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by

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Abstract

We investigate the four-jet production processes $e^+e^- \rightarrow q\bar{q}gg$ and $e^+e^- \rightarrow q\bar{q}q\bar{q}$ in lowest order QCD perturbation theory. We estimate that four-jet events should be detectable at a rate of about 5 % at the highest PETRA energy. The acoplanarity distribution is calculated and compared to nonperturbative effects.

There is increasing evidence that quantum chromodynamics (QCD), the gauge theory of coloured quarks and gluons, is the underlying theory of the strong interactions. However, QCD is not yet a well defined theory. The confinement problem, e.g., is still in the dark, and we also do not know how to compute the hadron spectrum. But there is a large number of computable cross sections which can serve as tests and applications of QCD.

One of the striking predictions of QCD is the existence of hadronic jets in e^+e^--annihilation resulting from the primordial production of quarks and gluons [1)2)]. At moderate energies this means basically a two-jet final state coming from the lowest order diagram $e^+e^- \rightarrow q\bar{q}$ while gluons radiating off the quark and antiquark remain hidden in the nonperturbative jet spread. Experiments at SPEAR [3)] and DORIS [4)] have confirmed this prediction. The angular distribution of the jet axis with respect to the beam direction was found to be very close to $1 + \cos^2\Theta$ [4)5)] as one would expect for the production of a pair of point-like spin-1/2 particles.

Quarks and gluons are expected to materialize as jets in which the particles have limited transverse momentum $\langle p_T \rangle_{\text{nonpert.}} \simeq 0.3$ GeV. In contrast to this the mean transverse momentum of a bremsstrahlung gluon (with respect to the jet axis) grows with increasing energy like $\langle p_T^2 \rangle \sim q^2/\ln q^2$ [6)-10)]. Therefore the production of quarks and gluons becomes visible at high enough energy as final states containing three or more jets. This is perhaps the most direct way to "detect" quarks and gluons and their interactions at short distances.

QCD predictions for three-jet final states have been given in detail in the literature [6)-12)]. One can estimate that three-jet events will be detectable at a rate [+)] of 10-20 % at $q^2 \gtrsim 20$ GeV. This is plenty to "unveil" the gluon.

QCD shows its full gauge structure, however, only in second or higher order perturbation theory (order $\gtrsim \alpha_s^2$) where the triple-gluon coupling comes in. In second order we will be led to four-jet final states. We shall argue in this letter [++)] that four-jet events are detectable at the upper PETRA energy. This will be an interesting test of perturbative QCD.

To order α_s^2 the four-jet cross section is given by the two sets of diagrams shown in Fig. 1 corresponding to the final (jet) states

$$e^+e^- \to q(p_1)\bar{q}(p_2)g(p_3)g(p_4) \tag{1}$$

and

$$e^+e^- \to q(p_1)\bar{q}(p_2)q(p_3)\bar{q}(p_4) \quad . \tag{2}$$

Four-jet events stand out against two- and three-jet final states by having a nonvanishing acoplanarity A. [9)] Thus $d\sigma/dA$ is the canonical quantity to analyse as it allows one to cut off the dominant two- and three-jet events experimentally.

+) E.g., $\frac{1}{\sigma_0}\int_{0.66}^{0.9} \frac{d\sigma}{dT}\,dT \simeq$ 0.15 at 20 GeV, cf. Ref. [9] and [12].

++) A detailed discussion of the physics of four-jet final states will be presented elsewhere [14)].

Since hadrons come out as jets with finite p_T it will only make sense to talk about four-jet final states if the angle between either two jet momenta p_i, p_j ($i \neq j$) is larger than the nonperturbative jet spread. This is equivalent to requiring that in $d\sigma/dA$ the acoplanarity is larger than [9]

$$A \geq \langle A \rangle_{nonpert.} = \frac{16}{\pi^2 q^2} \langle n \rangle^2 \langle p_T^2 \rangle_{nonpert.} \tag{3}$$

where $\langle n \rangle$ is the mean multiplicity. Condition (3) is sufficient to avoid the infrared singularities associated with collinear emission of quarks, antiquarks and gluons, which appear at $A = 0$. We find [14]

$$\left.\frac{d\sigma}{dA}\right|_{A \to 0} \sim \frac{1}{A} \left| \ln A \right|^3 \tag{4}$$

for the leading singularity. Thus we shall be concentrating on $d\sigma/dA$.

