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FINAL STATES IN NON-LEPTONIC BOTTOM MESON DECAYS

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Abstract

We use Isospin-Statistical Models to study many body final states in the non-leptonic decays of bottom mesons in the framework of the Kobayashi-Maskawa weak current. Estimates of charge multiplicities, branching ratios and inclusive hadron momentum distributions are presented.

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I - Introduction

The discovery of Υ (9.46) by Herb et al. [1] and its confirmation by three groups working at DORIS [2] is a strong indication that a new quark, called bottom b with charge Q = -1/3 exists. Mesons with nonzero bottom quantum number should also exist. These would consist of bound states of the b quark with the known antiquarks $\bar{u}$, $\bar{d}$, $\bar{s}$ and $\bar{c}$. The lowest lying multiplet of these new bottom mesons must decay weakly.

Recently, there have been many attemptsin the framework of the Kobayashi-Maskawa model [3] to study the leptonic and hadronic final states in the decay of the bottom mesons [4,5,6]. The masses of the bottom mesons (quarks) lie in a kinematic region where the jet embryo from the decay $Q \to qq\bar{q}$ is not likely to be observed though this mechanism may reliably be used to calculate the jet-broadening effects that a $Q\bar{Q}$ (or $B\bar{B}$) production will induce in an e^+e^- experiment [6]. On the other hand, scaling the two body and quasi two body branching ratios from charm to bottom, the exclusive two body states are not expected to dominate either [7]. Since the expected masses of the bottom mesons are large enough to allow multi-pionic and-kaonic production in the decay process, one could employ with some justification a statistical model approach to estimate some average quantities like particle multiplicities and inclusive pion energy spectrum etc. Hopefully, this should work approximately for the non-leptonic decay.

The dominant piece of the bottom changing weak current in the KM model induces the transitions

$$b \to c + (\bar{u} d) \qquad (1.1)$$

$$b \to c + (\bar{c} s) \qquad (1.2)$$

However, phase space in the transition (1.2) is very much reduced due to the heaviness of the charm quark (meson) pairs and we expect (1.2) to be dominated by two body and quasi two body modes. The transition (1.1) leads to the following final states in the decay of the various bottom mesons

$$B_u^- \to (D\pi, \eta_c K) + n\pi \qquad (1.3)$$

$$B_d^o \to (D\pi, D\eta, D\eta', \eta_c \bar{K}^o, \eta_c D, DK) + n\pi \qquad (1.4)$$

$$B_s^o \to (F\pi, DK, \eta_c\eta, \eta_c\eta') + n\pi \qquad (1.5)$$

$$B_c^- \to (\eta_c\pi^-, \pi^o\pi^-, K^oK^-, \bar{D}^oK^-, K^oD^-, \eta F^-, \eta' F^-, \eta_c F^-) + n\pi \qquad (1.6)$$

We remark that we have not separately treated the decays involving vector mesons, for example $B_u^- \to D^*\pi$, $D\rho$ etc. Since D^* decays into $D\pi$ and ρ into two pions dominantly, the end product contains a D (or F) meson and a number of pions, contributing to $D + n\pi$ (or $F + n\pi$) modes in our approach. In the same philosophy the decays of B_c^- do pose a problem since the modes $B_c^- \to \eta_c\pi^-$, $J/\psi\,\pi^-$ etc. are all allowed. However, only a very small fraction of J/ψ decays involve η_c. So one has to determine the relative rates of $B_c^- \to J/\psi + n\pi$ and $B_c^- \to \eta_c + n\pi$. Because of this complication, the paucity of data on η_c and the expected small production of $B_c^+ B_c^-$ in e^+e^- experiments (as compared to $B_u^+ B_u^-$ or $B_d^o \bar{B}_d^o$) we don't present any detailed results for the B_c^- decay modes.

To determine the relative strengths of the groups of final states in the decays (1.3) - (1.5), we resort to the specific two body decays i.e. states with n=o. Presumably, the relative strength of the various two-body and quasi two-body decay modes is reliably given by the renormalisation group improved quark recombination methods [8]. It can be checked [7] that in the two body decays all but the first entries in (1.3)-(1.5) are colour suppressed. One expects that any subsequent quark pair production on top of the skeleton (2 body) quark decay diagrams (thus producing multipions) should maintain this colour suppression. In this sense our model amounts to a hybrid quark-statistical description where we take into account intrinsic colour suppression factors in neglecting otherwise allowed final states. We, therefore, concentrate only on the colour allowed multibody decays, namely

$$\begin{aligned} B_u^- &\to D\pi + n\pi \\ B_d^o &\to D\pi + n\pi \\ B_s^o &\to F\pi + n\pi \end{aligned} \qquad (1.7)$$

We also discuss the multikaonic states in the bottom meson decays namely

$$B \longrightarrow D\pi + n\pi + m(K\bar{K}) \qquad (1.8)$$

and calculate the relative contributions of the processes (1.7) and (1.8). We find that the total and charged multiplicities are not very sensitive to the contribution of (1.8).

In Section II we use the Fermi-Statistical Model and the Poisson distribution to calculate the relative branching ratios in (1.7). We then combine these estimates with the isospin considerations [9,10] to predict branching ratios in specific charge states and the average charge multiplicities in the decay of bottom mesons. Both the particle and charge multiplicities turn out to be large, and we argue that they distinguish the weak transition $b \to c + \bar{u}d$ from the transition $b \to u + \bar{u}d$.

