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## Evading the 'Axion' without Massless Quarks

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# Evading the 'Axion' without Massless Quarks

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Abstract

We point out a possible mechanism which avoids the presence of axions without having a massless quark. Some of the quarks must obtain their masses through the dynamical spontaneous breakdown of a chiral symmetry.

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Recently it was pointed out by Peccei and Quinn (1) that the possible large CP violation due to the instanton (2) effects can be avoided by imposing a U(1) symmetry on strong, electromagnetic and weak interactions (including the Higgs interactions). If this U(1) is to be violated spontaneously then we expect the existence of a Nambu-Goldstone (3) boson or a pseudo-Nambu-Goldstone (4) boson since the chiral symmetry is violated by the instanton anyway. The existence of such a particle was pointed out by Weinberg (5) and by Wilczek (6). In their recent articles they call it the 'axion'. An alternative without an axion was suggested in which they must have at least one massless quark.

In this note we want to point out the existence of a natural model with chiral symmetry, and also a way to avoid the existence of axions without massless quarks. The model was already discussed by S. Pakvasa and by myself in a paper entitled "Discrete Symmetry and the Cabibbo Angle" (7). The model Lagrangian is invariant under the group $S_3$ in addition to the ordinary SU(2) x U(1) of Weinberg and Salam (8). The origin of the $S_3$ symmetry was not stated in the paper. We now have the following reasoning. If we want to violate CP through the vacuum expectation values of Higgs bosons which we take to be all SU(2) doublets we need at least three of them.

Nature does not seem to have any means to distinguish between these three doublets, which means that the Lagrangian is invariant under the arbitrary transformations among the three Higgs. To avoid the appearance of Nambu-Goldstone bosons we restrict ourselves to discrete symmetries. Then follows the $S_3$ symmetry automatically. Note that this symmetry is defined through the Higgs bosons, not through quarks. In case of a four quark model the left handed doublets $\left\{ \binom{p_1}{n_1}_L, \binom{p_2}{n_2}_L \right\}$ constitute an $S_3$ doublet and $\{ n_{1R}, n_{2R} \}$ also transforms like a doublet. But to obtain a result which is consistent with the observation we are forced to assign $p_{1R}$ and $p_{2R}$ to singlets separately. This results in a massless up-quark providing the freedom $u_R \to e^{i\alpha} u_R$ necessary to avoid the large CP violation due to the instanton. This U(1) symmetry is not violated by the Higgs bosons. Thus the up-quark seems to stay massless which is not consistent with the current algebra calculations [9]. Our aim here is to show that this may not be the case, due to the dynamical spontaneous breakdown [10] which presumably avoids the appearance of a Nambu-Goldstone boson [11]. To be able to compare our results with the closely related results of massless Q.E.D. [12] we work in the lepton sector. Here $\left\{ \binom{\nu_e}{e}_L, \binom{\nu_\mu}{\mu}_L \right\}$ is an $S_3$ doublet and $e_R$ and $\mu_R$ are separately singlets in $S_3$. We have $m_e = 0$ just as we have $m_u = 0$ in the quark sector. The customary way to study the dynamical spontaneous break down is to write down the Dyson-Schwinger equation for the electron propagator and to look for a symmetry breaking solution to the equations. If we do this, only the diagrams with electron propagators will be important. The reason is to cancel a small coupling constant in front of the momentum integral in the D-S equation, we need a very slow damping (when $p^2 \to \infty$ ) propagator [12]. We expect only the electron propagator can have this property since this is the only one which corresponds to the dynamical breaking. To clarify the approximation involved we use a different approach, instead of



directly referring to the D-S equation.

Neglecting all the contributions other than the photon and the Z-boson exchange, the effective Lagrangian can be written as

$$\mathcal{L}_{eff} = \bar{e}(i\gamma^\mu \partial_\mu) e + \frac{e_0^2}{2} \int T\left(J_\mu(x) D_\gamma^{\mu\nu}(x,y) J_\nu(y)\right) d^4x\, d^4y$$
$$+ \frac{G_{T_0}^2}{8} \int T\left( J_\mu^5(x) D_Z^{\mu\nu}(x,y) J_\nu^5(y) \right) d^4x\, d^4y , \tag{1}$$

where the path integrals over the photon field and the Z-boson field are already performed, neglecting non-abelian terms. Here

$$J_\mu = \bar{e} \gamma_\mu e \qquad \text{and} \qquad J_\mu^5 = \bar{e} \gamma_\mu \gamma_5 e \tag{2}$$

and $D_\gamma$ and $D_Z$ are the photon and the Z-boson propagators in a certain gauge. We assumed $\sin^2\theta_W = \frac{1}{4}$ to simplify the Z-boson coupling. For this value of the Weinberg angle the electron-Z-boson coupling is pure axial vector and there is no parity violating effect in the electron propagator. After Fiertz-transforming the four-fermion part we set $\rho(x-y) = \langle 0 | T(\bar{\psi}_e(x) \psi_e(y)) | 0 \rangle$ to be the order parameter and use the mean field approximation or the Hartree-Fock-Bogoliubov approximation [13]. Let us work in the Feynman gauge and put $D^{\mu\nu}_{(\gamma,Z)} = g^{\mu\nu} D_{(\gamma,Z)}$

We then define

$$A(k^2) = \int \left\{ e_0^2 D_\gamma(x) + \frac{G_{T_0}^2}{2} D_Z(x) \right\} \rho(-x)\, e^{ikx} d^4x \tag{3}$$

The gap equation in terms of $A(k^2)$ takes the following form:

$$A(k^2) = -4ie_0^2\int\frac{d^4k'}{(2\pi)^4}\frac{1}{(k'-k)^2}\frac{A(k'^2)}{k'^2-A^2(k'^2)} - i\,G_0^2\int\frac{d^4k'}{(2\pi)^4}\frac{1}{(k'-k)^2-m_Z^2}\frac{A(k'^2)}{k'^2-A^2(k'^2)} \quad (4)$$

The mass of the electron is defined by the solution of

$$m_e + A(+m_e^2) = 0 \quad (5)$$

Equation (4) is exactly the same as the one we obtain from the D-S equation using the rainbow-graph approximation as was done by Baker, Johnson and Willey [12] in the case of massless Q.E.D. The point here is that I did not have to rely on the Gell-Mann-Low condition [14] to justify the approximation. We studied the equation (4) in detail and found the following: (1) The equation certainly gives a symmetry breaking solution with $m_e \neq 0$. (2) Although the equation (4) contains the scale parameter $m_Z$ unlike in the case of massless Q.E.D., we still have a set of solutions with one free parameter and the value of $m_e$ in terms of $m_Z$ cannot be determined. This situation was already pointed out by Maskawa and Nakajima [15] some time ago. Whether this is due to the approximation we took or of a more general nature we do not know.

A natural continuation of the discussion given above is to look for a finite unified theory as an extension of finite Q.E.D. [12] We expect that some of the masses are of Higgs' origin and some are of dynamical origin.



We are of course far from achieving this goal. I would like to thank J. Arafune, S. Pakvasa for discussions, S.F. Tuan for the hospitality at the University of Hawaii, T. Truong for his introducing me to the papers on axions in the Physical Review Letters and for his hospitality at the Ecole Polytechnique where part of this work was done and H. Joos and T. Walsh for their hospitality at DESY.

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