The trace calculations have been done using REDUCE [15]. The subsequent integration over quark, antiquark and gluon momenta involves a six-dimensional integral which has been performed using a Monte Carlo program.

In Fig. 2 we show the A-distribution normalized to the zeroth order cross section $\sigma_0 = 4\pi\alpha^2 \sum_a Q_a^2 / q^2$ for $\sqrt{q^2} = 40$ GeV. Since the q^2-dependence of $\sigma_0^{-1} d\sigma/dA$ is determined by $\alpha_S^2(q^2)$ the result can easily be extrapolated to other energies by the known q^2-dependence of $\alpha_S(q^2)$ where

$$\alpha_S(q^2) = 12\pi \Big/ ((33-2F)\ln(q^2/\Lambda^2)) \tag{5}$$

We take F = 5 and Λ = 0.5 GeV. Note that the nonperturbative A-distribution shrinks as $\ln^2 q^2/q^2$ whereas the perturbative QCD distribution does not. Already at this energy ($\sqrt{q^2}$ = 40 GeV) a significant fraction of events lies outside the non-perturbative A-regime which is characterized by $\langle A\rangle_{nonpert.}$ = 0.007. +) Such large A events signal the onset of order α_S^2 perturbative QCD effects.

In Fig. 3 we show the average A defined by

$$\langle A\rangle = \int dA\, A \frac{d\sigma}{dA} \Big/ (\sigma_0 (1+\frac{\alpha_S}{\pi})) \tag{6}$$

where we have used the cross section up to first order in α_S, i.e. $\sigma_1 = \sigma_0(1+\alpha_S/\pi)$ in the denominator of (6). For comparison we have also included the $\langle A\rangle_{nonpert.}$ as given by (3) with the replacement $\langle n\rangle \rightarrow \langle n-2\rangle$. +) One should be careful in the interpretation of $\langle A\rangle$ as calculated from (6) in that use of the second order QCD differential cross section formula is extended to the very small A region where finite order perturbation theory is known to break down. Although the moment integral in (6) is formally finite the use of the divergent perturbative expression $d\sigma/dA \sim |\ln A|^3/A$ in the small A region leads to an overestimate of A . It would therefore be more appropriate physically to define an

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+) In order to incorporate nonasymptotic effects we use $\langle n-2\rangle$ in (3) rather than $\langle n\rangle$ since at least two particles lie in the acoplanarity plane and therefore do not contribute to the acoplanarity. Furthermore we have assumed $\langle n\rangle = \frac{3}{2}(2+0.7\ln q^2)$ and $\langle p_T\rangle$ = 0.3 GeV.

average A in an interval excluding the small A region. In this vein the average A in Fig. 3 should rather be interpreted as providing an upper bound of $\langle A \rangle$ in perturbative QCD. The cross-over of $\langle A \rangle_{4\text{-jet}}$ with $\langle A \rangle_{\text{nonpert.}}$ occurs at $\sqrt{q^2} = 15$ GeV. This cross-over will be shifted to higher center-of-mass energies when one takes into account the cut on the four-jet cross section. One also should notice that $\langle A \rangle_{\text{nonpert.}}$ is not without ambiguities due to uncertainties in the experimental values of $\langle p_T \rangle_{\text{nonpert.}}$ and $\langle n \rangle$. Furthermore experimental measurements of $\langle A \rangle$ include also acoplanarity effects originating from weak decays.

The total four-jet cross section is not defined due to the $A \to 0$ singularity in (4) [+)]. Introducing a cut-off A_c one can calculate a cut-off dependent 4-jet cross section $\sigma(A_c)$ which we show in Fig. 4 for $\sqrt{q^2} = 40$ GeV. [++)] Again the energy dependence of $\sigma(A_c)$ is given by $\alpha_s^2(q^2)$.

Using a cut-off of $A_c = 0.05$ we have calculated the fraction of $(q\bar{q}q\bar{q})$ jet events in the total 4-jet rate. We obtain

$$\sigma(q\bar{q}q\bar{q})/\sigma(4\text{-jet}) = 0.08 \tag{7}$$

which means that most of the 4-jet events consist of two-gluon jets in addition to the quark-antiquark jets. We find that the ratio (7) is only mildly cut-off dependent.

+) Of course, the order α_s^2 correction (jets and loops) to the total cross section is finite.