We also calculate the inclusive energy distributions for the D- and π-mesons from the non-leptonic bottom meson decays, based on a statistical phase space model. The D-meson energy distribution is compared with the correspondending distributions from the parton model calculations based on the processes $b \to c + \bar{u}d$ and $b \to c + l\nu_l$, assuming a $c \longrightarrow D$ quark fragmentation function. We find that the pions and D's have rather soft energy spectra, which should be indicated by a shift in the inclusive hadronic distributions $\frac{1}{\sigma}\frac{d\sigma}{dx}$ towards smaller x $(= 2E_h/\sqrt{s})$ as the bottom meson threshold opens up in e^+e^- annihilation experiments.

Section III contains a brief discussion of our results.

II. Isospin - Statistical models

The rationale of the Isospin-Statistical Models lies in the observation [9] that the number of invariant isospin amplitudes grows rapidly with the number of particles. The particle distribution is given by assuming a thermodynamic statistical model which lets the debris of a bottom meson (or any other heavy meson) evolve into pions and additional mesons to conserve the quantum numbers in accordance with a transition, like e.g. (1.1). The charge distributions are then calculated by letting each of the isospin amplitudes have equal magnitude and assuming that the interference terms can be neglected when averaged over kinematic variables. Clearly such an averaging is not expected to work for low n. However, one may take some comfort in the remark that the two body and quasi two body modes, which contribute to $n = o$ and 1 respectively, are small in specific pole model calculations [7] as well as in statistical models (see below). The equal weight (for each isospin state) approximation is expected to work better for $n \gg 2$ which are the dominant decay modes.

We shall study in detail the consequences of imbedding the transition

$$b \rightarrow c + (\bar{u}d) \qquad (1.1)$$

in a statistical approach. As mentioned in the introduction, only the states (1.7) are expected to contribute dominantly to the many body final states in bottom meson decays. Later, we comment on the effect of considering the phase space suppressed transition $b \rightarrow c + (\bar{c}s)$. We also present multiplicities following from the transition $b \rightarrow u + \bar{u}d$.

The transition (1.1) leads to the selection rules

$$\Delta B = -\Delta C = -\Delta Q$$
$$|\Delta I| = |\Delta I_3| = +1$$

Since $I(B_u^-, B_d^o) = 1/2$ and B_s^o, B_c^- are isosinglets, we need consider only $I = 3/2,\ 1/2$ isospin states in the decays of (B_u^-, B_d^o) and $I = 1$ final states in the decays of B_s^o and B_c^-.
The needed charge coefficients to determine the branching ratios into specific charge states can be calculated from the existing literature [10].

Following [11,12], we have used two statistical (thermodynamic) distributions: (1) The Fermi Statistical Model [9], which states that the Lorentz invariant matrix element for the decay of $B \rightarrow D + (n+1)\pi$ is a constant times that for $B \rightarrow D + n\pi$.

$$\mathcal{M}(B \rightarrow D + (n+1)\pi) = c\,\mathcal{M}(B \rightarrow D + n\pi) \qquad (2.2)$$

where the constant c depends on the masses involved.

(2) Poisson distribution: The probability $P_{\langle n \rangle}(n)$ of observing $B \rightarrow D\pi + n\pi$, when the average value of n is $\langle n \rangle$, is given

by

$$P_{<n>}(n) = <n>^{n} \frac{e^{<n>}}{n!} \qquad (2.3)$$

$<n>$ can be calculated either from thermodynamic considerations [11]

$$<n> = 2 + 0.528\,(E/E_0)^{3/4} \qquad (2.4)$$

($E = M - M_1 - M_2$, $E_0 = \hbar c/R_0$, with R_0 the radius of the bottom meson)

where M_1 , M_2 are the masses of the particles in the decay $B \rightarrow 1+2+ n\pi$. Alternatively, $<n>$ can be calculated for the Fermi Statistical Model. Assuming

$$m_B = 5.2 \text{ GeV}$$
$$m_D = 1.865 \text{ GeV}$$
$$m_\pi = 0.14 \text{ GeV}$$

we get $<n> = 3.1$, using the Fermi Statistical Model. This corresponds to E_0 = 300 MeV in (2.4).

The two resulting particle distributions for the same value of $<n> = 3.1$ are shown in Fig. 1 . We remark that the Fermi Statistical distribution is narrower than the Poisson distribution and it leads to smaller branching ratios for n = 0 and n = 1. The relative branching ratios for the decay $(B_u^-, B_d^0) \rightarrow D\pi + n\pi$ are presented for convenience also in Table 1. The entries also hold for the decays $B_s^0 \rightarrow F^+ + n\pi$ and presumably approximately for the decays $B_c^- \rightarrow J/\psi + n\pi$. We thus find for the average particle multiplicity in the non-leptonic decay of bottom mesons using (1.7) ($<n>_{total} = D\pi + <n>$)

$$<n_{(B_d^0, B_u^-), B_s^0, B_c^-}>_{total} = 4.1 + <n_{D,F,J/\psi}> \qquad (2.5)$$

where $<n_{(D,F,J/\psi)}>$ are the average particle multiplicities in the decays of D,F and J/ψ mesons.

