

DESY 79/15
February 1979



MEASURING THE TRIPLE GLUON VERTEX

by

E. Reya

NOTKESTRASSE 85 · 2 HAMBURG 52

To be sure that your preprints are promptly included in the
HIGH ENERGY PHYSICS INDEX ,
send them to the following address (if possible by air mail) :

DESY
Bibliothek
Notkestrasse 85
2 Hamburg 52
Germany

Measuring the Triple Gluon Vertex

E. Reya

Deutsches Elektronen-Synchrotron DESY, Hamburg

Abstract

It is proposed to measure $F_L(x, Q^2)$, or moments thereof, in order to determine the Q^2 dependence of the gluon distribution. In contrast to $F_2(x, Q^2)$, the theoretical predictions for the Q^2 evolution of the moments of the gluon density depend critically on the gluon self-couplings and thus provide us with a clean test of the Yang-Mills structure of QCD.

Most of the "direct" tests of QCD done or suggested so far are mainly sensitive to the structure of the quark-gluon coupling, but not to the non-abelian self-couplings of colored vector gluons. These tests include the well known scaling violations in electro- and neutrino production, jets in e^+e^- annihilation, Drell-Yan processes, and many others more. Although present deep inelastic scattering data eliminate^{1,2} already all strong interaction (fixed point) field theories which do not include colored quarks as well as colored (non-abelian) gluons, such as QCD or an asymptotically non-free fixed point version of QCD (with no local color gauge invariance), present high-statistics experiments are insensitive² to the triple gluon vertex of asymptotic freedom. This is not too surprising, however, since the triple gluon coupling enters the Q^2 evolution of $F_2(x, Q^2)$, for example, only in a very indirect way via the singlet Wilson operator mixing (the term proportional to $C_2(G)$ in the gluonic anomalous dimension $\gamma_{VV}^V(n)$ - we follow closely the notation of Ref. 1.). Or, in the language of Altarelli and Parisi,³ the Q^2 evolution of $F_2(x, Q^2)$ is not directly proportional to the gluon \rightarrow gluon decay probability P_{gg} . This is of course in contrast to the Q^2 development of the gluon distribution $G(x, Q^2)$ itself. Thus, although the measured effects of the breakdown of scaling require² a fundamental strong interaction theory to have colored quarks and colored vector gluons, the (local) gauge structure, i.e. possible self-couplings of the gluons, has not been seen and tested at all so far. Needless to say that this last feature is essential for asymptotic freedom.

Possible direct tests of the gluon self-couplings, suggested so far, are either exceedingly small,⁴ and therefore non-perturbative effects due to

final state interactions may totally mask these predictions, or⁵ are experimentally not accessible in the near future. For this latter test of the three-gluon (3g) vertex it has been proposed⁵ to look for the Q^2 evolution of gluon jets in heavy quarkonium decay, $e^+e^- \rightarrow Q\bar{Q} \rightarrow 3g \rightarrow h + \text{anything}$, where the gluon decay function $D_g^h(z, Q^2)$ could be measured experimentally. This requires, however, detailed measurements of $D_g^h(z, Q^2)$ for $0.3 \lesssim z \lesssim 1$ and $100 \text{ GeV}^2 \lesssim Q^2 \lesssim 1000 \text{ GeV}^2$, in order to delineate the triple-gluon contribution.

In this note we propose to study the Q^2 evolution of the gluon distribution $G(x, Q^2)$ itself, and to determine experimentally this distribution by measuring the longitudinal structure function $F_L(x, Q^2)$ in deep inelastic electro(muon)- or neutrino-production. Measurements of this latter quantity should become available in the near future.⁶ The idea is the following. In order to avoid any ambiguities on the theoretically ill understood x -dependence of $G(x, Q_0^2)$ at a certain input momentum Q_0^2 , we discuss only moments of structure functions, which weight the large Bjorken- x region and are partly already known experimentally.⁷⁻⁹ Of course, our subsequent discussion applies to the full x -dependence as well. To leading order in perturbation theory, the n -th moment of the longitudinal structure function $F_L \equiv F_2 - 2xF_1$ is given by