++) The acoplanarity cut is sufficient to cut off the infrared region, but of course, is not necessary. Sufficient and necessary cuts are discussed in [14)].

We also show in Fig. 4 the ratio of 4-jet to 3-jet production at $\sqrt{q^2} = 40$ GeV as a function of A_c, where the three-jet cross section is calculated with a thrust cut-off at $T_c = 0.9$. We see that this ratio is appreciable if the A cut-off on σ(4-jet) is not chosen too large. On the other hand the A and T cut-offs cannot be chosen too small in order to retain the applicability of perturbative QCD. For the three jets we have taken the cut-off at $T_c = 0.9$ in order to have a reasonable ratio of σ(3-jet) to σ_{tot} (see footnote on p. 3). In order to obtain a similar ratio of σ(4-jet) to σ(3-jet) the cut-off for A in σ(4-jet) is $A_c = 0.05$.

In conclusion we find that σ(4-jet) with a reasonable cut-off is about 5 % of the total cross section and might be measurable at highest PETRA and PEP energies. $\sigma(q\bar{q}q\bar{q})$ is only a small fraction of σ(4-jet). An experimental study of the acoplanarity distribution $d\sigma/dA$ should serve as an additional important check on higher order QCD corrections.

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Figure Captions

Fig. 1: Tree diagrams for 4-jet production:

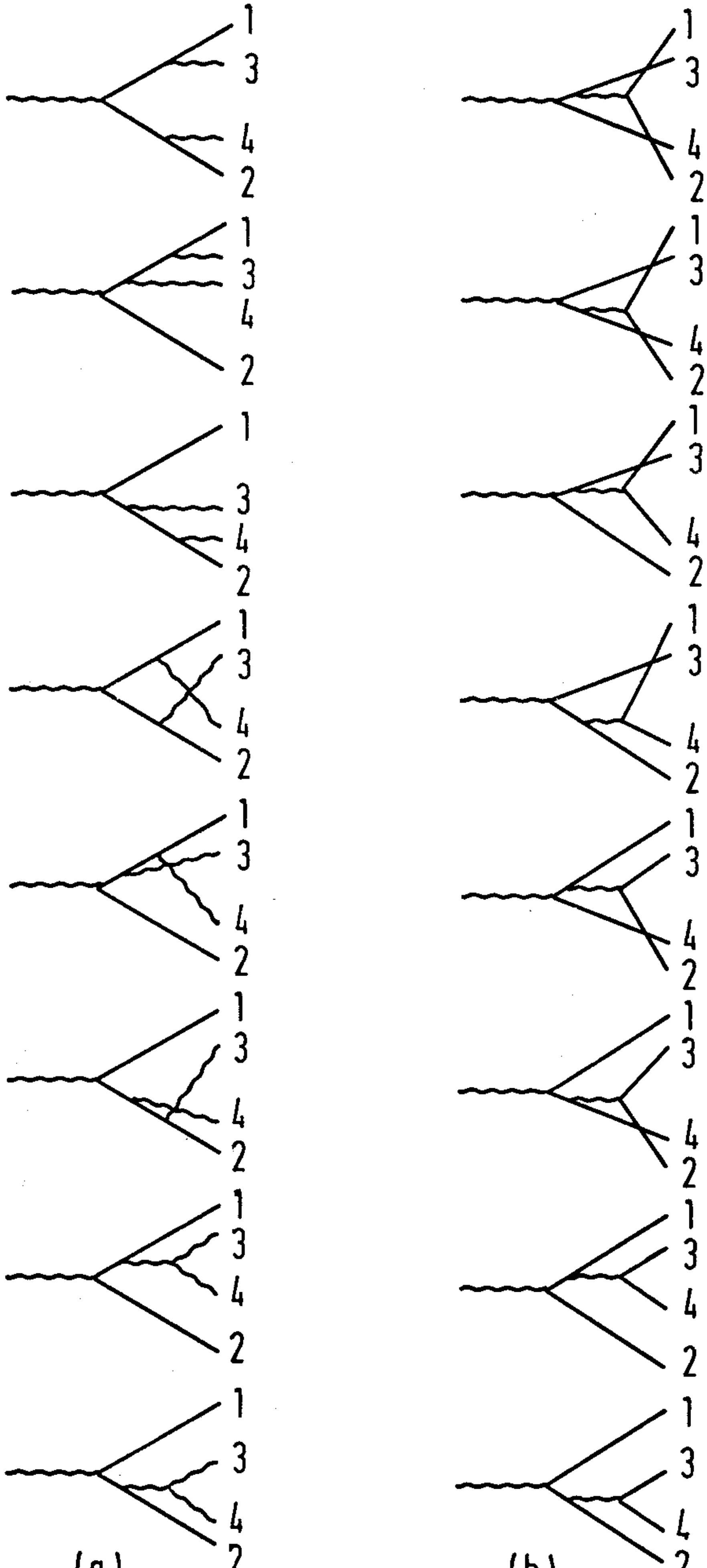
(a) $q\bar{q}gg$ and

(b) $q\bar{q}q\bar{q}$.