We now discuss the effect of including the multikaonic modes in the decay of bottom mesons. We use the Fermi Statistical Model to calculate the relative rates of the processes (1.8) as compared to (1.7). The results are shown in table 2. The numbers reflect pure phase space differences. The Fermi Statistical Model leads to an interesting prediction (SU(3) symmetry is implicit here):

$$\frac{B \longrightarrow D\text{+pions + kaons}}{B \longrightarrow D + \text{pions}} = 0.2$$

SU (3) symmetry breaking can reduce this ratio by roughly a factor 2. We find the average particle multiplicity from the processes (1.8) to be:

$$\langle n_B \rangle_{\substack{\text{non-leptonic} \\ \text{(processes (1.8))}}} = 3.7 + \langle n_D \rangle \qquad (2.6)$$

Combining Eqs. (2.5) and (2.6) and using the ratio 4: 1 for the processes (1.7) and (1.8) we get

$$\langle n_B \rangle^{\text{non-leptonic}}_{\text{total}} = 4.02 + \langle n_D \rangle$$

Since the effect of including the multikaonic processes (1.8) on the multiplicities is rather small, we shall concentrate on the processes (1.7) and use Table 1 for the subsequent calculations of charge multiplicity and inclusive momentum spectra.

The entries in Table 1 can be combined with isospin considerations to calculate the relative branching ratios for specific charge states. The results are contained in Tables 2,3 and 4 for the decays $B_d^o \to D + n\pi^{\pm,o}$, $B_u^- \to D + n\pi^{\pm,o}$ and $B_s^o \to F^{\pm} + n\pi$, using the Poisson distribution. The corresponding numbers for the Fermi Statistical model can be obtained using Table 1. Since the average charge multiplicities in the charm meson decays are now measured [13] , namely

$\langle n_D \rangle_{charged} = 2.3 \pm 0.3$ for both D^o and D^+

one could use them to predict the charge multiplicities in the non-leptonic bottom decays. We find [F 1]

$$\langle n_{B_d^o} \rangle^{non-leptonic}_{charged} = 2.63 + \langle n_D \rangle_{charged}$$
$$\langle n_{B_u^-} \rangle^{non-leptonic}_{charged} = 2.77 + \langle n_D \rangle_{charged}$$
$$\langle n_{B_s^o} \rangle^{non-leptonic}_{charged} = 2.65 + \langle n_F \rangle_{charged}$$

Predictions of the overall charge multiplicities in the decays of bottom mesons necessarily require a description of the semi-leptonic decays. There are good reasons to expect that the average particle multiplicities in the semi-leptonic decays of the bottom mesons are smaller (as compared to the non-leptonic decays).This is suggested from a quark model description of the two decay modes $Q \to q(\bar{q}q)$ and $Q \to q(\ell\nu_\ell)$ since in the latter case only one of the quarks is fragmenting. In paricular, in the decay

$$b \to c + \ell\nu_\ell \qquad (2.7)$$

we find $\langle E_c \rangle = 2.23$ GeV for $m_b = m_B = 5.2$ GeV and $m_c = m_D = 1.86$ GeV [7]. It is evident that the charm quark produced in (2.7) will fragment only softly thus leading to small particle multiplicity. The softness of the $c \to D$ fragmentation is moreover suggested by the high energy e^+e^- and ν - dimuon production data [13,14]. Alternatively, one could argue that in the processes

$$(B_d^o,\ B_u^-) \to D + n\pi + \ell\nu_\ell \qquad (2.8)$$

$$\left.\begin{array}{lll} B_s^o & \to & F^+ + n\pi + \ell\nu_\ell \\ B_c^- & \to & (J/\psi, \eta_c) + n\pi + \ell\nu_\ell \end{array}\right\} \text{ n even}$$

the $I = 0$ nature of the weak current

$$j_\lambda^{\Delta B = \pm 1} \sim \bar{b}\gamma_\lambda(1-\gamma_5)c + h.c. \qquad (2.9)$$

suppresses the emission of multipions in the low energy limit. This suppression should be more evident for large n. Moreover, the suppression is expected to work much better for the B_s^o and B_c^- semi-leptonic decays due to the I = o nature of the $F^{\pm}$,J/ψ and η_c mesons which forbids any odd number of pion emission. We guess that $\langle n \rangle_{charged}$ in the semi-leptonic decays of the bottom mesons is well described by the expressions; [F 2] (1.o is due to the charged lepton)

$$\langle n\, (B_d^o,\, B_u^-),\, B_s^o,\, B_c^- \rangle_{charged}^{semi-leptonic} = \langle n_{D,\ F,\ J/\psi} \rangle_{charged} + 1.o \qquad (2.1o)$$

Combining (2.1o) with the entries in Tables 2-4 and using a ratio of 1 : 2 for the semi-leptonic to non-leptonic decays [6] , we get

$$\langle n\ B_d^o \rangle_{charged} = \langle n_D \rangle_{charged} + 2.1 = 4.4 \pm o.3$$

$$\langle n\ B_u^- \rangle_{charged} = \langle n_D \rangle_{charged} + 2.2 = 4.5 \pm o.3$$

$$\langle n\ B_s^o \rangle_{charged} = \langle n_{F^\pm} \rangle_{charged} + 2.1 \qquad (2.11)$$

where for B_d^o and B_u^- we have used the experimental numbers (2.6). Thus we anticipate an average charged multiplicity of $\sim$ 9 to 1o in the process

$$e^+e^- \longrightarrow B + \bar{B} \quad + \text{anything}$$

This has to be contrasted with the number $\sim$ 6.0 which is obtained by extrapolating the measured charged multiplicity at 9.4 GeV [15] [F 3]

$$\langle n \rangle_{charged} (9.4\ \text{GeV}) = 4.9 \pm o.1 \qquad (2.12)$$

to $E_{c.m.}$ = 1o.4 GeV. Thus, the onset of $B\bar{B}$ threshold will be indicated by a jump in the charged multiplicity.

