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SU(4) Weak Currents

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SU(4) WEAK CURRENTS

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Abstract

We suggest $SU_L(4) \otimes U(1)$ as the gauge symmetry of weak and electromagnetic interactions for quartets of quarks and leptons. We analyze how the (additional) $SU_L(4)$ weak currents (besides the $SU_L(2)$ subgroup) could affect the weak interactions of ordinary particles, the atomic parity violation, the neutral-current neutrino reactions and the decays of the τ heavy lepton and the charmed mesons. The suppression of neutral-current parity violation in atomic experiments can be naturally incorporated in this model while at the same time the success of the Weinberg-Salam model with respect to the inclusive neutral current data is kept. The model has limited freedom and therefore many definite prediction.

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I. Introduction

The recent discovery of the "charmed" mesons[1] and baryons[2], the inclusive electron spectrum in e^+e^--annihilation[3] all support the picture of at least four quarks[4]. The standard $SU(2) \otimes U(1)$ model of Weinberg and Salam[5] (W-S) inbeds the four quarks in a very elegant manner (the GIM scheme[4]). As a gauge model, it predicts a definite structure of the weak neutral currents. Compared with the measured cross sections of the neutrino induced neutral current reactions, the Weinberg-Salam model fits the data reasonably well. This leads us to believe that it represents at least a partial truth.

Nevertheless, there are some experimental evidences which suggest modification of the basic Weinberg-Salam model. The first is the "heavy lepton" τ observed at SLAC[6] and DESY[7]. In the framework of the $SU(2) \otimes U(1)$ model, it has been pointed out that τ can not be a singlet[8]. This means, in this framework, the existence of a new doublet of quarks (or leptons) in order that the model is anomaly free. A new quark of charge $-1/3$, the b-quark, has been introduced in the literature, which couples to the u-quark with right-handed currents[9]. Either a "y anomaly" or a rising anti-neutrino to neutrino charged-current cross-section ratio or both would be evidence for the production of the right-handed b-quark in neutrino reactions. Experimentally, however, there is no evidence for a right-handed b-quark. Recent dimuon data from CDHS[10] and CF collaboration[11] are consistent with only the charm production[12]. The CDHS data shows no evidence for the "y anomaly", nor does the ratio of anti-neutrino to neutrino charged-current cross sections rise with energy. In short , the right-handed b-quark, if it exists, has not been found experimentally. Another alternative is to introduce a left-handed b-quark which has to couple with another new quark of charge 2/3, the t-quark, to form a doublet[13]. The recent observed enhancement in μ pair mass around 9.5 GeV in hadronic collision[14] has been interpreted as composed of several narrow resonances and could thus be possible evidence for the (left-handed) b-quark. Independent of the outcome of that experiment, the $SU_L(2) \otimes U(1)$ models face the following problem with respect to the neutral-current induced parity violation in atomic experiments.

The recent atomic Bi experiments[15] indicate that the parity violation in atomic Bi is a factor of $5 \sim 10$ smaller than expected from the Weinberg--Salam model. One source of the discrepancy could be that the theoretical atomic physics calculation have overestimated the effect of parity-violation in atoms. However, a factor beyond 2 is considered unlikely by some experts. If we take the atomic neutral current results seriously, this means that the neutral currents of the Weinberg-Salam model need to be modified. Barring the mixing between the quarks in doublets with the quarks in singlets (by the "naturalness" arguments[16]), only two alternatives are open to us: either introducing right-handed currents or enlarging the left-handed gauge symmetry (note that the left-handed t- or b-quark could not change the atomic neutral current up to some small mixing angles).

One possibility is to assume that both the left-handed and right-handed electrons belong to doublets[17]. Because of the new right-handed component, the electron neutral currents are pure vector and no parity violation is expected in atomic Bi experiments. We have seen above that there is no experimental evidence for right-handed charged quark currents. We note also

that the Weinberg-Salam model is in good agreement with the recent CDHS data on the inclusive neutral-current cross sections in neutrino reactions, which we interpret to mean no additional right-handed quark neutral currents[18]. In view of these experimental facts, the right-handed electron current seems unmotivated. Note that with respect to flavor, we would expect for the leptons and quarks to have the same weak interactions. In any case, this model could be unambiguously tested in elastic $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering. Preliminary data on comparing the events ratio below and above $y = 0.5$ disfavor the vector electron neutral-current model[19], but no definite conclusion could be drawn at this time.

From these remarks, we conclude that the $SU_L(2) \otimes U(1)$ model could not explain the atomic neutral current Bi experiments, and since there is no experimental evidence for right-handed currents, this means that we need to enlarge the left-handed gauge symmetry beyond $SU_L(2)$.

The purpose of this paper is to present unified weak and electromagnetic interactions in terms of a $SU_L(4) \otimes U(1)$ model. Above, we have given some phenomenological reasons for considering such a model. We like to add a theoretical remark. Consider a world of n-quarks which possess a strong interaction symmetry SU(n). The quarks eventually acquire different masses which break the SU(n) symmetry. At the moment, we are not concerned with the symmetry breaking. The most natural gauge symmetry to consider for n-quarks is naturally SU(n) - for if we are to consider a smaller gauge symmetry group, the quarks could belong to different irreducible representations: for example, under the SU(2) symmetry, we do not, a priori, know why the quarks should belong to doublets rather than triplets and singlets. On the other hand, experimentally we have only evidence for SU(2) charged currents from the weak decays of ordinary mesons and hyperons - conventional wisdom suggests $SU_L(2) \otimes U(1)$ as the gauge symmetry for the ordinary weak interactions.

How do we reconcile this experimental fact with the above philosophy?

If the weak interactions could indeed be explained with a $SU_L(2) \otimes U(1)$ symmetry, in view of its simplicity, the other consideration may be only a secondary problem. But since the atomic neutral current results could not be accomodated in the $SU_L(2) \otimes U(1)$ models, we abandon them. We believe that the cause of this problem is because we have not taken into consideration the full gauge symmetry group. Note that even though the ordinary particles decay via the charged $SU_L(2)$ currents, we in general could not draw any implication on the weak interactions of the (new) heavy leptons or the neutral currents. In fact, both of these areas are only recently being studied experimentally. Thus even if the underlying symmetry of the weak interactions is larger than the $SU_L(2)$, the difference from the $SU_L(2)$ symmetry prediction could be small in some cases, and we would have no way of knowing it until it is searched for in the future. In this paper, we take the following attitude: the fact that we only know about SU(2) weak currents could very well be the artifact of low energy phenomena.

Armed with these remarks, we predict in the $SU_L(4) \otimes U(1)$ model proposed below that the decays of heavy leptons will show evidence for weak currents outside the conventional $SU_L(2) \otimes U(1)$ model. In particular, the τ heavy lepton could decay via strangeness-changing neutral currents with a much suppressed rate, which could nevertheless be searched for experimentally. Because of the additional neutral currents in the $SU_L(4) \otimes U(1)$ model, we could incorporate the absence of large parity violation in atomic experiments in a natural manner.

For some models employing the $SU_L(4) \otimes U(1)$ symmetry, the additional SU(4) weak currents have to be extremely suppressed.[21] This has to do with the lepton representation of their choice. We consider these models unsatisfactory for several reasons as we shall see below.

Our model is different from the others in the lepton representation which is determined by requiring the model to be anomaly free with respect to the full $SU_L(4) \otimes U(1)$ group. The τ lepton and ν_τ are naturally incorporated in this lepton representation; they belong to the electron family (ie. e^-, ν_e, ν_τ, τ^-) but they act almost like sequential leptons. In fact, the cancellation of anomaly requires eight "new" leptons: τ^-, ν_τ; M^-, M^0 (the corresponding members of the muon family); and a third family of 4 leptons. This implies rich phynomenology for high energy e^+e^- annihilation experiments and neutrino reactions. This subject is worth studying in its own right.

Since the leptons are introduced to cancel the anomaly of the quarks, there is in principle no need for introducing more heavy quarks. If experimental data[14] requires additional heavy quarks, we should enlarge the flavor group. For the purpose of this paper, we proceed with the assumption of only 4 quarks.

From the quark and lepton representations, (u, d, s, c) and $(e^-, \nu_e, \nu_\tau, \tau^-)$ we shall first see that there is no experimental reason to require that all of the SU(4) - SU(2) gauge bosons to be very heavy. Strangeness changing neutral currents do not appear in the sector of ordinary particles. Indeed some of them can be as light as 160 GeV. This number is deduced from the small parity violation in atomic Bi experiments.

The predictions for the decays of charmed mesons and the heavy leptons are obtained after relating the masses of the various w bosons by certain symmetry.