$$F_{L,n-1}(Q^2) = C_n^q F_{2,n-1}(Q^2) + a C_n^g G_n(Q^2) \quad (1)$$

where C_n^q and C_n^g are the moments of the longitudinal projections of the fundamental processes¹⁰ $Vq \rightarrow gq$ and $Vg \rightarrow q\bar{q}$ ($V = \gamma^*$ or W), respectively,

$$C_n^q = \frac{4\alpha_s(Q^2)}{3\pi(n+1)}, \quad C_n^g = \frac{2\alpha_s(Q^2)}{\pi(n+1)(n+2)} \quad (2)$$

and, in the four quark model, $a = \sum_q e_q^2 = \frac{10}{9}$ for electroproduction and $a = 4$ for ν and $\bar{\nu}$ scattering on matter, and $\alpha_s(Q^2) = 12\pi/25 \ln(Q^2/\Lambda^2)$ with $\Lambda \approx 0.5$ GeV. The moments of the structure functions are defined by $G_n(Q^2) \equiv \int_0^1 dx x^{n-1} G(x, Q^2)$. From Eq. (1) we see that good data on $F_{L,n-1}(Q^2)$, together with the experimental knowledge of⁷⁻⁹ $F_{2,n-1}(Q^2)$, $n = 2, 3, \dots$, can be translated into a reasonable knowledge of the gluon density

$$G_n(Q^2) = \frac{\pi(n+1)(n+2)}{2a\alpha_s(Q^2)} F_{L,n-1}(Q^2) - \frac{2(n+2)}{3a} F_{2,n-1}(Q^2). \quad (3)$$

It should be noted that, for this case, $F_L(x, Q^2)$ needs only to be measured accurately for $x \gtrsim 0.3$, since $n > 2$ moments are sensitive mainly to the large- x region of structure functions. Furthermore, as we shall see, measurements in the region $10 \text{ GeV}^2 \lesssim Q^2 \lesssim 100 \text{ GeV}^2$ will be required in order to clearly pin down the triple gluon coupling. At these large values of Q^2 , non-perturbative contributions to F_L can be safely neglected¹¹ since they are of the order k_T^2/Q^2 or m^2/Q^2 , with k_T being the intrinsic transverse momentum of partons and m some typical hadronic mass scale.

Theoretically the Q^2 evolution of the moments of the gluon distribution is predicted by the renormalization group to be¹

$$\frac{G_n(Q^2)}{G_n(Q_0^2)} = \left[1 - \alpha_n + \frac{\alpha_n}{\beta_n} (1 - \alpha_n) \rho_n(Q_0^2) \right] e^{-S\alpha_-(n)} + \left[\alpha_n - \frac{\alpha_n}{\beta_n} (1 - \alpha_n) \rho_n(Q_0^2) \right] e^{-S\alpha_+(n)} \quad (4)$$

where $\alpha_n = (\gamma_{FF}^F - \gamma_+)/(\gamma_- - \gamma_+)$ and $\beta_n = \gamma_{VV}^F/(\gamma_- - \gamma_+)$ with the anomalous dimensions in the well known notation as given for example in

Ref. 1. The eigenvalues γ_{\pm} of the singlet anomalous dimension matrix are simply related to the renormalization group exponents in Eq. (4) for an asymptotically free QCD by¹

$$a_i = \frac{\gamma_i}{\alpha_s b}, \quad s = \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \quad (5)$$

with $2\pi b = 11 - 2f/3$, f being the number of flavors; for an asymptotically non-free fixed point theory these exponents read

$$a_i = \frac{\gamma_i}{2}, \quad s = \ln \frac{Q^2}{Q_0^2} \quad (6)$$

where now the value of the UV finite fixed point α^* , appearing in γ_i , has to be determined from experiment: a moment analysis¹² of F_3^{vN} with non-colored (abelian) vector gluons yielded a fixed point of about 0.5 and therefore, in the only relevant remaining² fixed point theory with colored (non-abelian) gluons which survived all tests so far, we have to take $\alpha^* = \frac{3}{4} 0.5$ in order to account for the different value^{1,2} of the group invariant $C_2(R)$. For this fixed point QCD all anomalous dimensions are the same as for an asymptotically free QCD except^{1,2} taking the group invariant $C_2(G) = 0$ in γ_{VV}^V .