Next we calculate the anticipated particle and charge multiplicities in the bottom meson decay processes that arise from the weak transition

$$b \longrightarrow u + \bar{u}d \qquad (2.13)$$

Assuming the same value of E_o (=3oo MeV) in (2.4) that we have used earlier,one gets $\langle n_B \rangle_{total}^{non-leptonic} = 6.3$ assuming the weak current (2.13), leading to a charge multiplicity $\langle n_B \rangle_{charged}^{non-leptonic} \simeq 4.o$. The corresponding

numbers for the weak current (1.2) are $\langle n_B \rangle^{\text{non-leptonic}}_{\text{total}} = 4.1 + \langle n_D \rangle \simeq 8$ and $\langle n_B \rangle^{\text{non-leptonic}}_{\text{charged}} = 2.7 + \langle n_D \rangle_{\text{charged}} = 5.0 \pm 0.3$.
Thus, we anticipate larger multiplicities for the transition $b \to c + q\bar{q}$ as compared to the transition $b \to u + q\bar{q}$.

The entries in Table 1 can also be combined with a phase space model to predict the inclusive hadron energy spectrum in the non-leptonic decays of the bottom mesons. The rationale of undertaking this exercise again lies in the oberservation that the average energy per quark in the decay $b \to c + q\bar{q}$ does not allow a jet-oriented topology. The pion energy distribution in the process

$$(B^o_d \ , B^-_u) \longrightarrow D + n\pi \qquad (2.14)$$

is shown in Fig.2. The inclusive pion energy distribution from the process $B^o_s \longrightarrow F^+ + n\pi$ is very similar and hence not shown separately. We remark that the pion energy distribution in the process (2.14) is very soft and corresponds to an average pion energy, $\langle E_\pi \rangle = 0.74$ GeV in the rest frame of the B meson (with $m_B = 5.2$GeV). The D-meson energy distribution for the process (2.14) is presented in Fig.3, where for comparison we have also shown the distributions from the process [5]:

$$b \longrightarrow \underset{\hookrightarrow D + \cdots}{c}\, \ell^- \nu_\ell \, , \ \underset{\hookrightarrow D + \cdots}{c}\, \bar{u} d \qquad (2.15)$$

where $c \to D + \cdots$ denotes charm quark fargmentation. The distributions correspond to the following two choice of the fragmentation function

$$D\ (Z) = Z^n \qquad \text{with} \quad n = 1,2 \qquad (2.16)$$

with $Z = E_D/E_c$. The choice (2.16) have been motivated by the fact that the fragmentation $c \to D$ is supposed to take into account inelasticity in the decay process alone, for example, $B \to (D, D^*, D\pi, \cdots) + (\ell\nu_\ell / \nu\bar{\nu})$. It is clear that the use of fragmentation in this region is not kinematically justified since the charm quark in the decay (2.14) is produced with a momentum distribution over a large fraction of which no fragmentation $c \to D + n\pi$ is allowed due to phase space. However, not knowing any other reliable way to incorporate the inelasticities in the decay process, we keep using (2.16) with the hope that it describes the D-energy distribution in (2.15) in an average sense.

The distributions corresponding to the free quark decay mode (also shown in Fig. 3) and the statistical phase space model represent two extremes, with presumably D (Z) = Z a more likely description of the D meson energy distribution in the decay of a bottom meson at rest. We would like to close this section by an obvious remark that the large total multiplicities

$$\langle n_{e^+e^- \to B\bar{B}} \rangle = 2\left(\frac{2}{3}\langle n_B\rangle^{\text{non-leptonic}} + \frac{1}{3}\langle n_B\rangle^{\text{semi-leptonic}}\right) \simeq 15\text{-}16$$

would lead to a very low $\langle x\rangle = 2\,E_h/\sqrt{s}$, resulting in a shift of the inclusive hadronic distribution $\frac{1}{\sigma}\frac{d\sigma}{dx}$ to low x as compared to the background which gives $\langle n\rangle_{\text{total}}$ = 10 - 11 , extrapolating the measured particle multiplicity.

III. Conclusions

We have presented estimates of the many body decay modes of the anticipated bottom mesons namely $(B_u^-, B_d^o) \longrightarrow D + n\pi$ and $B_s^o \longrightarrow F^+ + n\pi$, based on two statistical models. These are combined with the assumption of equal weights for the different isospin final states in the bottom decays to predict branching ratios for specific charge states. We have then presented estimates for the charge multiplicity and anticipated hadron energy distributions in the decay of bottom mesons. In doing this, we have ignored all colour suppressed decay modes as well as some two body decay modes like $B_u^- \longrightarrow D^oF^-$, $B_d^o \longrightarrow D^+F^-$, $B_s^o \longrightarrow F^+F^-$ etc. These later decay modes presumably will not introduce any appreciable error in inclusive estimates of multiplicities since their contribution to $\langle n_B\rangle$ and $\langle n_B\rangle$ charged is approximately the same as the ones obtained from the processes (1.7). Processes like $B_u^- \longrightarrow DF + n\pi$, $B_d^o \longrightarrow DF + n\pi$ which lead to higher particle multiplicites are suppressed due to phase space. We have refrained from using statistical models to estimate the final states in the semi-leptonic decays of the bottom mesons. This is based partly on the failure of such an exercise in describing the charm decays and partly on the specific calculation that the average D meson energy in the semi-leptonic decay of the bottom mesons is rather modest. Pole models of the type $B \longrightarrow (D, D^*, D\pi) + \ell\nu_\ell$ etc. as well as the soft fragmentation quark models $b \longrightarrow c\,\ell\nu_\ell$, $c \longrightarrow D + \cdots$ should adequately account for the semi-leptonic decays. Based on the KM transition (1.1), we find $9 \lesssim 2\langle n_B\rangle_{\text{charged}} \lesssim 10$ as a reasonable range of charged particle multiplicity in the process $e^+e^- \longrightarrow B\bar{B}$. This is higher by at least two units as compared to the background estimated by extrapolating the measured charged multiplicity at $E_{c.m.}$ = 9.4 GeV to $E_{c.m.}$ = 10.5 GeV. The corresponding numbers for the transition $b \longrightarrow u + qq$ are estimated to be $7 \lesssim 2\langle n_B\rangle_{\text{charged}} \lesssim 8$. Thus, average particle and charge multiplicities, in principle, are sensitive to the underlying weak current.

