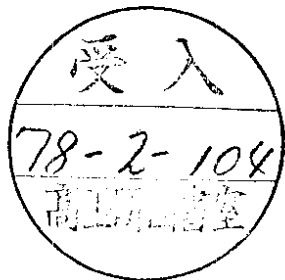


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A Polarization Prediction from Two Gluon Exchange  
for  $1^{--}(Q\bar{Q}) \rightarrow \gamma + 2^{++}(q\bar{q})$

by

Margarete Kramer

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A POLARIZATION PREDICTION FROM TWO GLUON EXCHANGE

$$\text{FOR } 1^{--}(Q\bar{Q}) \rightarrow \gamma + 2^{++}(q\bar{q}) \text{ } ^{+)}$$

by

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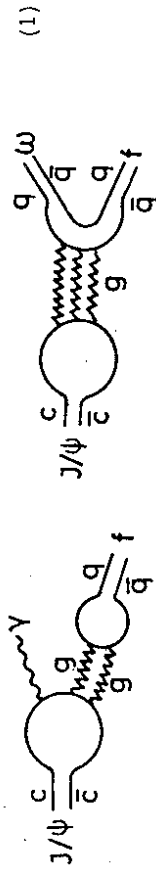
Abstract

An exclusive Zweig suppressed radiative decay of heavy  $1^{--}$  vector mesons is considered in the approximation  ${}^3S_1(Q\bar{Q}) \rightarrow \gamma + [g + g \rightarrow] {}^3P_2(q\bar{q})$  with on-shell gluons. The  $2^{++}$  meson will appear to be in the helicity zero state if the mass ratio  $M_{2^{++}}/M_{1^{--}}$  is small. A comparison with the decays  $J/\psi(3.1)$ ,  $\psi'(3.7)$ ,  $\Upsilon(9.4), \dots \rightarrow \gamma + f(1.3)$ ,  $f'(1.5)$ , .... is suggested.

+ ) I would like to dedicate this paper to Dr. Kurt Mellentin, who set up the High Energy Physics Index and the Scientific Retrieval System at DESY, and who headed the Scientific Documentation here until his sudden death, in December, 1977.

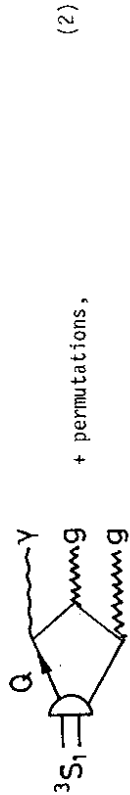


The experimental observations of the decay  $J/\psi \rightarrow \gamma + f$  <sup>1,2</sup> with a rate similar to the decay  $J/\psi \rightarrow \omega + f$  <sup>3,4</sup> nicely support the gluon counting rules which are abstracted from the asymptotic freedom limit of QCD <sup>5</sup> and contradict superficial VDM estimates. The strength of Zweig suppressed decays of heavy  $Q\bar{Q}$  bound states increases with decreasing minimal number of gluons. Zweig suppressed decays involving an "isoscalar" photon may thus become "large" compared to the vector dominance estimate:  $\Gamma(J/\psi \rightarrow \gamma f)/\Gamma(J/\psi \rightarrow \omega f) = \pi\alpha/\gamma_\omega^2 = 10^{-3}$ , as might be seen from the comparison of the following graphs:



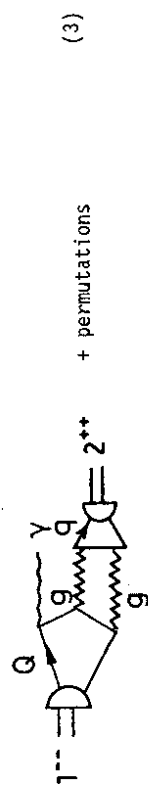
(a) (b)

The necessary additional gluon suppresses  $\omega$ -dominance of the photon. For this reason an exploration of the typical QCD graph (a) seems worthwhile. Conceptually QCD is a theory like QED, however such diagrams are calculable only in principle. For instance,  $1^{--}(Q\bar{Q})$  annihilation into one photon and two gluons has been studied with appropriate approximation of the  $Q\bar{Q}$  bound state



in particular its application to the decay width  $J/\psi \rightarrow \gamma +$  all old hadrons <sup>6</sup> or the decay distribution  $\gamma \rightarrow \gamma +$  two gluon jets. <sup>7</sup>

In this paper I consider the diagram



which is related to the decays  $J/\psi \rightarrow \gamma + f$ ,  $\psi' \rightarrow \gamma + f$ , etc. The  $1^{--} \rightarrow \gamma + 2^{++}$  transition depends, for a real photon, on three amplitudes which are independent by kinematics but which get specified by dynamics.