The Q^2 evolution of the gluon distribution is now uniquely predicted by Eq. (4) provided we know the input wave functions at Q_0^2 , i.e.

$$g_n(Q_0^2) \equiv \Sigma_n(Q_0^2)/G_n(Q_0^2) \quad (7)$$

with the fermionic flavor-singlet component $\Sigma(x, Q^2) \equiv \sum_f [q_f(x, Q^2) + \bar{q}_f(x, Q^2)]$, the sum runs over all quark flavors f , being measured directly by^{8,9}

$F_2^{\nu N}(x, Q^2) = x \Sigma(x, Q^2)$ above charm threshold, and where we have taken $s \approx \bar{s}$ and $c \approx \bar{c}$ for the small strange and charmed parton distributions.

In order to avoid any theoretical prejudices as far as the input gluon distribution $G(x, Q_0^2)$ is concerned, we only use the lowest moments $G_n(Q_0^2)$ determined experimentally^{8,9} from the Q^2 variation of the $n = 2, 3, \dots$ moments $F_{2, n-1}^{\nu N}(Q^2)$. It should be noted that such a determination is insensitive² to the triple gluon vertex, which enters the Q^2 evolution predicted by the renormalization group only in a very indirect way (via the singlet mixing). The actual values obtained for $G_n(Q_0^2)$ in this way are therefore practically independent of whether one uses the common asymptotically free QCD or a fixed point version of QCD with no gluon self-couplings. The input quantities in Eq. (7) are then estimated to be^{8,9}

$$\rho_3 = 1.08 \pm 0.15, \quad \rho_4 = 2 \pm 0.5, \quad \rho_5 = 1.6 \pm 0.4 \quad (8)$$

at $Q_0^2 \approx 4 \text{ GeV}^2$. The $n = 2$ moments are not very interesting for our purpose since they refer just to energy-momentum conservation.

Figure 1 shows the predictions for the Q^2 evolution of the lowest gluon moments as predicted by Eq. (4). It can be seen that the effects due to the triple gluon coupling, the term proportional to the group invariant $C_2(G)$ in the purely gluonic anomalous dimension¹ γ_{VV}^V , are enormous (this is in contrast² to the fermionic singlet quantity $\Sigma(x, Q^2)$ measured by $F_2^{\nu N}(x, Q^2)$). Already at $Q^2 \approx 50-100 \text{ GeV}^2$, the predictions in Fig. 1(b) of a fixed point version of QCD (a non-Yang-Mills theory with colored gluons

and colored quarks but with no gluon self-interactions) differ by more than a factor of two from the results of QCD in Fig. 1(a). But even within asymptotically free QCD the Q^2 evolution of $G_n(Q^2)$ is entirely dominated by the triple gluon vertex as can be seen from the difference between the solid and dashed curves in Fig. 1(a), the latter being the result without the contribution stemming from the triple gluon coupling ($C_2(G) = 0$ in $\gamma_{\nu\nu}^V$). At $Q^2 \approx 10^4 \text{ GeV}^2$, a region appropriate for future ep colliding beam facilities, the effects due to the triple gluon coupling become as large as one order of magnitude. These results depend rather weakly on the poorly known higher input moments in Eq. (8): Taking the extreme values of $g_n(Q_0^2)$ as allowed by experiment in Eq. (8), the results shown in Fig. 1 differ by less than 3 %.

Thus measuring, in addition to $F_2(x, Q^2)$, the lowest moments of $F_L(x, Q^2)$ up to $Q^2 \approx 50\text{-}100 \text{ GeV}^2$, which should be feasible⁶ with the μ -beam and neutrino experiments at CERN in the near future, the Q^2 dependence of the moments of gluon distributions will provide us with clean unambiguous tests of the Yang-Mills structure of QCD, i.e., of the gluon self-couplings which are so very essential for asymptotic freedom.