A brief summary of our results has been presented in a short note[20]. We fill in the details here. Particular attention is paid to the Higgs sector to demonstrate how the different routes of spontaneous symmetry breaking can be realized. In section II, we discuss how the lepton representations are determined, we give our estimate for the heavy lepton masses and the cabibbo angle. In section III, we present the Higgs mechanisms. The interaction Lagrangian is given in section IV. Its implication for the ordinary weak interactions, and the neutral current neutrino cross sections are discussed in sections V and VI, respectively. Our predictions for the charmed meson decays and the heavy lepton τ decays are presented in section VII. A summary is given in section VIII. Details of the Higgs potential are given in the appendix.

The phenomena of the heavy leptons in high energy neutrino reactions and e^+e^- annihilation experiments will be discussed separately elsewhere.

II. The Fermion Representations

The Weinberg-Salam Model assigns the left-handed leptons and quarks in doublets, i. e. (ν_e, e^-), (ν_μ, μ^-), (u,d) and (c,s) under the $SU_L(2)$ symmetry. With respect to $SU_L(4)$ symmetry, these two quark doublets form a quartet, namely (u,d,s,c) (neglecting the Cabibbo angle for the moment) which transforms like the 4 representation. If we assign the lepton doublets in a SU(4) quartet as the quarks do, i. e. $(\nu_e, e^-, \mu^-, \nu_\mu)$ (which was first proposed by Pati and Salam[21], many of the W bosons which mediate either the lepton-number non-conserving transitions or strangeness-changing neutral currents (e. g. $\bar{\psi}_L \gamma_\mu \lambda_{6+i7} \psi_L$) will have to be extremely heavy, since such phenomena are untolerable in the weak interactions of ordinary particles. We consider this lepton representation unsatisfactory for several reasons. First, the quark lepton analogy (they behave the same under SU(4)) would imply similar mass breaking for the quarks and the leptons, which is obviously not true in this model. (for example, ν_μ is almost massless whereas c-quark is much heavier than the ordinary quarks). Secondly, this assignment leaves no room for a "heavy lepton" τ whose properties have been extensively studied at SLAC and DESY. The third reason is that this assignment is not anomaly free with respect to the full SU(4) gauge symmetry unless one introduces in addition a set of 16 mirror fermions to cancel the triangle anomalies which seems to us arbitrary. Furthermore, if we propose that some of the SU(4) - SU(2) W bosons could be as light as 160 GeV (in order to cancel the parity violation in atomic experiments), then this lepton representation is totally unacceptable because of the induced strangeness-changing neutral current and lepton-number nonconservation processes. We mention these points in order to show in contrast that the model proposed in this paper does not have such difficulties.

We propose to assign the (e^-,ν_e) and (μ^-,ν_μ) in two different SU(4) multiplets. The τ lepton is an excited electron state. The electron family consists of $(e^-,\nu_e,\nu_\tau,\tau^-)$ (where ν_τ is the τ neutrino) which transforms as a $\underline{4}$ representation. (It is also acceptable to identity τ and ν_τ as axcited muon and ν_μ with the obvious consequence that the muon number should be conserved in τ decays instead of the electron number). In reality, because of the fact that the τ mass is very close to the charmed meson masses, the τ decays just like a sequntial heavy lepton (see section VII). There are still small differences which could be used to trace down the lepton number of the τ lepton. We give definite predictions later in this paper (see section VII).

We find two reasons for assigning τ and ν_τ in the electron (or muon) family: one has to do with the handedness of the τ interaction, the other has to do with the fact $m_\tau \sim m_c$. These questions will be examined in this section. The question of strangeness-changing neutral currents will be examined in section V. The main observation is that the strangeness-changing neutral currents $\bar{\Psi}_L \gamma_\mu \lambda_{6\pm i7} \Psi_L$ always couple to one of the "new" leptons and are thus not seen in the weak decays of ordinary particles (if ν_τ, for example, is heavy enough). The masses of these W bosons which couple to one of the "new" leptons are hence not constrained by the ordinary weak interactions. We also find a very restrictive way of generating the Cabibbo angle in this model as we shall see below.

To determine the lepton representation, we find that the criteria for cancelling the triangle anomaly is very useful, to which we turn our attention first.

A. Cancellation of Anomaly and the Lepton Representations

Since the quarks (u,d,s,c) belong to the 4 representations under SU(4), in order to cancel anomaly, the leptons must belong to the $\underline{4}$ representations, and the sum of the quark and lepton charges must equal zero. This results in a proliferation of the lepton families. The leptons can have charges either in the sequence as (-1, 0, 0, -1) or (0, +1, +1, 0), barring doubly charged leptons. The electron and muon family can only take the 1st charge assignment. We must also have a third family of lepton with the 2nd charge assignment in order for the sum of quark and lepton charges to be zero. Thus we have the following quark and lepton representation.

$$\begin{pmatrix} u^r & u^g & u^b \\ d^r & d^g & d^b \\ s^r & s^g & s^b \\ c^r & c^g & c^b \end{pmatrix} \qquad \begin{pmatrix} e^- & \mu^- & \ell^0 \\ \nu_e & \nu_\mu & \ell^+ \\ \nu_\tau & M^0 & L^+ \\ \tau^- & M^- & L^0 \end{pmatrix}$$

where r, g, b denote the colors of the quarks.

The quarks have supposedly unbroken quark color symmetry. The leptons also seem to have a lepton color symmetry, but the lepton color is necessarily broken because the leptons have integer charges. We note that once the electron and muon belong to two different representations, their charges have to be negative to that of the proton. (This follows from anomaly cancellation.) We also note that a "lepton-quark symmetry" (with respect to the flavor) is maintained naturally in this scheme. The excited electrons or muons have V-A interactions as the electron and muon do, whereas the negative charged leptons in the third lepton family (namely ℓ^-, L^-) decay via V+A interactions. The data from τ decays favor V-A over V+A. Thus we assign τ to be an excited electron.

B. The Fermion Masses

The gauge theory provides a rational for the quark and lepton masses. The quark and lepton masses originate from the non-zero vaccum expectation values of the Higgs scalars. Since they transform the same way under SU(4), a unified picture of the quark and lepton masses could emerge from the Higgs mechanism. We assume, for simplicity, that the same set of Higgs scalars couple to the quarks and leptons. We then expect roughly the same mass pattern for the lepton and the bare quark masses (up to the radiative corrections to the lepton masses). For example, from $m_u^{bare} \sim m_d^{bare} < m_s^{bare} \ll m_c^{bare}$, we expect $m_e \sim m_{\nu_e} < m_{\nu_\tau} \ll m_\tau$. This seems reasonable enough. In fact, the relation between the quarks and the electron family may be an approximate equality: $m_e \sim m_{\nu_e} \sim m_u^{bare} \sim m_d^{bare} \sim 0$ and $m_\tau \sim m_c^{bare}$ are consistent with strong PCAC and the observed τ mass and the charmed meson masses. We thus conclude that $m_{\nu_\tau} \sim m_s^{bare}$.

Turning the above argument around, from the mass relation $\frac{m_{\nu_\tau}}{m_\tau} \sim \frac{m_s^{bare}}{m_c^{bare}}$ if we could estimate m_{ν_τ} and m_s^{bare}, we would have some handle on the mass ratio m_τ/m_c^{bare}. Theoretically, the m_s^{bare} is believed (estimated) to be a couple of hundred MeV[23]. Phenomenologically, m_{ν_τ} has a limited range in this model, namely, 350 MeV $\lesssim m_{\nu_\tau} <$ 500 - 700 MeV. The upper limit comes from the measured lepton spectrum of the τ decay. The lower limit comes from the absence of $K^+ \to \pi^+ \nu_\tau \bar{\nu}_e$, etc. (See section V. A stronger prediction is $m_{\nu_\tau} \sim m_{K^\pm}$). (This phenomenological range of m_{ν_τ} is consistent with the above theoretical estimate). If we now put these numbers together, we are not surprised that the τ mass and the charmed meson masses are almost degenerate.

Applying the above arguments to the muon family, we arrive at the prediction $m_\mu, m_{\nu_\mu} < m_{M^0} \ll m_{M^-}$. If the reproted tri-muon events are indeed due to the production of M^- which subsequently decays into M^0, with M^0 decaying in chain, the mass ratio of M^- and M^0 estimated from the data are consistent with the above relation. The formulas should, strictly speaking, be applied to the bare masses of the leptons. For light mass leptons, electron or muon, the radiative corrections to the bare masses can be as big as the bare masses themselves. But for heavy mass leptons, we presumably could expect the radiative correction to be negligible comapaed with the (bare) masses. We then expect the relation $\frac{m_{\nu_\tau}}{m_\tau} \sim \frac{m_{M^0}}{m_{M^-}}$ to give a reasonable estimate for the ratio of the M^0, M^- masses.