In this spirit I shall obtain a polarization prediction from lowest order QCD which I suggest to apply to these decays, to be tested in


$$e^+e^- \rightarrow J/\psi, \psi', (\Upsilon), \dots \rightarrow \gamma + f, f', (\Upsilon), \dots \rightarrow \gamma + \pi\pi, K\bar{K}. \quad (4)$$

These are the dynamical assumptions and the procedure of my calculation:

i) A nonrelativistic  $^3S_1(Q\bar{Q})$  state annihilates into one photon and two gluons. This leads to the familiar QCD analogue in QED <sup>8</sup>, the decay amplitudes  $\mathcal{M}_1$  take the form

$$\begin{aligned} \mathcal{M}_1^*(k_1, k_2, k_3; \epsilon_1, \epsilon_2, \epsilon_3; K, \epsilon) \propto & [(k_1 + k_2, k_3)(k_2 + k_3, k_1)(k_3 + k_1, k_2)]^{-1} \cdot \\ & \cdot \left\{ (\epsilon_1 \epsilon_2) [-(k_1 k_3)(\epsilon_3 k_2)(\epsilon^* k_1) - (k_1 k_3)(k_2 k_3)(\epsilon^* \epsilon_3) - (k_2 k_3)(\epsilon_3 k_1)(\epsilon^* k_2)] + \right. \\ & \left. + (\epsilon^* \epsilon_3) [(k_3 k_1)(\epsilon_1 k_2)(\epsilon_2 k_3) - (k_1 k_2)(\epsilon_1 k_2)(\epsilon_2 k_3) + (k_3 k_2)(\epsilon_2 k_1)(\epsilon_1 k_3)] + \right. \\ & \left. + 1 \leftrightarrow 3 + 2 \leftrightarrow 3 \right\} \quad (5) \end{aligned}$$

where the  $k_i$  denote the photon or gluon four-momenta,  $\epsilon_i^{(\lambda_i)}$  their polarization four-vectors and where  $K = k_1 + k_2 + k_3$ ,  $K^2 = M_1^2$ , and  $\epsilon^{(\lambda)}$  refer to the vector meson,  $(\epsilon K) = 0$ .


If I consider Eq. (5) for real quanta ( $k_i^2 = 0$ ) and take the limit  $k_2 \rightarrow k_1$ :  $(k_1 + k_2)^2 = M_X^2 \rightarrow 0$ ,  $(\epsilon_1 k_2) \rightarrow 0$ ,  $(\epsilon_2 k_1) \rightarrow 0$ , then only the first term  $(\epsilon_1 \epsilon_2) \cdot [---]$  survives. The physical rule is that the pair of the quanta has  $J_Z = 0$ :  denotes

helicity). Note, that for this pair  $J^{PC} = 0^{+-}$  is forbidden!

ii) The gluons create a quark pair in the nonrelativistic  $^3P_2$  state. -  $^3P_2(Q\bar{Q})$  annihilation into gluons 9,10) has, again, its analogue in the Positronium decay into  $\gamma\gamma$  11). - The  $g + g \rightarrow 2^{++}$  vertex  $\mathcal{M}_2$  becomes

$$\mathcal{M}_2(k_1, k_2; \epsilon_1, \epsilon_2; P, \epsilon) \propto (k_1 k_2)^{-2} \cdot E_{\mu\nu}^* \{ 4(k_1 k_2)_\mu \epsilon_1^\nu \epsilon_2^\nu - (\epsilon_1 \epsilon_2)(k_1 - k_2)^\mu (k_1 - k_2)^\nu + 2(k_1 \epsilon_2) \epsilon_1^\mu (k_1 - k_2)^\nu + 2(k_2 \epsilon_1) \epsilon_2^\mu (k_2 - k_1)^\nu \} \quad (6)$$

where  $P = k_1 + k_2$ ,  $P^2 = M_2^2$ , and where  $E^{(\lambda)}$  denotes the symmetric, traceless spin two polarization tensor,  $E_{\nu\mu}^{HP} = 0$ .