References

1. M. Glück and E. Reya, Phys. Rev. D16, 3242 (1977).
2. M. Glück and E. Reya, DESY 79/13 (1979), to be published in Nucl. Phys. B.
3. G. Altarelli and G. Parisi, Nucl. Phys. B126, 298 (1977).
4. A. de Rújula, R. Petronzio and B. Lautrup, Nucl. Phys. B146, 50 (1978).
5. K. Koller, T.F. Walsh and P.M. Zerwas, DESY 78/79 (1978), to be published in Phys. Lett. B.
6. See, for example, E. Gabathuler, XIX International Conference on High Energy Physics, Tokyo, 1978, and private communication; CDHS coll., F. Eisele and K. Kleinknecht, private communication.
7. H.L. Anderson, H.S. Matis and L.C. Myriantopoulos, Phys. Rev. Lett. 40, 1061 (1978).
8. BEBC coll., P.C. Bosetti et al., Nucl. Phys. B142, 1 (1978).
9. CDHS coll., J.G.H. de Groot et al., QCD Analysis of Charged Current Structure Functions, to be published in Phys. Lett. B.
10. A. Zee, F. Wilczek and S.B. Treiman, Phys. Rev. D10, 2881 (1974);
A. de Rújula, H. Georgi and H.D. Politzer, Ann. of Phys. 103, 315 (1977);
I. Hinchliffe and C.H. Llewellyn Smith, Nucl. Phys. B128, 93 (1977).
11. M. Glück and E. Reya, Nucl. Phys. B145, 24 (1978).
12. E. Reya, DESY 79/02 (1979), to be published in Phys. Lett. B.

Figure Caption

Fig. 1. Predictions for the Q^2 evolution of gluon moments according to Eq. (4) for (a) asymptotically free QCD and (b) a non-Yang-Mills colored gluon fixed point version of QCD. The dashed curve for $n = 3, 4$ in (a) is the contribution with the triple gluon coupling turned off ($C_2(G) = 0$ in γ_{VV}^V); this contribution is similar but slightly larger for $n = 5$.