Fig. 2: $d\sigma/dA$ normalized to σ_o for 4-jet production

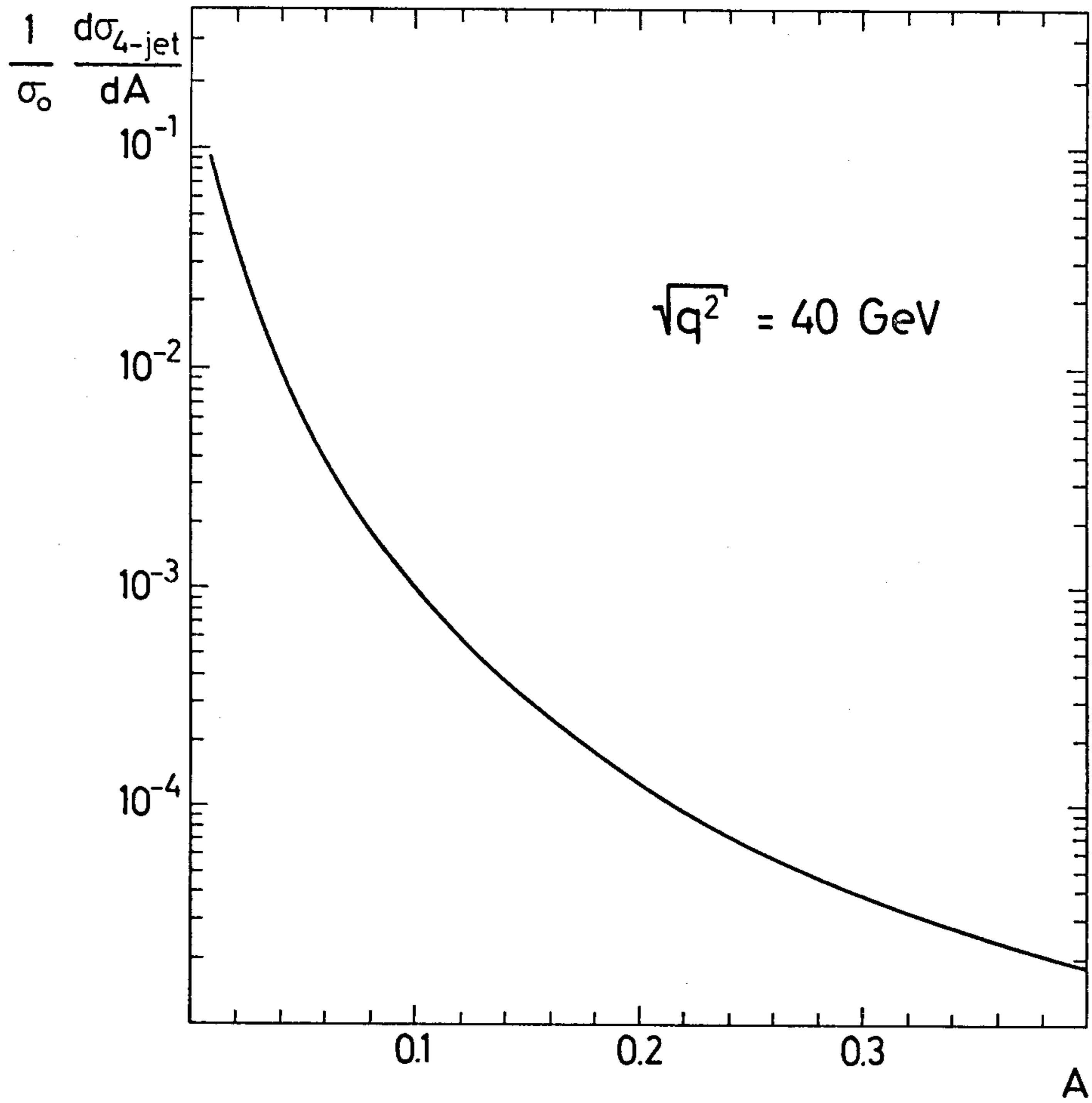
Fig. 3: Average acoplanarity. Full line: perturbative $\langle A \rangle$; dotted line: nonperturbative $\langle A \rangle$.