From (6) follows the selection rule that two on-shell gluons couple to the spin two meson in the  $J_Z = \pm 2$  states only 10): 

The non-relativistic approximation for the  $2^{++}$  mesons made of light quarks, i.e. the  $f$  and  $f'$ , might be here a disputable dynamical assumption.

iii) The intermediate gluons are kept on their mass shell. With this prescription ( $k_1^2 = k_2^2 = 0$ ) and with  $M_2^2$  fixed the loop integral in (3) reduces actually to an integration over the angle between the three-momenta  $\mathbf{k}_1 - \mathbf{k}_2$  and  $\mathbf{k}_3 = \mathbf{K}$  in the  $2^{++}$  rest frame.

For the kinematically independent quantities one can choose the helicity amplitudes

$$1^{--}(1, 0, -1) \rightarrow \gamma(1) + 2^{++}(0, 1, 2) \equiv (A^0, A^1, A^2) \quad (7)$$

The result of my calculation along the steps i) - iii) is that the ratios of the helicity amplitudes depend on the mass ratio  $M_2/M_1$  and are given by

$$x \equiv A_1/A_0 = \frac{2}{\sqrt{3}} \left( \frac{z-1}{z+1} \right)^{1/2} \cdot \frac{-2 - 6z^2 + 3z^{-1} \cdot (1-z)^4 \cdot (\ln(z-1) - \ln(z+1))}{-10 + 6z^2 + 3z^{-1} \cdot (1-2z^2+z^4) \cdot (\ln(z-1) - \ln(z+1))}$$

$$y \equiv A_2/A_0 = \frac{1}{\sqrt{6}} \frac{z-1}{z+1} \cdot \frac{38 + 6z^2 + 3z^{-1} \cdot (1+6z^2+z^4) \cdot (\ln(z-1) - \ln(z+1))}{-10 + 6z^2 + 3z^{-1} \cdot (1-2z^2+z^4) \cdot (\ln(z-1) - \ln(z+1))} \quad (8)$$

$$z = (M_1^2 + M_2^2)/(M_1^2 - M_2^2)$$

shown graphically in Figure 1.

For the physical discussion of the result Eq. (8), I consider first the Limiting cases:

a) If  $M_2/M_1 \rightarrow 1$  then  $(x,y) \rightarrow (\sqrt{3}, \sqrt{6})$ : it corresponds to an electric dipole (E1) radiation pattern. This limit would be physically realized if there were two different heavy quarks with nearly equal masses.

b) If  $M_2/M_1 \rightarrow 0$ , then  $(x,y) \rightarrow 0$ : the spin two meson is in the helicity zero state. This limit should be satisfied rather well in the decays

$$\Upsilon(9.4) \rightarrow \gamma + f, f'$$

The application of Eq. (8) to the observed 1,2) decays gives the following values to the ratios of the helicity amplitudes:

	x	y
$J/\psi \rightarrow \gamma f$	0.76	0.54
$J/\psi \rightarrow \gamma f'$	0.88	0.70

The amplitudes, x, y, can be experimentally determined from the angular distribution of the reaction

$$e^+e^- \rightarrow J/\psi, \psi', (\Upsilon), \dots \rightarrow \gamma + f, f', (\pi\pi), \dots \rightarrow \gamma + \pi\pi, K\bar{K}. \quad (4)$$

It has the form <sup>+) 12)</sup>

$$W(\theta_\gamma; \theta_p, \phi_p) = (1 + \cos^2 \theta_\gamma) \cdot \left[ (3 \cos^2 \theta_p - 1)^2 + 3/2 y^2 \sin^4 \theta_p \right] + 3x^2 \cdot \sin^2 \theta_\gamma \cdot \sin^2 2\theta_p + \\ + \sqrt{3} \cdot x \cdot \sin 2\theta_\gamma \cdot \sin 2\theta_p \cdot (3 \cos^2 \theta_p - 1 - \sqrt{3}/\sqrt{2} \cdot y \cdot \sin^2 \theta_p) \cdot \cos \phi_p \\ + \sqrt{6} \cdot y \cdot \sin^2 \theta_\gamma \cdot \sin^2 \theta_p \cdot (3 \cos^2 \theta_p - 1) \cdot \cos 2\phi_p \quad (10)$$

<sup>+) The decay distribution of a  $2^{++}$  meson into a pair of pseudoscalar mesons with momenta  $p_1, p_2$  follows uniquely from  $\mathcal{M}_{\mu\nu} \propto E_{\mu\nu}(\lambda) (p_1^- p_2^-)^\mu (p_1^- p_2^-)^\nu$ .</sup>

where  $\theta_\gamma$  is the angle between the photon and the  $e^+e^-$  beam axis,  $\theta_p, \phi_p$  are the angles of the pseudoscalar mesons, in the  $2^{++}$  rest frame, relative to the photon with  $\phi_p = 0$  defined by  $e^+e^-$ .