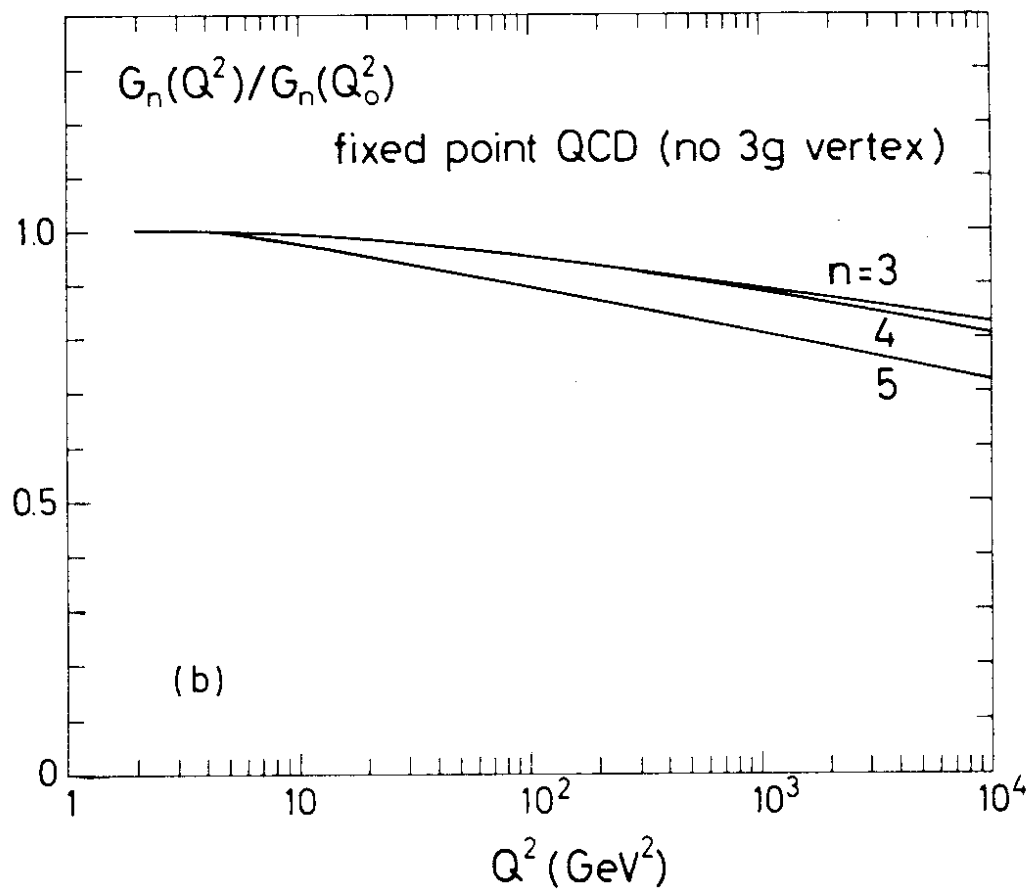
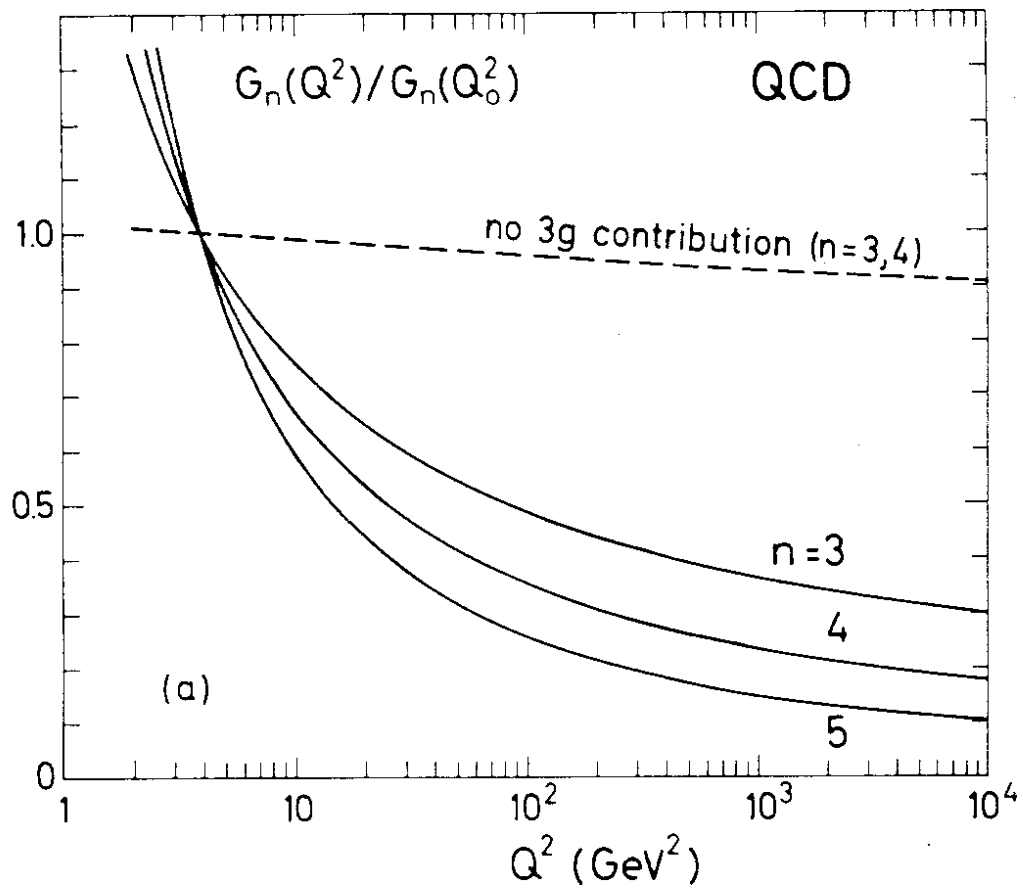


Fig. 1