C. The Cabibbo Angle and the Lepton Representation

The main reason for introducing charm is to cancel the strangeness-changing neutral currents within the SU(2) ⊗ U(1) sector[4]. When we extend the gauge symmetry beyond SU(2) ⊗ U(1), we need to study this question regarding the "new" currents. We find that it restricts severely the structure of the weak currents.

With respect to the SU(2) sub-symmetry, the quarks are in coublets $\binom{u}{d}$ and $\binom{c}{s}$, the Cabibbo angle could appear either as a mixing between the d and s states (the GIM scheme) or between the u and c states (namely,

$$u(\theta) = u \cos\theta_c - c \sin\theta_c \,, \quad c(\theta) = u \sin\theta_c + c \cos\theta_c \,)$$

They are equivalent to each other - both give the same SU(2) ⊗ U(1) weak currents. However, in a fully gauged SU(4) model, the two shemes have quite different phenomenological implications. The origin of the Cabibbo angle also appears quite different. We'll discuss these two subjects in turn.

The first thing we note is that if the d and s quarks are mixed, then a neutral W boson coupling to a diagonal neutral current, e. g.

$$\bar{\psi}_L \gamma_\mu \left(\sqrt{\tfrac{2}{3}}\lambda_8 + \tfrac{1}{\sqrt{3}}\lambda_{15}\right)\psi_L \sim \bar{\psi}_L \gamma_\mu \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \psi_L$$

would generate strangeness-changing neutral currents and induce $K \to \pi e e$ decays. Thus assuming GIM mixing, this neutral W boson mass has to be very heavy. This difficulty is absent, if the Cabibbo mixing appears in the u and c quark sector and d, s remain unmixed. Since in this paper we have postulated that the neutral W boson coupling to the above diagonal neutral currents is not so heavy, $m_W \sim 160$ GeV, (the absence of parity violation in atomic experiments is attributed to this weak current). We conclude that d, s mixing is unacceptable. We postulate that the Cabibbo currents are generated by c, u mixing. Note that in the SU(3) limit we can always define the physical d and s state, such that they are unmixed (by performing a rotation). We assume that SU(3) symmetry breaking does not induce mixing between the d and s quarks.

The interpretations for the Cabibbo angle in this sheme is quite different from the GIM scheme (d, s mixing). For the GIM scheme, the Cabibbo angle does not exist in the exact SU(3) limit. The Cabibbo angle is thus somehow related to the (chiral) SU(3) symmetry breaking. In the present shceme, the Cabibbo angle could not be rotated away even in the exact SU(3) limit, since $m_u \neq m_c$. The magnitude of the Cabibbo angle, in this cheme, could be traced back to the differnet orders of the spontaneous symmetry breaking. Since the strong interaction symmetry SU(4) is badly broken to SU(3), and the breaking appears only in the quark mass term, this means that the charm quark acquires a large mass. (This can be accomplished with a Higgs scalar transforming as a 4 representation, which couples to the left-handed quarks, the right-handed quarks being SU(4) singlets can always be rotated into the diagonalized state).

We assume that a second stage of symmetry breaking, on a much smaller scale, is responsible for the masses of the u quark and the mass mixing of the u and c quarks (Δm_{uc}). One finds by diagonalizing the u, and c quark mass matrix, ($m_u, \Delta m_{uc} \ll m_c$) that

$$\sin\theta_c \sim \frac{\Delta m_{uc}}{m_c - m_u} \sim \frac{m_u}{m_c - m_u}$$

where m_u, m_c refer to the "physical" quark masses for the Cabibbo angle. Taking $m_u \sim \frac{1}{3} m_N \sim 0.31\ GeV$, $m_c \sim \frac{1}{2} m_{J/\psi} \sim 1.55\ GeV$ as commonly assumed, we arrive at $\sin\theta_c \sim 0.24$. At least in order of magnitude, this provides a reason why the Cabibbo angle is small. If the second stage of symmetry breaking is due to the radiative correction, one might hope to be able to calculate the Cabibbo angle. This point has not been demonstrated in models, but the possibility seems appealing from the theoretical point of view.

III. The Breaking of the SU(4) Gauge Symmetry

The weak interaction phenomenology is crucially determined by the masses (eigenvalues) and the eigenstates of the W bosons. In order to know that within the context of renormalizable gauge theory, one must work out how the gauge symmetry is broken. Fortunately, even though the gauge symmetry is large, most of the Higgs mechanism could be understood by studying how the SU(4) gauge symmetry is broken (it should not be confused with the strong interaction symmetry breaking present in the quark mass terms which is also induced by Higgs scalars). We shall first study the hierarchy of SU(4) symmetry breaking. It remains a nontrivial matter to show later that how the different routes of symmetry breaking can be realized via the Higgs scalars.

A. The SU(2) Subgroup

The SU(2) symmetry here refers to the symmetry of the conventional charged currents a la Weinberg-Salam model. In terms of the quartet representation $(u(\theta_c), d, s, c(\theta_c))$, where $u(\theta_c) = u\cos\theta_c - c\sin\theta_c$, $c(\theta_c) = u\sin\theta_c + c\cos\theta_c$, the SU(2) consists of the following generators:

$$\lambda_{W^+} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & K & 0 \end{pmatrix} = \frac{1}{2}[\lambda_{1+i2} + K\lambda_{13-i14}] = [\lambda_{W^-}]^+$$

(3.1)

$$\lambda_3 = \frac{1}{2}[\lambda_{W^+}, \lambda_{W^-}] = \frac{1}{2}[\lambda_3 + \frac{\lambda_8}{\sqrt{3}} - \sqrt{\tfrac{2}{3}}\,\lambda_{15}] = \begin{pmatrix} \frac{1}{2} & & & \\ & -\frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & \frac{1}{2} \end{pmatrix}$$

where $K = \pm 1$, λ's are the SU(4) λ-matrices. The sign of K is not determined experimentally.

B. The Hierarchy of the SU(4) Symmetry Breaking

First, we note that the SU(2) of (3.1) is formed by the direct sum of the generators of two SU(2) groups operating in the (u,d) and (s,c) spaces. The SU(4) group contains another SU(2) subgroup in addition to (3.1) which containes the following generators

$$\lambda_{(K\bar{D})^0} \equiv \begin{pmatrix} 0 & 0 & 0 & -h \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}[\lambda_{6+i7} - h\lambda_{9+i10}] = [\lambda_{(\bar{K}D)^0}]^+$$

(3.2)

$$\lambda_Y = \frac{1}{2}[\lambda_{(K\bar{D})^0}, \lambda_{(\bar{K}D)^0}] = \frac{1}{\sqrt{3}}\lambda_8 + \frac{1}{\sqrt{6}}\lambda_{15} = \begin{pmatrix} \frac{1}{2} & & & \\ & \frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & -\frac{1}{2} \end{pmatrix}$$

where $h = \pm 1$. The SU(4) has only one simple subgroup which contains the SU(2) of (3.1) as a subgroup. This group is O(5) (or SP(4)) which consists of the generators given in (3.1) and (3.2) $(h = K = \pm 1)$ and in addition the following generators

$$\lambda_{K^+} \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\lambda_{4+i5} = [\lambda_{K^-}]^+$$

(3.3)

$$\lambda_{D^+} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \frac{1}{2}\lambda_{11-i12} = [\lambda_{D^-}]^+$$

The $SU(2) \otimes SU(2)$ of (3.1) and (3.2) are (with $K = h = \pm 1$) also subgroups of $O(5)$. But the two subroups with $K = -h = \pm 1$ are subgroups of $SU(4)$ but not $O(5)$.

The rest of the $SU(4)$ generators are given by

$$\lambda_{H^+} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -K & 0 \end{pmatrix} = \frac{1}{2}\left[\lambda_{1+i2} - K\lambda_{13-i14}\right] = \left[\lambda_{H^-}\right]^+ \tag{3.4}$$

$$\lambda_{H^0} \equiv \begin{pmatrix} 0 & 0 & 0 & h \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{2}\left[\lambda_{6+i7} + h\,\lambda_{9+i10}\right] = \left[\lambda_{\bar{H}^0}\right]^+$$

and

$$\lambda_X \equiv \begin{pmatrix} \frac{1}{2} & & & \\ & -\frac{1}{2} & & \\ & & \frac{1}{2} & \\ & & & -\frac{1}{2} \end{pmatrix} = \frac{1}{2}\left[\lambda_3 - \frac{1}{\sqrt{3}}\lambda_8 + \sqrt{\frac{2}{3}}\,\lambda_{15}\right] \tag{3.5}$$

From the above remarks, we conclude with the following three typical routes of breaking the $SU(4)$ symmetry:

(1) $SU(4) \to O(5) \to SU(2)$. The "super" symmetry-breaking provides us with $SU(4) - O(5) = 5$ W bosons of super heavy masses and $O(5) - SU(2) = 7$ W bosons of intermediate masses which are heavier than the $SU(2)$ W bosons. The breaking of $SU(2)$ with the additional $U(1)$ gives the Weinberg-Salam model.

