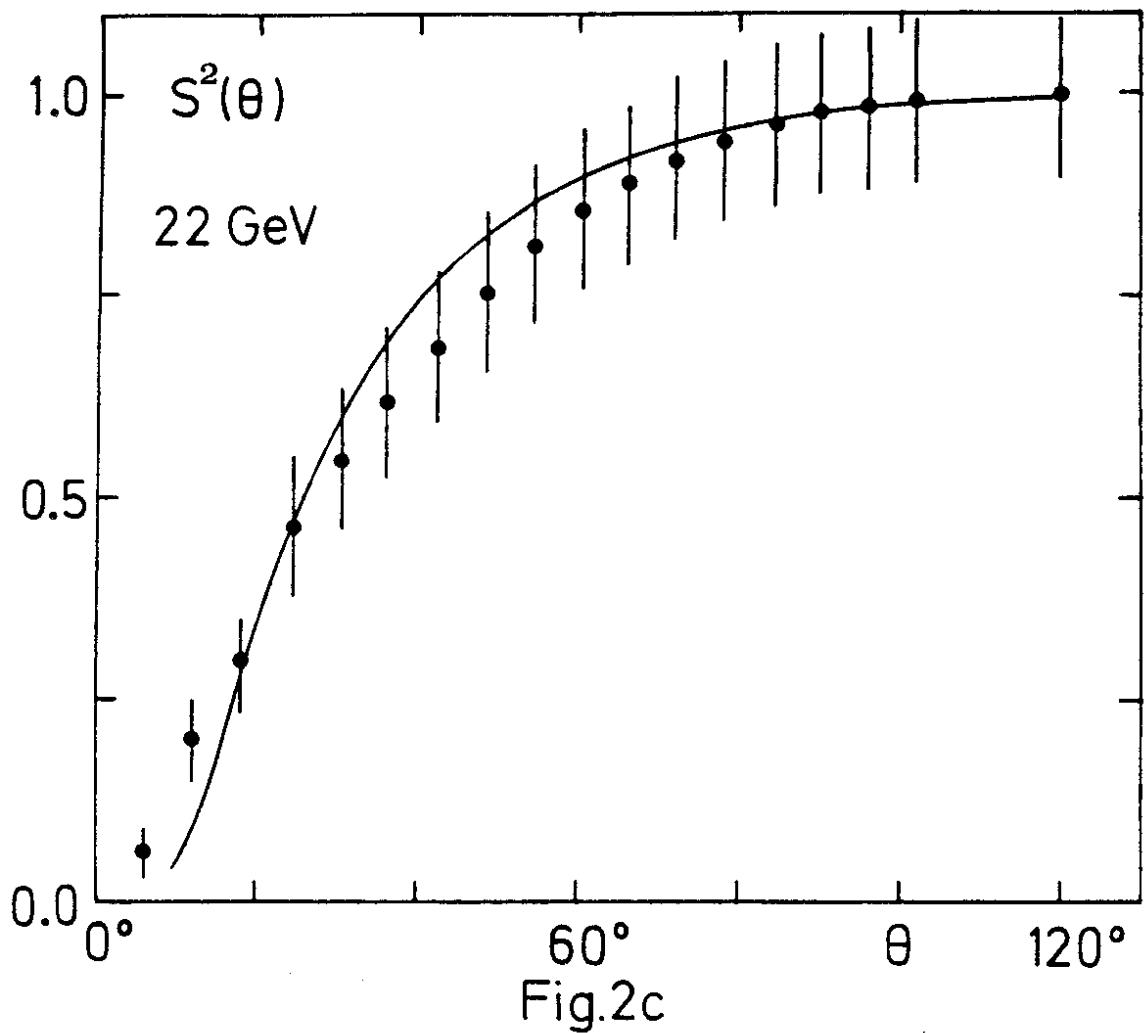