The hadronic energy distribution from the non-leptonic decays are rather soft. This will result in shifting the cross section $\frac{1}{\sigma}\frac{d\sigma}{dx}$ ($x = 2\,E_h/\sqrt{s}$) towards lower values of x, indicating essentially the rise in particle multiplicity as the bottom meson threshold is crossed.

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Footnotes

F 1 Strictly speaking $\langle n_D \rangle$ involves both $\langle n_{D^o} \rangle_{charged}$ and $\langle n_{D^+} \rangle_{charged}$. However, since experimentally $\langle n_{D^o} \rangle_{charged} \simeq \langle n_{D^+} \rangle_{charged}$, we ignore any possible small difference between $\langle n_{D^o} \rangle_{charged}$ and $\langle n_{D^+} \rangle_{charged}$.

F 2 The multiplicity of ordinary hadrons (pions) from the fragmentation of c quark can be estimated, $n_c \simeq a \ln (\bar{E}_c / m_c)$ with $a \simeq 2$. This gives for the bottom decay $b \rightarrow c \ell \nu_\ell$, $n_c \simeq 0.5$. We have ignored n_c in the estimates of $\langle n \rangle^{semi\text{-}leptonic}_{charged}$ quoted in the text.

F 3 The result quoted in Eq. (2.12) is not corrected for acceptance. In estimating the background at 10.4 GeV, we have multiplied the measured multiplicity at 9.4 GeV by 1.2 and used the formula $\langle n \rangle_{charged}(10.4 \text{ GeV}) = \langle n \rangle_{charged}(9.4 \text{ GeV}) + 1.4 \ln (10.4/9.4)$ which gives $\langle n \rangle_{charged}$ (10.4 GeV) = 6. We acknowledge discussions with G.Alexander, H.J.Meyer and G. Zech on this point.

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Table Captions

Table 1: Particle multiplicities for the Fermi and Poisson Statistical Models with same $\langle n_B \rangle$ in the non-leptonic bottom meson decays. For the Fermi Statistical Model pion masses are not neglected and we have used $m_B = 5.2$ GeV, $m_D = 1.865$ GeV.

Table 2: Relative Contribution of multikaonic states in the process (1.8) using Fermi Statistical Model. The numbers correspond to SU (3) symmetric case.

Table 3: Relative (with repect to the non-leptonic modes) branching ratios for the specific charge states in the decays $B_d^o \rightarrow D +$ n pions, assuming an Isospin-Statistical Model with a Poisson distribution. Relative rates for the Fermi Statistical Model can be obtained using table 1. The entries lead to an average charged multiplicity $\langle n_{B_d^o} \rangle^{\text{non-leptonic}}_{\text{charged}} = 2.63 + \langle n_D \rangle_{\text{charged}}$.

Table 4: Same as Table 2 for the process $B_u^- \rightarrow D + n$ pions. The entries give an average charge multiplicity $\langle n_{B_u^-} \rangle^{\text{non-leptonic}}_{\text{charged}} = 2.77$ $+ \langle n_D \rangle$ charged .

Table 5: Same as Table 2 for the process $B_s^o \rightarrow F + n$ pions. The entries give an average charge multiplicity $\langle n_{B_s^o} \rangle^{\text{non-leptonic}}_{\text{charged}} = 2.65 +$ $\langle n_{F^\pm} \rangle$ charged.

Figure Captions

Fig. 1 Particle multiplicity distributions from the non-leptonic decays of the bottom mesons $(B_d^o, B_u^-) \longrightarrow D + n\pi$ with the same $\langle n \rangle_{total} = 5.1$. The distributions are calculated for $m_B = 5.2$ GeV, $m_D = 1.865$ GeV and $m_\pi = 0.14$ GeV. Solid line refers to Fermi statistical model and the dashed line to the Poisson distribution. Note D is included in $\langle n \rangle_{total}$ and is counted as one particle.

Fig. 2 Inclusive pion energy distribution from the non-leptonic decays $(B_d^o, B_u^-) \longrightarrow D + n\pi$ using statistical phase space model.

Fig. 3 Inclusive D-meson energy distribution from the non-leptonic and semi-leptonic decays of the bottom mesons. Solid curve corresponds to using phase space statistical model for $(B_d^o, B_u^-) \longrightarrow D + n\pi$. Dashed curve is calculated using free quark model $b \rightarrow c + \ell\nu_\ell$, $cq\bar{q}$ with $m_b = m_B = 5.2$GeV and $m_c = m_D = 1.865$ GeV. The other two curves show the effect of using a $c \rightarrow D$ fragmentation function D (Z) with dash-dot curve referring to D (Z) = Z and cross hatched to D (Z) = Z^2.