(2) $SU(4) \to SU(2) \otimes SU(2) \to SU(2)$. The "super" symmetry-breaking provides us with $SU(4) - SU(2) \otimes SU(2) = 9$ W bosons of super heavy masses and $SU(2) \otimes SU(2) - SU(2) = 3$ W bosons of intermediate masses. The breaking of $SU(2)$ with the additional $U(1)$ leads to the Weinberg-Salam model.

(3) $SU(4) \to U(1) \otimes SU(2) \to SU(2)$. In this case, we have 11 super-heavy W bosons and one medium-heavy W boson (which is neutral and conserves flavor, i. e. Y) plus the W bosons of the Weinberg-Salam model.

In all the above three cases, we have at least one medium heavy W boson coupling to diagonal currents, which cancels the parity violation in atomic experiments. The trivial case of $SU(4) \to SU(2)$ does not interest us und need not be mentioned here.

We note that the W bosons coupling to (3.4) will distort the Cabibbo structure of weak interaction as given by (3.1). Thus to preserve the weak interaction universality, we estimate that their masses are $>$ 500 GeV, the detail depends on the uncertainty of the Cabibbo angle.

C. The Higgs Scalars

We introduce first two Higgs scalar multiplets, M and N, which transform under $SU(4)$ as anti-symmetric second rank tensors and another Higgs scalar multiplet, L, which transforms as symmetric second rank tensor under $SU(4)$. Denoting the $SU(4)$ transformation as R, then they transform under $SU(4)$ as

$$\phi_{ij} \to R_{il} R_{jm} \phi_{lm} \qquad \text{where} \qquad \phi = M, N, L$$

Let g and W^i_μ (i =1,....15) be the gauge coupling and the gauge bosons, the gauge invariant Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{2}\sum_{\phi=M,N,L} \mathrm{Tr}\,\Big|\,\partial_\mu\phi + \frac{ig}{2}(\lambda_i\cdot W_\mu^i)\phi + \frac{ig}{2}\phi(\lambda_i^T W_\mu^i)\Big|^2 + \cdots \tag{3.6}$$

We find that :

(1) The first route of SU(4) symmetry breaking can be done with two anti-symmetric 2nd rank tensors[22]. SU(4) is broken to O(5) with following non-zero vaccum expectation value

$$\langle M\rangle = A\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -h \\ -1 & 0 & 0 & 0 \\ 0 & h & 0 & 0 \end{pmatrix}, \quad h = \pm 1 \tag{3.7}$$

O(5) is broken to SU(2) with $\langle N\rangle$ given by

$$\langle N\rangle = a\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.8}$$

(2) SU(4) is broken to SU(2) ⊗ SU(2) (with $h = -K = \pm 1$) via the Higgs scalar L

$$\langle L\rangle = B\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -h \\ 1 & 0 & 0 & 0 \\ 0 & -h & 0 & 0 \end{pmatrix}, \quad h = \pm 1 \tag{3.9}$$

A Remark: In general, we would expect that the two matrix elements in (3.7) or (3.9) need not be equal (i. e. $|h| \neq 1$). However, by minimizing the most general potential of M and N, for example,(which is even under $M \to -M$, or $N \to -N$) we find that the solutions require that $h^2 = 1$. If $h^2 \neq 1$, we would find that the mass relation of the Weinberg-Salam model, namely, $m_Z = m_W/\cos\theta_W$ is spoiled. Because of the minimization condition, the above mass relation turns out to hold also in this $SU_L(4)$ ⊗ U(1) model. What this means is that the group theoretical analysis of the SU(4) symmetry breaking can be realized. The above mass relation has indirect experimental support from the inclusive

neutral-current neutrino-reactions[10].

To break the SU(2) ⊗ U(1) symmetry a la Weinberg-Salam model, we need 4 Higgs scalars each transforming as 4 representation with the following vacuum expectation values

$$\langle\phi_1\rangle = \begin{pmatrix} \lambda \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ \lambda \\ 0 \\ 0 \end{pmatrix}, \quad \langle\phi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ \lambda \\ 0 \end{pmatrix}, \quad \langle\phi_4\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \lambda \end{pmatrix} \tag{3.10}$$

where ϕ_1, ϕ_4 have charge in the order (0, -1, -1, 0) and ϕ_2, ϕ_3 have charge (1, 0, 0, 1). The gauge invariant Lagrangian is given by

$$\mathcal{L}_\phi = -\frac{1}{2}\sum_j \Big|\,\partial_\mu\phi_j + \frac{ig}{2}(\lambda_i\cdot W_\mu^i)\phi_j \pm \frac{ig'}{2}B_\mu\phi_j\Big|^2 + \cdots \quad \begin{matrix} + \text{ for } j = 2,3 \\ - \text{ for } j = 1,4 \end{matrix} \tag{3.11}$$

where g' is the coupling constants for U(1), and B the corresponding boson .

From (3.6) - (3.11), we obtain the following grand formulas for the eigen-states and masses of the W bosons. (For definiteness, we take $K = 1$.)

$$\begin{aligned}
W_{H^\pm} &\equiv \tfrac{1}{2}\left[W_{1\pm i2} - W_{13\mp i14}\right], & m^2_{W_{H^\pm}} &= g^2\left(A^2 + B^2 + \frac{\lambda^2}{4}\right) \\
W^\pm &\equiv \tfrac{1}{2}\left[W_{1\pm i2} + W_{13\mp i14}\right], & m^2_{W^\pm} &= \frac{g^2}{4}\lambda^2 \\
&\tfrac{1}{2}\left[W_{6\pm i7} + W_{9\pm i10}\right], & m^2 &= g^2\left(A^2 + \frac{a^2+\lambda^2}{4}\right) \\
&\tfrac{1}{2}\left[W_{6\pm i7} - W_{9\pm i10}\right], & m^2 &= g^2\left(B^2 + \frac{a^2+\lambda^2}{4}\right)
\end{aligned} \tag{3.12a}$$

(In conformity with the previous notation, we define

$$W_{(K\bar{D})^\circ,(\bar{K}D)^\circ} \equiv \frac{1}{2}[W_{6\pm i7} - h\, W_{9\pm i10}]$$

$$W_{H^\circ,(\bar{H}^\circ)} \equiv \frac{1}{2}[W_{6\pm i7} + h\, W_{9\pm i10}]\ , \quad \text{with } h = \pm 1\ ).$$

$$W_{K^\pm} \equiv \frac{1}{\sqrt{2}} W_{4\pm i5}\ , \qquad m^2_{W_{K^\pm}} = g^2\left(B^2 + \frac{a^2+\lambda^2}{4}\right)$$

$$W_{D^\pm} \equiv \frac{1}{\sqrt{2}} W_{11\mp i12}\ , \qquad m^2_{W_{D^\pm}} = g^2\left(B^2 + \frac{a^2+\lambda^2}{4}\right)$$

$$X \equiv \frac{1}{\sqrt{2}}\left(W_3 - \frac{1}{\sqrt{3}} W_8 + \sqrt{\frac{2}{3}}\, W_{15}\right), \quad m_X^2 = g^2\left(A^2 + B^2 + \frac{\lambda^2}{4}\right) \tag{3.12b}$$

$$Y \equiv \sqrt{\frac{2}{3}}\, W_8 + \frac{1}{\sqrt{3}} W_{15}\ , \qquad m_Y^2 = g^2\left(\frac{a^2}{2} + \frac{\lambda^2}{4}\right)$$

$$Z = \cos\theta_W A_3 - \sin\theta_W B\ , \qquad m_Z^2 = m_W^2/\cos^2\theta_W$$

$$A = \sin\theta_W A_3 + \cos\theta_W B\ , \qquad m_A^2 = 0$$

where $\tan^2\theta_W = g'^2/\left(\frac{g^2}{2}\right)$, $A_3 \equiv \frac{1}{\sqrt{2}}\left(W_3 + \frac{1}{\sqrt{3}} W_8 - \sqrt{\frac{2}{3}}\, W_{15}\right)$.

From (3.12), we immediately see that:

(1) SU(4) $\rightarrow$ O(5) (h=K=1) $\longrightarrow$ SU(2) is realized with $A \gg (a,B) > \lambda$.

(2) SU(4) $\rightarrow$ SU(2) $\otimes$ SU(2) (h=-K=-1) $\rightarrow$ SU(2) is realized with $B \gg (A,a) > \lambda$.

(3) SU(4) $\longrightarrow$ U(1) $\otimes$ SU(2) $\rightarrow$ SU(2) is realized with $(A,B) \gg a > \lambda$.

The potential of the Higgs scalars and the stability of the vacuum are discussed in the appendix.