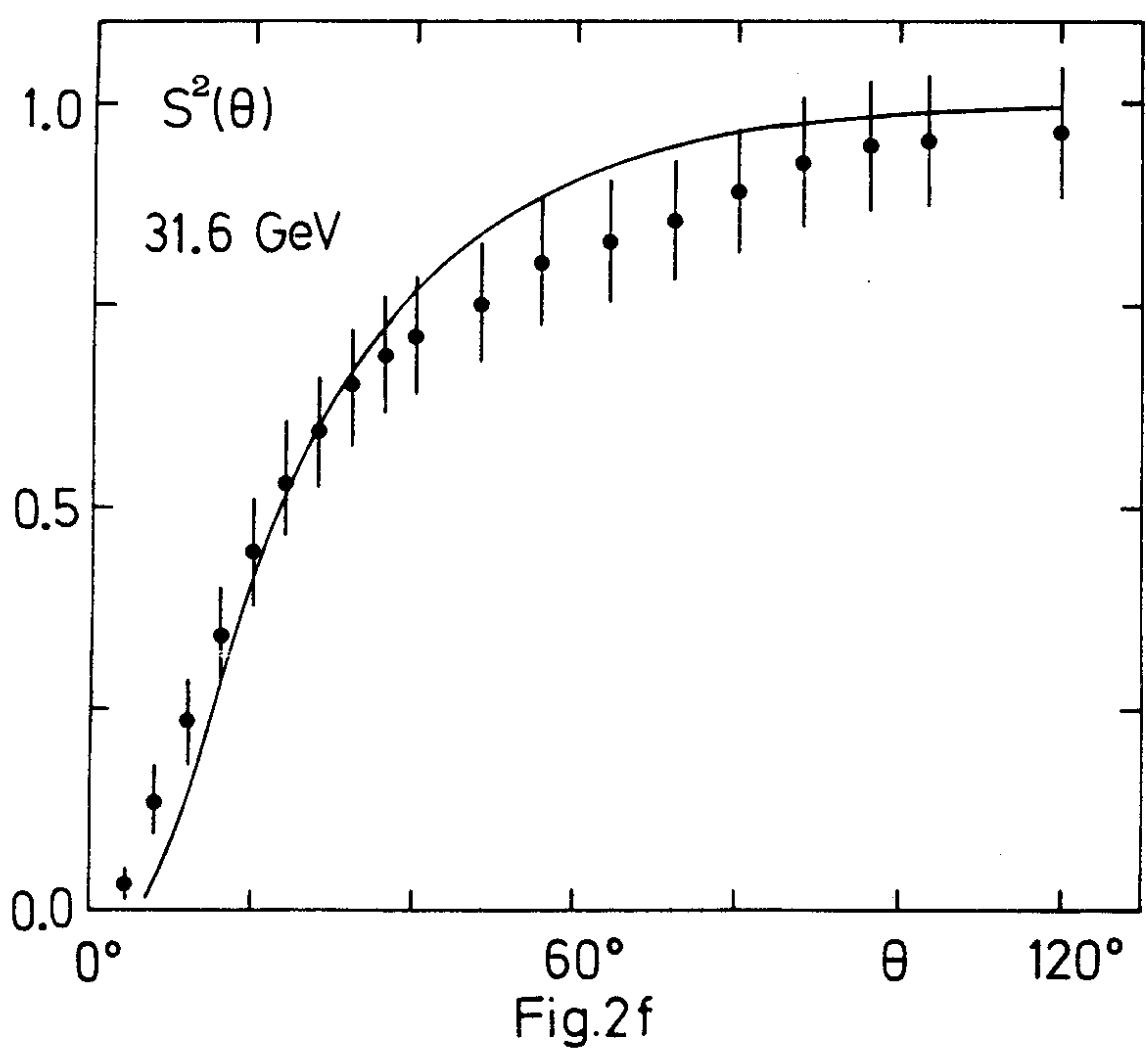
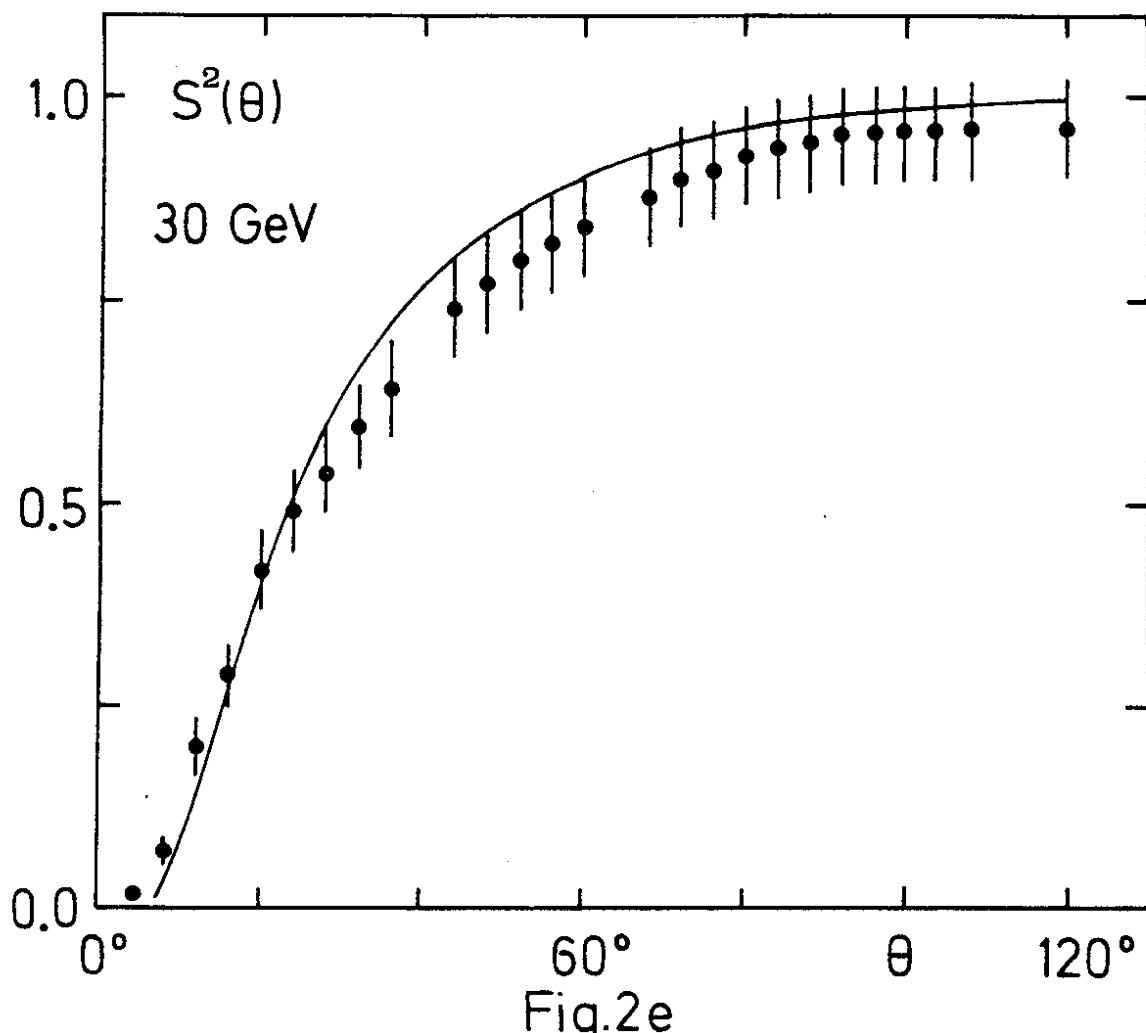