Decay Modes	Fermi Statistical Model	Poisson Distribution
$B \to D + \pi$	7 x 1o^{-3}	5 x 1o^{-2}
$B \to D + 2\pi$	8 x 1o^{-2}	o. 15
$B \to D + 3\pi$	o . 24	o . 22
$B \to D + 4\pi$	o . 33	o . 23
$B \to D + 5\pi$	o . 22	o . 17
$B \to D + 6\pi$	o . 1	o . 1
$B \to D + 7\pi$	2.2 x 1o^{-2}	o . o5
$B \to D + 8\pi$	7 x 1o^{-3}	o . o3
$\langle n_B \rangle$ non-leptonic	4.1 + $\langle n_D \rangle$	4.1 + $\langle n_D \rangle$

Table 1.

Decay Modes	Relative Rates (Fermi Statistical Model)
$\frac{B \to D + \pi + K\bar{K}}{B \to D + 3\pi}$	o.5
$\frac{B \to D + 2\pi + K\bar{K}}{B \to D + 4\pi}$	o.3
$\frac{B \to D + 3\pi + KK}{B \to D + 4\pi}$	o.17
$\frac{B \to D + \text{pions} + \text{kaons}}{B \to \text{pions}}$	o.2
$\langle n_B \rangle$ non-leptonic	3.7 + $\langle n_D \rangle$

Table 2.

$n(\pi^{+})$ =	o	1	2	3	4	5	6	7	8	Sum (Poisson)
D=	D^{o}	D^{+}	D^{o}	D^{+}	D^{o}	D^{+}	D^{o}	D^{+}	D^{o}	
$D\ \pi$	2.5	2.5								5.o
$D\ 2\pi$	2.25	6.o	6.25							15.o
$D\ 3\pi$	1.46	4.4	1o.27	5.87						22.o
$D\ 4\pi$	o.45	3.62	7.o5	7.96	3.92					23.o
$D\ 5\pi$	o.16	o.82	1.o	5.94	6.6	2.47				17.o
$D\ 6\pi$	o.o2	o.17	o.89	1.94	3.8	2.32	o.86			1o.o
$D\ 7\pi$	o	o.o1	o.o5	o.46	o.6	2.17	o.83	o.87		5.o
$D\ 8\pi$	o	o	o.o3	o.o8	o.58	o.32	1.65	o.18	o.16	3.o
Total	6.84	17.52	26.o4	22.25	15.5	7.28	3.34	1.o5	o.16	1oo