IV. Coupling of the Quark and Lepton Currents with the W Bosons

The interaction of the gauge W boson with the quarks and leptons is prescribed by the following gauge invariant Lagrangian.

$$\begin{aligned}\mathcal{L} = {} & \bar{\psi}^q_L \gamma_\mu \left(\partial^\mu + \frac{ig}{2}\lambda_i W_i^\mu + \frac{ig'}{2} y^q B^\mu\right)\psi^q_L + \bar{\psi}^q_R \gamma_\mu\left(\partial^\mu + ig' Q B^\mu\right)\psi^q_R \\ & + \bar{\psi}^\ell_L \gamma_\mu \left(\partial^\mu + \frac{ig'}{2}(-\lambda_i^T) W_i^\mu - \frac{ig'}{2} y^\ell B^\mu\right)\psi^\ell_L + \bar{\psi}^\ell_R \gamma_\mu \left(\partial^\mu + ig' Q B^\mu\right)\psi^\ell_R\end{aligned} \tag{4.1}$$

where $\psi_{L,R} = \frac{1\mp\gamma_5}{2}\psi$. The y^q, y^ℓ are constants (matrices) determined from the charges of the quarks and leptons, namely $y = Q - \frac{1}{2}(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8 - \sqrt{\frac{2}{3}}\lambda_{15})_L$. Thus $y = \frac{1}{3}$ for the quarks and $y = 1$ for the electron and muon families. Q is the charge matrix. Note that ψ^ℓ(the lepton representations) transforms as $\underline{4}$ under SU(4). In terms of the eigenstates of the W bosons given in Eq.(3.8–3.10), one finds the following couplings for the off-diagonal weak currents,

$$\begin{aligned}\mathcal{L}^{\text{off-diagonal}}_{int} = {} & \frac{g}{2} W^-_\mu \left(\bar{u}_\theta \gamma^\mu_L d + \bar{c}_\theta \gamma^\mu_L s - \bar{\nu}_e \gamma^\mu_L e^- - \bar{\nu}_\tau \gamma^\mu_L \tau^- - \bar{\nu}_\mu \gamma^\mu_L \mu^-\right. \\ & \left. - \bar{M}^\circ \gamma^\mu_L M^-\right) + \frac{g}{2} W_{(K\bar{D})^\circ,\mu}\left(-h\,\bar{u}_\theta \gamma^\mu_L c_\theta + \bar{d}\gamma^\mu_L s - \bar{\nu}_\tau \gamma^\mu_L \nu_e + h\,\bar{\tau}^- \gamma^\mu_L e^-\right. \\ & \left. - \bar{M}^\circ \gamma^\mu_L \nu_\mu + h\,\bar{M}^- \gamma^\mu_L \mu^-\right) + \frac{g}{\sqrt{2}} W_{K^-,\mu}\left(\bar{u}_\theta \gamma^\mu_L s - \bar{\nu}_\tau \gamma^\mu_L e^- - \bar{M}^\circ \gamma^\mu_L \mu^-\right) \\ & + \frac{g}{\sqrt{2}} W_{D^-,\mu}\left(\bar{c}_\theta \gamma^\mu_L d - \bar{\nu}_e \gamma^\mu_L \tau^- - \bar{\nu}_\mu \gamma^\mu_L M^-\right) + \text{hermitian conjugates} \\ & + \cdots\cdots\end{aligned} \tag{4.2}$$

where $\gamma^\mu_L \equiv \gamma^\mu \frac{1-\gamma_5}{2}$, u_θ and c_θ stand for $u\cos\theta_c - c\sin\theta_c$, and $u\sin\theta_c + c\cos\theta_c$ respectively with θ_c the Cabibbo angle. The interactions of super-heavy W bosons of Eq.(3.8) are omitted from (4.2). The coupling of the diagonal neutral currents can be expressed as

$$\begin{aligned}\mathcal{L}^{\text{Diagonal}}_{int} = {} & eA^\mu\left[j^{em,q}_\mu + j^{em,\ell}_\mu\right] + \frac{g}{\sqrt{2}} Y^\mu\left[j^{Y,q}_\mu - j^{Y,\ell}_\mu\right] \\ & + \sqrt{\frac{g^2}{2} + g'^2}\, Z^\mu\left\{j^{A_3,q}_\mu - \sin^2\theta_W\, j^{em,q}_\mu - j^{A_3,\ell}_\mu - \sin^2\theta_W\, j^{em,\ell}_\mu\right\} \\ & + \cdots\cdots\end{aligned} \tag{4.3}$$

$$j_\mu^{em,q} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$$

$$j_\mu^{em,\ell} = -\bar{e}\gamma_\mu e - \bar{\tau}\gamma_\mu \tau - \bar{\mu}\gamma_\mu \mu - \bar{M}^-\gamma_\mu M^- + \cdots$$

$$j_\mu^{A_3,q} = \bar{\psi}^q \gamma_L^\mu \begin{pmatrix} \frac{1}{2} & & & \\ & -\frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & \frac{1}{2} \end{pmatrix} \psi^q$$

$$= \frac{1}{4}\left(\bar{e}\gamma_\mu(1-\gamma_5)e + \bar{\tau}\gamma_\mu(1-\gamma_5)\tau - \bar{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e - \bar{\nu}_\tau\gamma_\mu(1-\gamma_5)\nu_\tau\right.$$

$$\left. + \text{(the muon family)}\right) + \cdots$$

$$j_\mu^{Y,q} = \bar{\psi}^q \gamma_L^\mu \begin{pmatrix} \frac{1}{2} & & & \\ & \frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & -\frac{1}{2} \end{pmatrix} \psi^q \tag{4.4}$$

$$= \frac{1}{4}\left[\bar{u}\gamma_\mu(1-\gamma_5)u + \bar{d}\gamma_\mu(1-\gamma_5)d - \bar{s}\gamma_\mu(1-\gamma_5)s - \bar{c}\gamma_\mu(1-\gamma_5)c\right]$$

$$j_\mu^{Y,\ell} = \bar{\psi}^\ell \gamma_L^\mu \begin{pmatrix} \frac{1}{2} & & & \\ & \frac{1}{2} & & \\ & & -\frac{1}{2} & \\ & & & -\frac{1}{2} \end{pmatrix} \psi^\ell$$

$$= \frac{1}{4}\left(\bar{e}\gamma_\mu(1-\gamma_5)e + \bar{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e - \bar{\nu}_\tau\gamma_\mu(1-\gamma_5)\nu_\tau - \bar{\tau}\gamma_\mu(1-\gamma_5)\tau\right]$$

$$+ \text{(the muon family)} + \cdots$$

We note that the quark and lepton currents coupling to $W^\pm$, Z are identical with the Weinberg-Salam model. The weak angle in this case is defined by $\tan^2\theta_W = g'^2/(\frac{g^2}{2})$ (the corresponding coupling constants are $\frac{g}{\sqrt{2}}$ and g').

V. SU(4) Currents and the Weak Interactions of the Ordinary Particles

A. Strangeness-Changing Neutral Currents

The reasons that the strangeness-changing neutral currents are absent in the sector of ordinary particles become transparent from the interaction Lagrangian obtained above. (1) As pointed out previously, the strangeness-changing neutral currents (coupling to $W^0_{(KD)}$) couple to at least one "new" lepton in semi-leptonic processes (see 4.2). Thus we'll not see any strangeness-changing neutral current in the weak interactions of ordinary particles if the "new" leptons are heavy enough. An immediate consequence of this assumption is the prediction of strangeness-changing neutral currents in the decays of the heavy leptons (see section VII). Note from (4.2) that the strangeness-changing neutral current (off-diagonal) are not coupled to the diagonal neutral currents (which couple to ee, $\mu\mu$, $\nu\nu$, for example) thus $K^+ \not\to \pi^+ ee$, etc. (2) The diagonal neutral currents do not change strangeness (from 4.3). This has been discussed above in connection with the Cabibbo structure. (3) The non-leptonic interactions in the order G_W has no $|\Delta S| = 2$ transition. This follows from the fact that the strangeness-changing neutral currents are not coupled to the diagonal neutral currents. For the question of strangeness-changing neutral current processes and K_1-K_2 mass-difference in higher order weak interactions, we must examine the box diagram with two charged W exchange etc. The GIM cancellation between the u quark and c quark intermediates states work here as it should. Not too surprisingly, the same kind of cancellation works for the additional SU(4) (charged) currents.