Fig. 4: Total 4-jet cross section normalized to σ_o as a function of the acoplanarity cut-off A_c. Ratio of σ (4 jet) to σ (3 jet) as a function of A_c. The 3-jet cross section is cut off at $T_c = 0.9$.



(a) (b)

Fig. 1



$\frac{1}{\sigma_o} \frac{d\sigma_{4-jet}}{dA}$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\sqrt{q^2} = 40$ GeV
0.1
0.2
0.3
A

Fig. 2

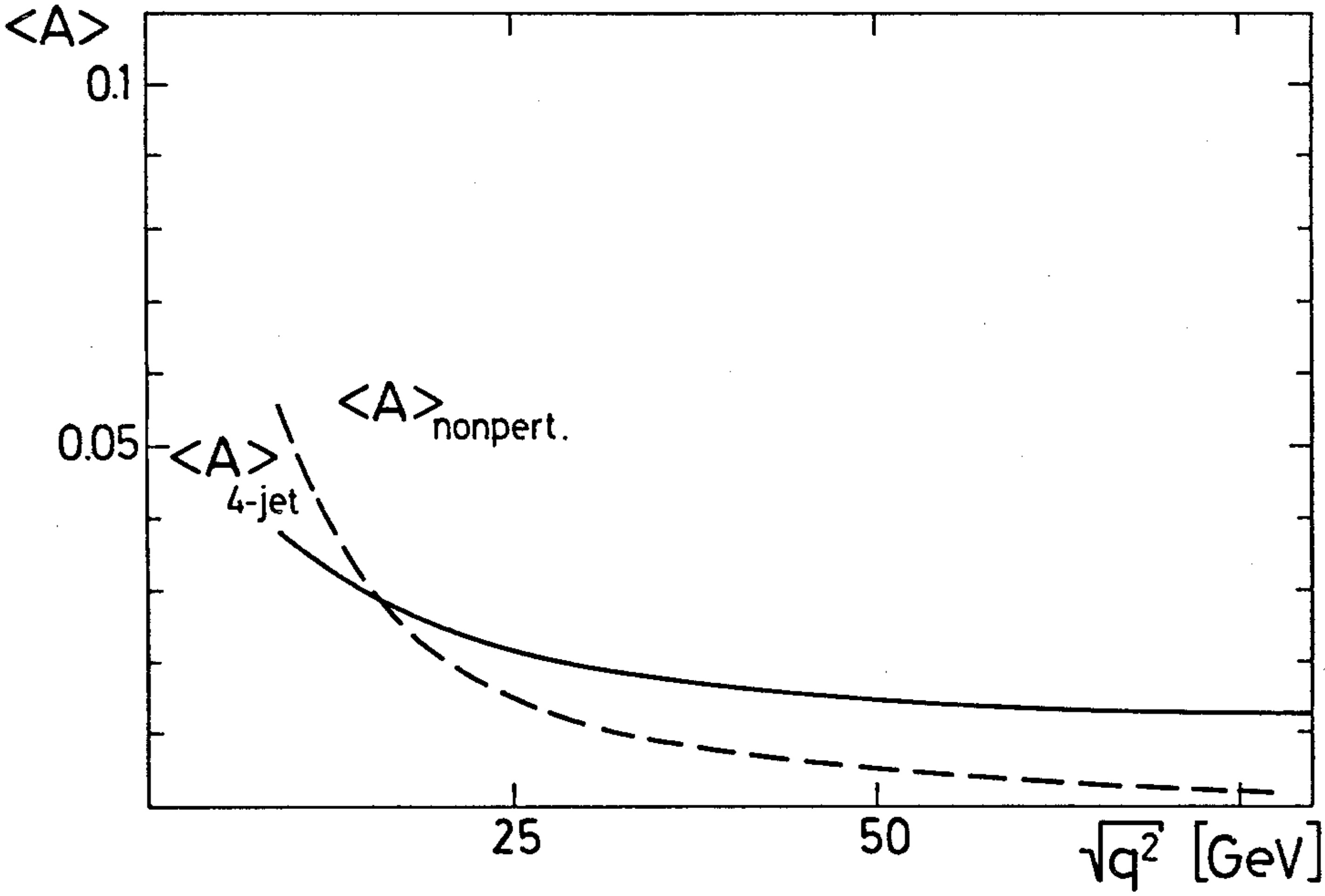


Fig. 3

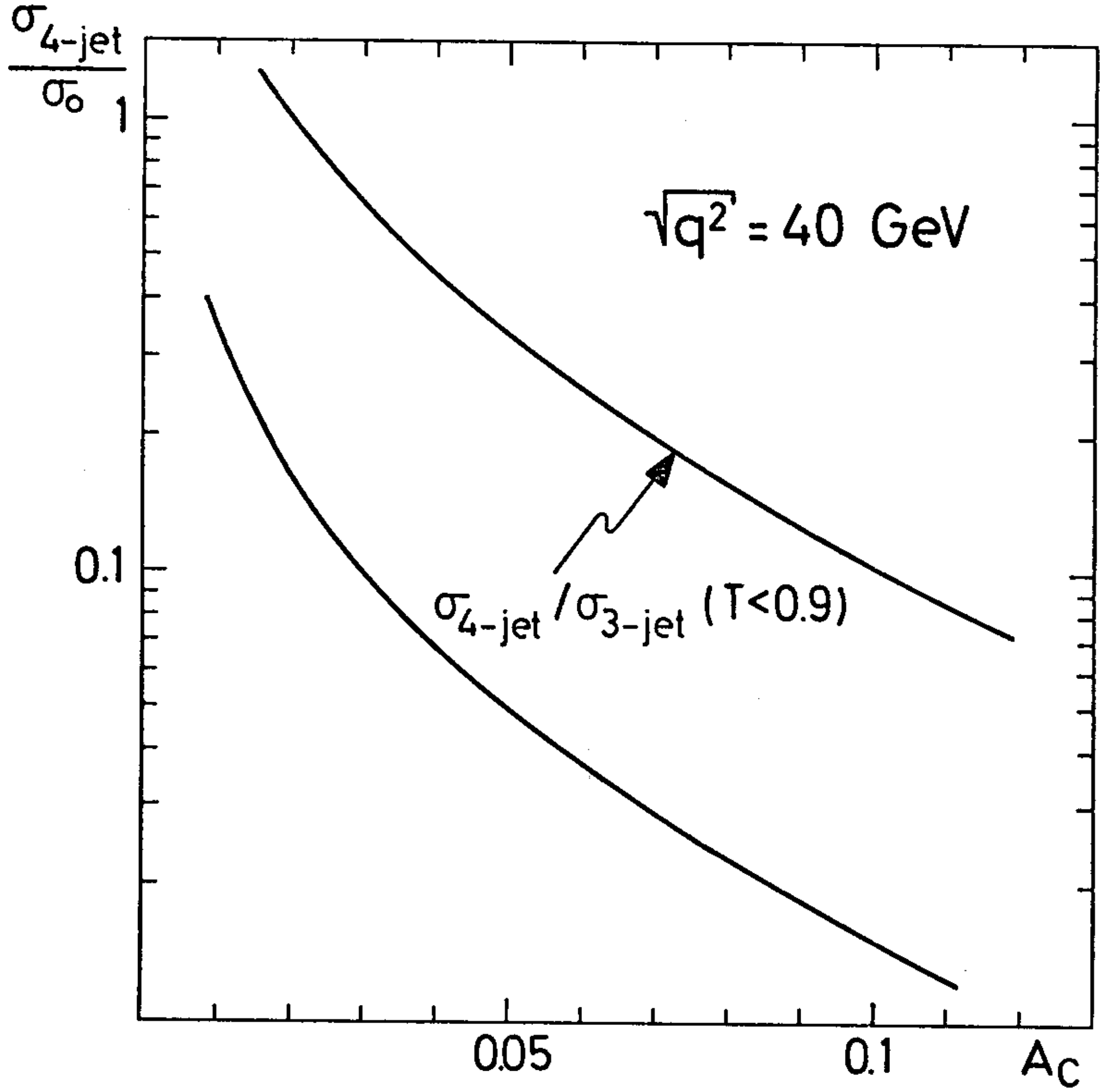


Fig. 4