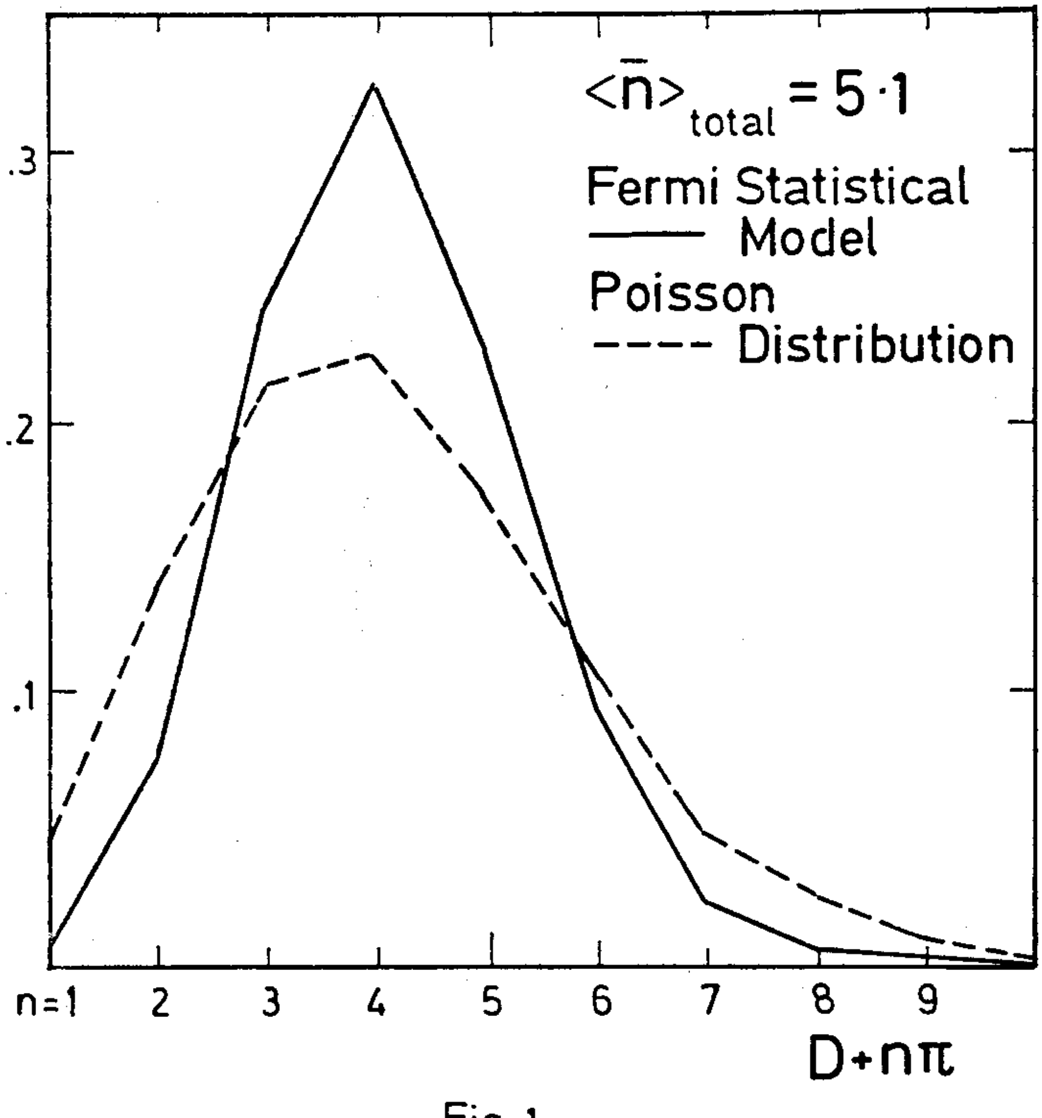
Table 3

$n(\pi^{\pm})$	1	2	3	4	5	6	7	8	Sum (Poisson)	
D =	D^0	D^+	D^0	D^+	D^0	D^+	D^0	D^+		
D π	5.0								5.0	
D 2 π	9.0	6.0							15.0	
D 3 π	6.16	7.04	8.8						22.0	
D 4 π	2.85	4.82	10.95	4.38					23.0	
D 5 π	0.9	1.94	6.47	4.85	2.84				17.0	
D 6 π	0.22	0.57	2.5	2.76	3.07	0.88			10.0	
D 7 π	~ 0	0.06	0.35	0.88	0.76	2.66	0.29		5.0	
D 8 π	~ 0	~ 0	0.17	0.11	1.16	0.2	1.32	0.04	3.0	
Total	24.13	20.43	29.74	12.98	7.83	3.74	1.61	0.04	100	

Table 4

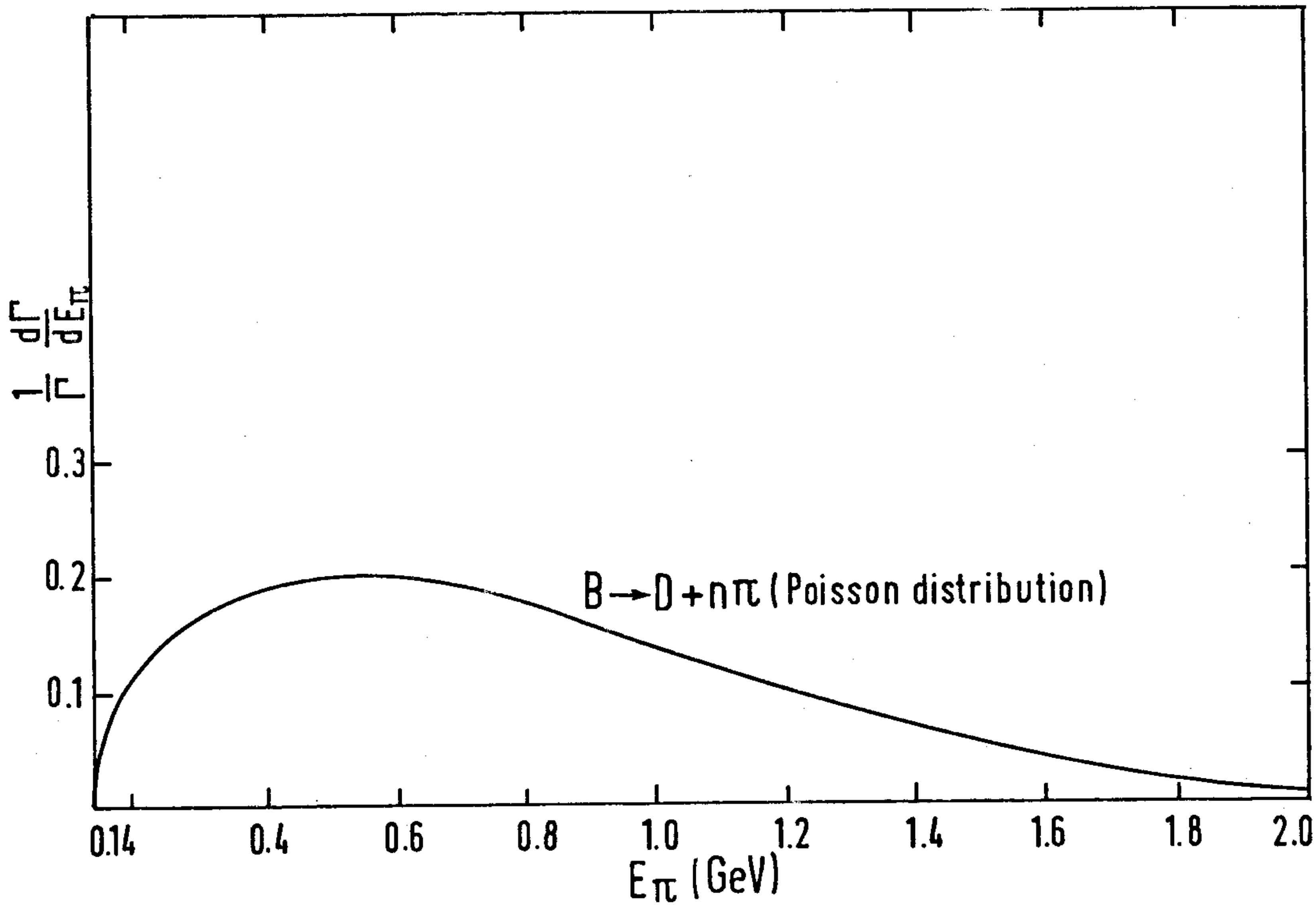
$n(\pi^{\pm})$	1	3	5	7	Sum (Poisson)
F π	5.o				5.o
F 2 π	15.o				15.o
F 3 π	8.8	13.2			22.o
F 4 π	4.6	18.4			23.o
F 5 π	1.45	1o.7	4.85		17.o
F 6 π	o.35	4.3o	5.35		1o.o
F 7 π	o.o8	1.26	3.o2	o.64	5.o
F 8 π	o.o2	o.42	1.6	o.96	3.o
Total	35.3	48.28	14.82	1.6	1oo.o