As remarked above, the "new" leptons must be heavy enough in order that strangeness-changing neutral currents do not appear in the decays of ordinary particles, (otherweise the $W^0_{(KD)}$ boson coupling to this current will have to be extremely heavy). This means a lower limit on m_{ν_τ}, namely, $m_{\nu_\tau} \gtrsim (m_{K^+} - m_{\pi^+})$, from the absence of $K^+ \to \pi^+ \nu_\tau \bar{\nu}_e$ etc. From the absence of $K^+ \to e^+ \nu_\tau$, we conclude that either $m_{\nu_\tau} \sim m_{K^+} \sim$ 500 MeV or the weak SU(4) symmetry must be badly broken to SU(2) $\otimes$ SU(2) such that $W_{K^\pm}$ (couling to $e\nu_\tau$) is extremely heavy. These alternatives have been discussed in section III. Experimentally, $m_{\nu_\tau} \sim$ 500 MeV is not ruled out at present. Allowing medium-heavy W bosons, we note that the mass of ν_τ is very much restricted in this model which could be easily checked by the experiments.

B. Modified Non-Leptonic Interactions

From (4.2) we see that the $|\Delta S|=1$ non-leptonic interactions of the ordinary particles have a new piece due to $W^0_{(KD)}$ exchange. After Fierz transformaion, one has

$$\mathcal{L}_{eff}^{|\Delta S|=1} = \frac{G_W}{\sqrt{2}}\left(1+\frac{m_W^2}{m_{W'}^2}\right)\left\{\bar{s}\gamma_\mu(1-\gamma_5)u\cdot\bar{u}\gamma^\mu(1-\gamma_5)d - \bar{s}\gamma_\mu(1-\gamma_5)c\cdot\bar{c}\gamma^\mu(1-\gamma_5)d + h.c.\right\}$$

where $m_{W'} \equiv m_{W^0_{(KD)}}$.

VI. Low Energy Neutral Current Reactions

From the interaction Lagrangian obtained in the previous section, we can study the experimental consequences of the medium-heavy W bosons. We first study here the neutral-current parity violation in atomic physics and examine how the interaction of the Y boson is able to cancel the parity violation effect in atomic Bi experiments. Next we examine the neutral current neutrino processes including the Y interaction. We shall see that the success of the Weinberg-Salam model with respect to the inclusive neutral current cross section in neutrino reactions is preserved in this model, partly because the Y boson is heavier than the Z boson, There exist however different predictions between this model and the Weinberg-Salam model which could be checked experimentally in the future.

A. Neutral Currents in Atomic Physics

In atomic Bi experiments, for example, one measures the parity violation induced by the weak neutral currents. Since the time-components of the vector hadronic currents (the weak charges) are enhanced by the atomic numbers, the dominant contribution comes from the vector quark currents and the axial-vector electron currents, which are given by (4.3) and one finds

$$\frac{G_W}{\sqrt{2}}\left\{\left[\bar{u}\gamma_\mu u-\bar{d}\gamma_\mu d-4X_W\left(\frac{2}{3}\bar{u}\gamma_\mu u-\frac{1}{3}\bar{d}\gamma_\mu d\right)\right](\bar{e}\gamma^\mu\gamma_5 e) + \left(\frac{m_W^2}{m_Y^2}\right)\left[\bar{u}\gamma_\mu u+\bar{d}\gamma_\mu d\right](\bar{e}\gamma^\mu\gamma_5 e)\right\} \tag{6.1}$$

where $X_W \equiv \sin^2\theta_W$ The first term is the Weinberg-Salam model prediction and the second term the new contribution from the Y exchange. The effective weak charge for Bi^{209}_{83} experiments is then

$$Q^W = Z(1-4X_W) - N + \left(\frac{m_W^2}{m_Y^2}\right)3(Z+N) \tag{6.2}$$

$$\sim -129 + \left(\frac{m_W^2}{m_Y^2}\right)627 \qquad \text{for } X_W \sim 0.26$$

where we used $X_W \sim 0.26$ from fitting the inclusive neutral current data (see below). If Q^W is almost zero as suggested by the experimental data, we then find $m_Y \sim 2\, m_W \sim 160\,GeV$ ($m_W \sim 80\,GeV$ for $X_W \sim 0.26$). But m_Y could of course be heavier, depending on the final experimental results and the theoretical calculation of the atomic physics.

From the SU(4) symmetry breaking, we noted that Y is heavier than the Z boson. Phenomenologically, it has to be heavy enough in order not to spoil the success of the Weinberg-Salam model with respect to the inclusive neutral current data (see below). On the other hand, we note above that the Y current, being an isoscalar, is very much enhanced by the atomic numbers. This is the reason why it is quite important in atomic neutral currents. The above two factors compensate each other to make the parity violation in atomic Bi experiments small.

B. Neutral Currents in Inclusive Neutrino Reactions

From (4.3) we find the following effective Lagrangian

$$\frac{G_W}{\sqrt{2}}\Big\{-\big(\bar{u}\gamma_\mu(1-\gamma_5)u-\bar{d}\gamma_\mu(1-\gamma_5)d-4X_W\big(\tfrac{2}{3}\bar{u}\gamma_\mu u-\tfrac{1}{3}\bar{d}\gamma_\mu d\big)\big] + \Big(\frac{m_W^2}{m_Y^2}\Big)\big[\bar{u}\gamma_\mu(1-\gamma_5)u+\bar{d}\gamma_\mu(1-\gamma_5)d\big]\Big\}\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\nu_\mu \qquad (6.3)$$

One immediately finds from (6.3) the ratios of neutral-current to charged-current cross sections on an isoscalar target (neglecting the sea contribution) as follows:

$$R^\nu = \frac{\sigma^\nu_{NC}}{\sigma^\nu_{CC}} = \frac{1}{4}\Big\{\big(1-\tfrac{4}{3}X_W-\tfrac{1}{\rho}\big)^2+\big(-1+\tfrac{2}{3}X_W-\tfrac{1}{\rho}\big)^2 + \tfrac{1}{3}\big[\big(\tfrac{4}{3}X_W\big)^2+\big(\tfrac{2}{3}X_W\big)^2\big]\Big\}$$

$$= R^\nu(W\text{-}S) - \frac{1}{2\rho}\Big(\frac{1}{\rho}-\frac{2}{3}X_W\Big)$$

$$R^{\bar\nu} = \frac{\sigma^{\bar\nu}_{NC}}{\sigma^{\bar\nu}_{CC}} = \frac{1}{4}\Big\{\big(1-\tfrac{4}{3}X_W-\tfrac{1}{\rho}\big)^2+\big(-1+\tfrac{2}{3}X_W-\tfrac{1}{\rho}\big)^2 + 3\big[\big(\tfrac{4}{3}X_W\big)^2+\big(\tfrac{2}{3}X_W\big)^2\big]\Big\} \qquad (6.4)$$

$$= R^{\bar\nu}(W\text{-}S) - \frac{1}{2\rho}\Big[\frac{1}{\rho}-\frac{2}{3}X_W\Big]$$

where $\rho^{-1} = \frac{m_W^2}{m_Y^2}$. We note that the new currents only modify the left-handed components. For $\rho^{-1} \simeq \frac{1}{4}$ as determined above, the correction terms are small compared with the W-S model predictions. This is partly because the Y boson is heavy, and partly because the Y current is isoscalar. For $X_W \sim 0.26$ the correction term is at the level of 2 %, thus both the W-S model and this model, are in good agreement with the data (see Fig. 1). The difference is well within the experimental error bars and the uncertainty of the sea contribution.

C. Elastic $\nu_\mu e$ and $\bar\nu_\mu e$ Cross Sections

From (4.3) one finds the following effective Lagrangian for $\nu_\mu e$ and $\bar\nu_\mu e$ elastic scattering

$$\frac{G_W}{\sqrt{2}}\Big[-\bar{e}\gamma_\mu(1-4X_W-\gamma_5)e+\frac{m_W^2}{m_Y^2}\,\bar{e}\gamma_\mu(1-\gamma_5)e\Big]\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\nu_\mu \qquad (6.5)$$

We note that the modification of the Weinberg-Salam model appears only in the left-handed components; the effect suppresses the left-handed coupling. We plot in Fig. 2 the following quantities $\sigma(\nu_\mu e\to\nu_\mu e)/\sigma(\nu_\mu e\to\mu\nu_e)$ and $\sigma(\bar\nu_\mu e\to\bar\nu_\mu e)/\sigma(\nu_\mu e\to\mu\nu_e)$ for various values of X_W to contrast our predictions with the Weinberg-Salam model. For $X_W\sim 0.26$ the predicted cross sections are quite different. For $X_W\sim 0.26$ we predict

$\sigma(\nu_\mu e)\sim 0.63\times10^{-42}E_\nu\,cm^2GeV^{-1}$ and $\sigma(\bar\nu_\mu e)\sim 1.25\times10^{-42}E_\nu\,cm^2GeV^{-1}$,

as compared with the Weinberg-Salam model which predicts

$\sigma(\nu_\mu e)\sim 1.4\times10^{-42}E_\nu\,cm^2GeV^{-1}$ and $\sigma(\bar\nu_\mu e)\sim 1.5\times10^{-42}E_\nu\,cm^2GeV^{-1}$

correspondingly.