Fig.2b





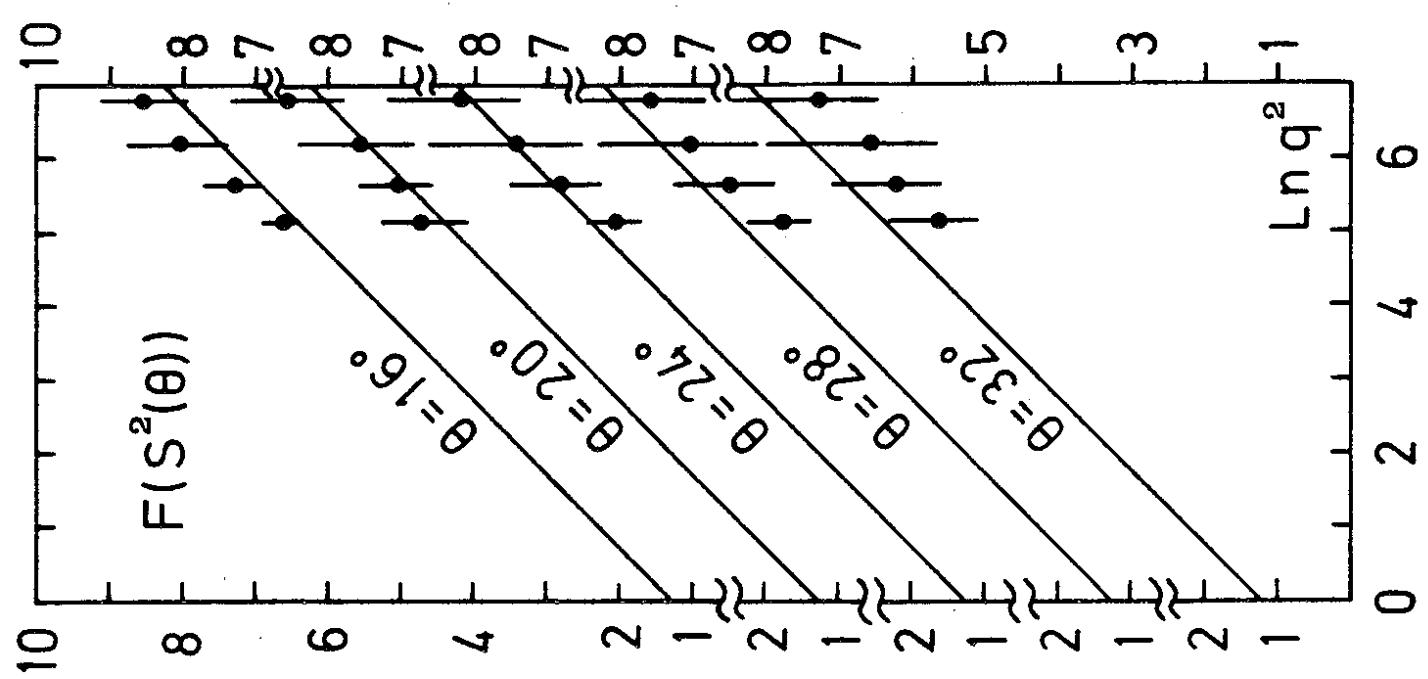