The predicted angular dependence does not seem to be inconsistent with a preliminary analysis <sup>13)</sup> of experimental data on  $J/\psi \rightarrow \gamma f$ . A precise measurement would be an interesting check of applied QCD.

#### Acknowledgement

I am indebted to H. Joos and T. Walsh for valuable discussions and to H. Krasemann for a nice argument and calculations. I thank G. Alexander and W. Koch for discussions of their experiments which initiated this investigation.

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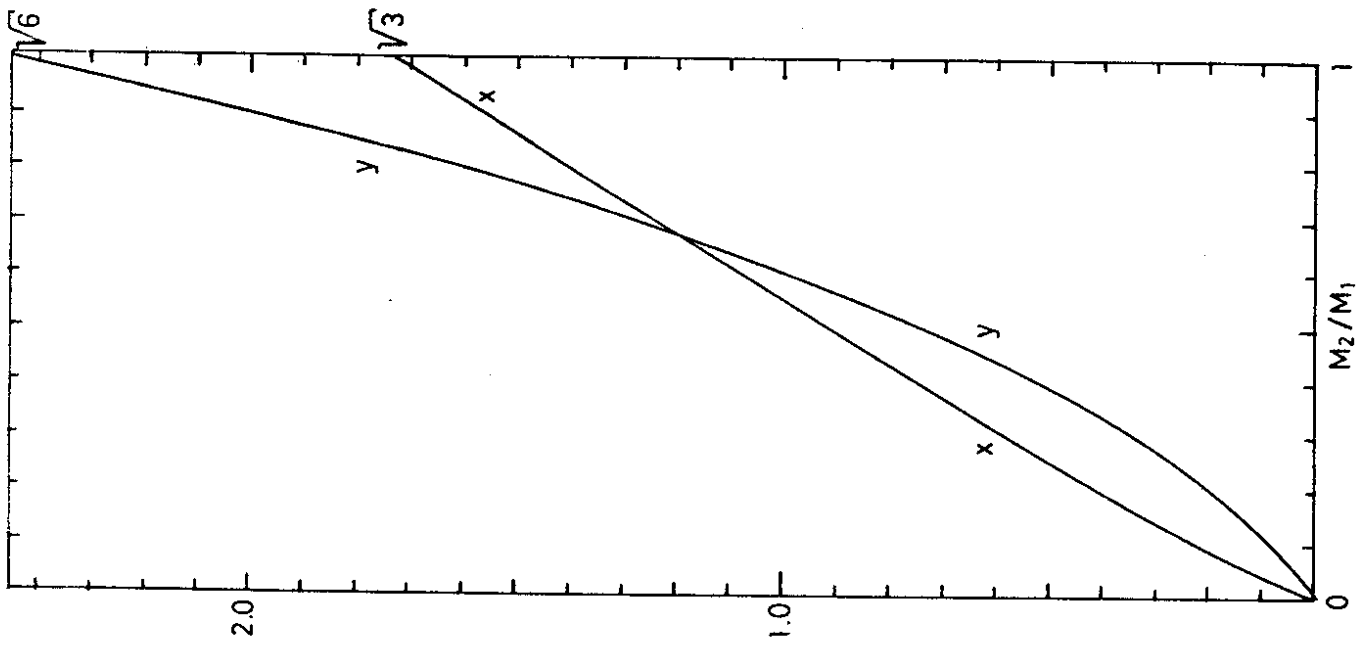


Fig. 1: Ratio of helicity amplitudes,  
 $x = A_1/A_0$ ,  $y = A_2/A_0$ , for  ${}^3S_1(Q\bar{Q}) \rightarrow \gamma + {}^3P_2(q\bar{q})$ ,  
 as function of the mass ratio  $M_2/M_1$ .