D. Elastic $\nu_\mu P$ and $\bar{\nu}_\mu P$ Cross Sections

From (6.4) the new contribution to $\sigma(\nu_\mu P)$ and $\sigma(\bar{\nu}_\mu P)$ is an isoscalar and left-handed current, where the form factors are least known. We make the following assumption as in ref. (25) in order to compare our model with the data. We assume that the isoscalar and isovector axial matrix elements to be the same as the ratio of the isoscalar and isovector magnetic contribution. To compare with the data, we average over the neutrino spectrum and kinematic acceptance of the experiment. For simplicity, we assume dipole form factors with $m_V^2 = m_A^2 = 0.7\,GeV^2$. The results are plotted in Fig. 3 and 4 together with the Weinberg-Salam model. For $x_W \sim 0.26$ the difference between the two models is not that big. Given the uncertainty in the form factors and the difficulty inherent in measuring the processes, the agreement with data is satisfactory.

VII. Implications for the Decays of the τ Heavy Lepton and the Charmed Mesons

In the last section, we saw how the neutral current reactions were modified by the "new" current coupling to the gauge boson Y. As discussed in section III, we could expect two bosons $W_{(K\bar{D})^0}$ and $W_{(\bar{K}D)^0}$ to have comparable masses as the Y boson, if the associated SU(2) symmetry is not badly broken (we'll also expect in addition the $W_{K^\pm}$, $W_{D^\pm}$ to have comparable masses, if O(5) is not badly broken). If this is the case, the decays of the τ and charmed mesons will be modified by the weak currents coupling to $W_{(K\bar{D})^0}$, $W_{(\bar{K}D)^0}$.

Notice first that with respect to the SU(2) weak currents (i. e. the Weinberg-Salam version), the decay properties of the τ lepton will be identical to a sequential heavy lepton. But because that τ is an excited electron, it would couple to the electron via $W^0_{(KD)}$. Although the strength of this coupling is suppressed ($m_W^2/m_{W'}^2 \sim 1/4$) the electron number of the τ lepton could only be discovered by searching for this coupling. Note that in this model $\tau^- \not\to e^-(e^+e^-)$ and $\tau^- \not\to e^-(\mu^+\mu^-)$. This is because that the non-diagonal neutral currents (coupling τ to e) are not coupled to diagonal neutral currents (coupling to $ee, \mu\mu$ etc.).

From (4.2) we find an additional Feynman diagram (see Fig. 5) via $W^0_{(KD)}$ exchange which is available to $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ but not to $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$. This will make the branching ratios of the two modes unequal. Using Fierz transformation, we find ($m_{W'} \equiv m_{W^0_{(KD)}}$)

$$\frac{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}{\Gamma(\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu)} = \left[1 - h\frac{m_W^2}{m_{W'}^2}\right]^2, \quad h = \pm 1$$

Assuming that $m_{W'} \sim m_Y \sim 2m_W$ as determined previously, we then find B.R. $(\tau^- \to e^- \nu_\tau \bar{\nu}_e)$ /B.R. $(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu) \sim 0.6$ or 1.5. The present experimental data for this ratio from the PLUTO collaboration(7) is ~ 0.9 assuming V-A, $m_{\nu_\tau} = 0$, or ~ 0.67 assuming V-A, $m_{\nu_\tau} \sim 300$ MeV(26). Since ν_τ is massive in this model, we find that the above prediction is consistent with the data. More statistics in the future could be a crucial test. The different branching ratios for the about two leptonic modes means that the cross section ratio $\sigma(ee)/\sigma(e\mu)$ is different from 0.5. Our prediction is 0.3 or 0.75 to be compared with the SLAC data $\sigma(ee)/\sigma(e\mu) \sim 0.57 \pm .3$(22).

For the semi-leptonic decays of the τ, the $W^0_{(KD)}$ exchange yields a decay mode

$$\frac{\Gamma(\tau^- \to e^- K_0)}{\Gamma(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu)} = \frac{12\pi^2 f_K^2}{m_\tau^2}\left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 \left(\frac{m_W^2}{m_{W'}^2}\right)^2 \frac{1}{f(z)}$$

where f_K is the K_0 coupling constant. $f(z) = (1-z^8)(1-8z^2+z^4) + 24z^4 \ln(1/z)$, with $z = m_{\nu_\tau}/m_\tau$. For $m_{W'} \sim 2m_W$, $m_{\nu_\tau} \sim (400-500)$ MeV, one finds

$$\frac{B.R.(\tau^- \to e K_S)}{B.R.(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu)} \sim 3-4\ \%$$

The decay branching ratio for eK_S is small, but still within experimental reach. To compensate for the small B.R., we suggest the following process

$$e^+e^- \longrightarrow \tau^+\tau^- \longrightarrow e^- K_S$$
$$\hookrightarrow \nu_\tau + 1 \text{ charged particle} + \ldots$$

The two-body kinematics of eK_S decay mode could be used to distinguish this process from that induced by the production and decays of charm mesons. If a resonance in eK_S is found, it would not only establish the existence of a heavy lepton, but also the lepton number of the heavy lepton. It will be a first experimental evidence for the strangeness-changing neutral currents

in the weak interaction (of τ decays). From (4.2) we note that τ could decay to charmed mesons via the "new" currents but these decay modes are severely suppressed by phase space.

In the quark sector we have charm-changing neutral currents coupling to $W^0_{(KD)}$. We estimate for the following decay modes,

$$\frac{\Gamma(D^+ \to \pi^+ \bar{\nu}_\tau \nu_e)}{\Gamma(D \to K e \nu)} \sim 7\ \%$$

and

$$\frac{\Gamma(D^0 \to \bar{\nu}_\tau \nu_e)}{\Gamma(D \to K e \nu)} \sim 2\ \%$$

for $m_{\nu_\tau} \sim 400 - 500$ MeV. We have assumed the same constant form factors for both processes and $f_D \sim f_\pi$. Although the B.R. for these two processes are very small, they could be searched for experimentally. Note that $W^0_{(KD)}$ exchange does not induce any D^0, $\bar{D}^0$ mixing. D^0, $\bar{D}^0$ mixing in this model is due to the Cabibbo mixing between the u and c quarks coupling to the gauge boson Y. But the rate is very small, i.e. $\left(G_W \sin\theta_c \frac{m_W^2}{m_Y^2}\right)^2$, and is well within the present experimental limit.

VIII. Summary

In this paper we have presented a $SU_L(4) \otimes U(1)$ model of unified weak and electromagnetic interactions. The essential difference between this model and the previous ones is in the lepton representation. We assumed anomaly cancellation between the quarks and the leptons to start with. Once we concluded that the electron, the muon and their associated neutrinos do not belong to the same SU(4) multiplet, we find that the lepton representation is uniquely fixed. The electron and the muon do not have the freedom of being positively charged in this model.

A necessary consequence of the above assumptions is the emergence of several heavy leptons. The τ lepton and its associated neutrino are both massive. They carry the electron quantum member; but their true identity will only show in a number of processes with much suppressed couplings. They behave almost like sequential leptons.

The almost degenerate masses of the τ lepton and the charmed mesons appear naturally as a consequence of the Higgs mechanism. The τ neutrino mass might also be degerenate with the kaon mass. Experimental measurement of the τ neutrino mass will not only serve as a first crucial test of this model, but also shed light on the route of the SU(4) gauge symmetry breaking.

The model also predicts two muon-type heavy leptons M^0 and M^-. The mass ratio between M^0 and M^- should be roughly equal to that of ν_τ and τ. From the coupling of the $SU_L(4)$ currents in eq. (4.2) we note that M^0 decay will always involve a strange particle in the final state. This striking prediction could be checked in high energy neutrino-reactions and e^+e^- annihilation.

We predict a third quartet of (heavy) leptons with a sequential lepton number. Analogous to three colors of quarks, the leptons also have three lepton-colors: electron, muon and sequential. The quark color binds the quarks strongly, the interaction of the lepton color is however not known (and thus very weak). This perhaps explains why the lepton color is broken; the integer charges of the leptons are manifestation of the broken lepton color. The difference between the quark color and the lepton color interactions is a mystery which we do not understand. But, in spite of that difference, the quarks and the leptons have identical weak interactions up to a small Cabibbo angle (they are equivalent under flavor). We call this a "lepton-quark symmetry"(27).

The Cabibbo angle in this model is generated by the u and c quark mixing. The mass ratio m_u/m_c provides the rational why the Cabibbo angle is small. This feature is different from the conventional approach which assumes the Cabibbo angle in the d and s quark mixing. We argue the Cabibbo angle from the Higgs mechanism.