Table 5



$\langle\bar{n}\rangle_{total} = 5\cdot1$
Fermi Statistical
Model
Poisson
Distribution
.3
.2
.1
n=1
2
3
4
5
6
7
8
9
$D+n\pi$

Fig·1



$\frac{1}{\Gamma}\frac{d\Gamma}{dE_\pi}$
0.3
0.2
0.1
$B \to D + n\pi$ (Poisson distribution)
0.14
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
E_π (GeV)

Fig. 2

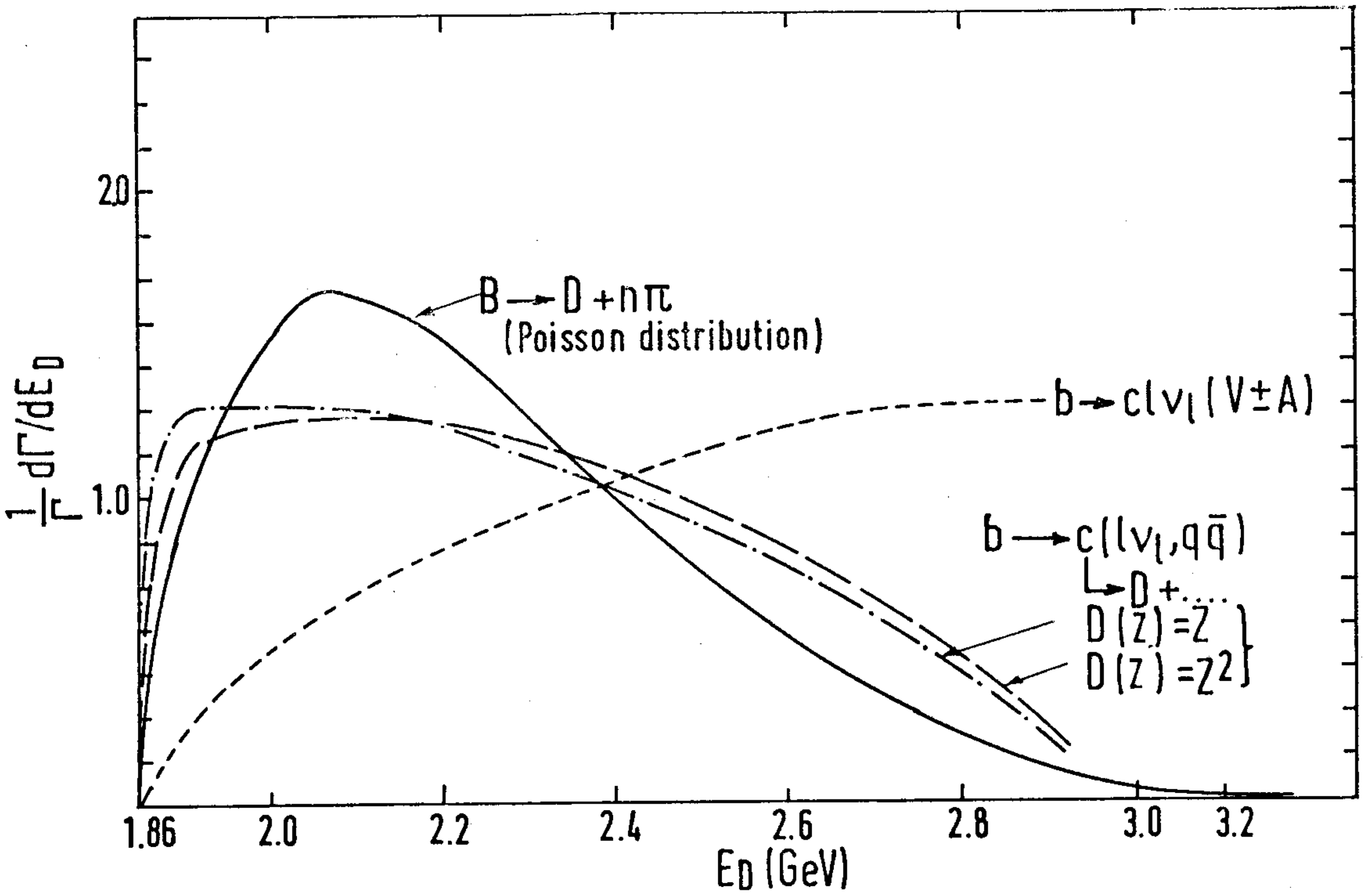

$\frac{1}{\Gamma} d\Gamma/dE_D$
2.0
1.0
$B \rightarrow D + n\pi$
(Poisson distribution)
$b \rightarrow c l \nu_l$ (V±A)
$b \rightarrow c (l\nu_l, q\bar{q})$
$\hookrightarrow D + \ldots$
$D(z) = z$
$D(z) = z^2$
1.86
2.0
2.2
2.4
2.6
2.8
3.0
3.2
E_D (GeV)

Fig.3