Fig. 4

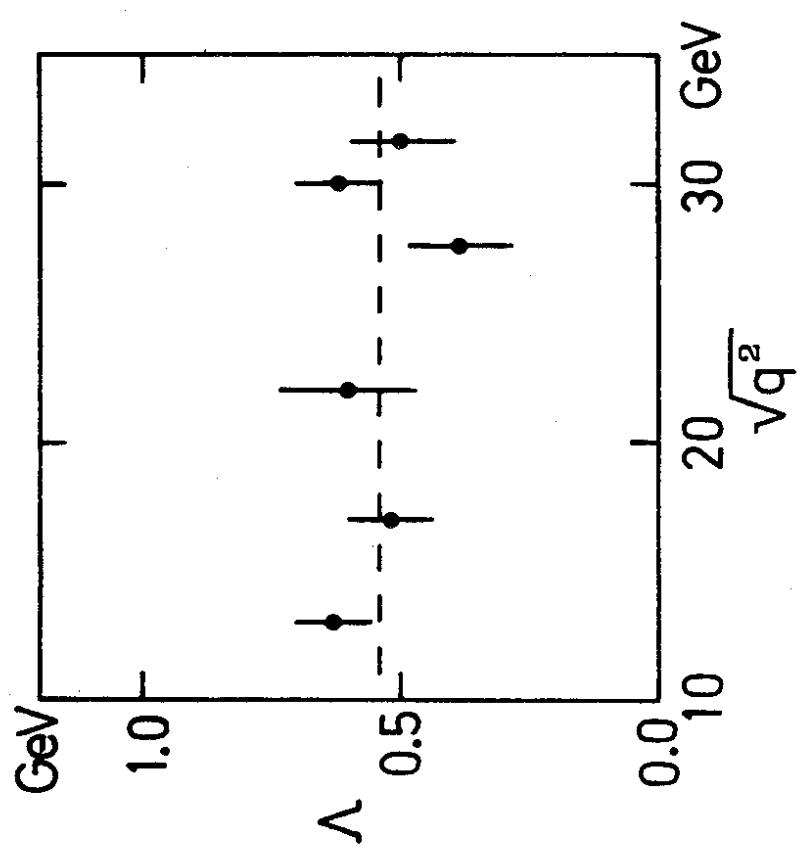


Fig. 3