We proceeded next to study the phenomenological implications of this model. First, we studied how the W bosons obtain masses via the Higgs mechanism. As we noted in section III, the answer is given by the structure of the SU(4) subgroups, in this sense the structure of the SU(4) weak currents is model independent. Nevertheless, we domonstrated explicitly the Higgs mechanism. Associated with the generators of the (definite) SU(4) subgroup we find the corresponding eigenstates of the W bosons; the relative magnitudes of the W boson masses do depend on the route of the SU(4) symmetry breaking. We notice that the W bosons which couple to strangeness-changing neutral currents are decoupled from the diagonal neutral currents. This is one crucial reason why strangeness-changing neutral currents do not appear in the weak interactions of ordinary particles. This is maintained naturally by the group structure as remarked above. If the Cabibbo angle were zero, we would find no strangeness-changing neutral currents even in order G_W^2 in the sector of ordinary particles. Our model is consistent with the low energy phenomena.

With respect to new aprticles (i. e. heavy leptons) we made several unconventional predictions. Our attitudes can be phrased as follows: the fact there is no experimental evidence for strangeness-changing neutral currents, for example, is perhaps, an artifact of the low energy phenomena (the ordinary particles). We predict that we will find evidence for strangeness-changing neutral currents in the heavy lepton decay, such as $\tau^- \to e^- K_S^0$ or $M^0 \to \nu_\mu K_S^0$. In contrast, the conventional models totally forbid any strangeness-changing neutral current to start with(16,28). The decay mode $\tau^- \to e^- K_S^0$ reveals the lepton number of the τ lepton.

The same remark applies to other $SU_L(4)$ weak currents. We predict that the heavy lepton decays will show evidence for $SU_L(4)$ weak currents which lie outside the conventional $SU_L(2)$ framework. These currents are suppressed compared with the $SU_L(2)$ weak currents. We argue that the atomic neutral-current parity-violation result is an evidence for more neutral currents than the Weinberg-Salam has predicted. We have seen that the inclusive neutral current neutrino cross sections are almost unaffected by the "new" neutral currents (coupling to the gauge boson Y) partly because their couplings are suppressed. But the "new" neutral currents make noticeable contribution to the elastic $\nu_\mu e$ and $\bar{\nu}_\mu e$ cross sections when the weak angle is small (say $\sin^2\theta_W \sim 0.26$). This should provide a definite test for this model. In the case of atomic neutral-curent parity-violation, this "new" piece of neutral current, being isoscalar, is enhanced by the atomic numbers about four times more than the $SU_L(2) \otimes U(1)$ neutral current (coupling to Z). This is why the parity violation is (almost) cancelled between the two neutral currents. A charm-changing neutral current in this model couples

to W bosons outside the $SU_L(2)$ sector and is thus suppressed. We have predictions for that also, such as $D^+ \to \pi^+ \bar{\nu}_\tau \nu_e$.

To conclude, we list several crucial tests of this model: (1) Measuring the elastic $\nu_\mu e$ and $\bar{\nu}_\mu e$ cross sections. We have definite predictions to be contrasted with the Weinberg-Salam model. (2) Searching for the $e^- K_S^0$ mode form τ^- decay. Although the branching ratio for this mode is very small, some evidence could be found with double or triple of the present statistics. (3) Measuring the mass of the τ neutrino via, for example, the two-body decay modes of τ. The present limits from measuring the lepton spectrum suffer from error bars and statistics. (4) Measuring the branching ratio of the leptonic modes. For this purpose one needs to know the QED background well in measuring the lepton pair (ee, $e\mu$, $\mu\mu$) cross sections or the underlying interaction (V-A, V+A) and the τ neutrino mass in measuring the inclusive lepton cross sections.

To test this model further, one has to find the additional heavy leptons. The M^0 and M^-, if they are not very heavy, could be found in high energy neutrino reactions or in e^+e^- annihilation. The special decay properties of M (e.g. $M^0 \to \mu^- K^+$, in contrast to the conventional $SU_L(2) \otimes U(1)$ model) should provide the signature for its detection. The analysis of these processes will be presented in a separate publication.

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G. Mikenberg

Appendix: The Higgs Potential

For simplicity, we consider first the most general quartic potential for two Higgs scalar multiplets M and N, both transforming as anti-symmetric second rank tensor under SU(4). We assume refection symmetry (i.e. invariance under $M \to -M$ or $N \to -N$). The potential can be written as

$$\begin{aligned} P(M,N) = {} & a_1 \,\mathrm{Tr}(\bar{M}M\bar{M}M) + a_2 \,\mathrm{Tr}(\bar{N}N\bar{N}N) + b_1 \left[\mathrm{Tr}(\bar{M}M)\right]^2 + b_2 \left[\mathrm{Tr}(\bar{N}N)\right]^2 \\ & + c\left[\mathrm{Tr}(\bar{M}M\bar{N}N) + \mathrm{Tr}(M\bar{M}N\bar{N})\right] + d\,\mathrm{Tr}(\bar{M}M)\,\mathrm{Tr}(\bar{N}N) + e\,\mathrm{Tr}(\bar{M}N)\,\mathrm{Tr}(\bar{N}M) \\ & + f_1 \,\mathrm{Tr}(\bar{M}M) + f_2 \,\mathrm{Tr}(\bar{N}N) \end{aligned} \tag{A.1}$$

Let us assume that M and N have the following nonvanishing vacuum expectation values

$$\langle M \rangle = \begin{pmatrix} 0 & 0 & x & 0 \\ 0 & 0 & 0 & y \\ -x & 0 & 0 & 0 \\ 0 & -y & 0 & 0 \end{pmatrix}, \qquad \langle N \rangle = \begin{pmatrix} 0 & z & 0 & 0 \\ -z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We find that the extreme point of the potential corresponds to x,y,z satisfying the following equations

$$2a_1 x^2 + 4b_1(x^2+y^2) + (c+2d)z^2 + f_1 = 0 \tag{A.2}$$

$$2a_1 y^2 + 4b_1(x^2+y^2) + (c+2d)z^2 + f_1 = 0 \tag{A.3}$$

$$2a_2 z^2 + 4b_2 z^2 + (c+2d)(x^2+y^2) + f_2 = 0 \tag{A.4}$$

From (A.2) and (A.3) one finds $x^2 = y^2$. The above equations can be simplified as

$$(2a_1 + 8b_1)x^2 + (c+2d)z^2 + f_1 = 0 \tag{A.5}$$

$$(2a_2 + 4b_2)z^2 + 2(c+2d)x^2 + f_2 = 0 \tag{A.6}$$

Thus the vacuum expectation values are determined. The implication of the constraint $x = \pm y$ has been discussed in section III.

We have to ask next whether the extreme point of the potential obtained above remains stable under small perturbations, for otherwise the vacuum expectation values do not correspond to a minimum of the potential and will shift under renormalization. Let $M = \langle M\rangle + \Delta M$ and $N = \langle N\rangle + \Delta N$. Since the first derivatives of the potential vanish at $\langle M\rangle$ and $\langle N\rangle$, we find after some algebra that

$$\begin{aligned} P(\langle M\rangle + \Delta M, \langle N\rangle + \Delta N) - P(\langle M\rangle, \langle N\rangle) &= \frac{c}{2}\left\{ \mathrm{Tr}\,|\alpha+\beta|^2 + \mathrm{Tr}\,|\alpha^T+\beta^T|^2 \right\} \\ &+ \frac{d}{2}\left[\mathrm{Tr}\,\alpha + \mathrm{Tr}\,\beta\right]^2 + \left(a_1-\frac{c}{2}\right)\mathrm{Tr}\,|\alpha|^2 + \left(a_2-\frac{c}{2}\right)\mathrm{Tr}\,|\beta|^2 \\ &+ \left(b_1-\frac{d}{2}\right)|\mathrm{Tr}\,\alpha|^2 + \left(b_2-\frac{d}{2}\right)|\mathrm{Tr}\,\beta|^2 + e\,|\mathrm{Tr}\,\gamma|^2 \end{aligned} \qquad \text{(A.))}$$

where α and β are hermitian matrices given by

$$\alpha \equiv \Delta\bar{M}\langle M\rangle + \langle\bar{M}\rangle\Delta M + \Delta\bar{M}\,\Delta M \qquad \text{(A.8)}$$

$$\beta \equiv \Delta\bar{N}\langle N\rangle + \langle\bar{N}\rangle\Delta N + \Delta\bar{N}\,\Delta N$$

and

$$\gamma \equiv \Delta\bar{M}\langle N\rangle + \langle\bar{M}\rangle\Delta N + \Delta\bar{M}\,\Delta N \qquad \text{A.9)}$$

The stability condition requires that the coefficients of each term in (A.7) should all be positive, namely

$$c,\ d,\ e \geq 0$$

$$a_1 \geq \frac{c}{2},\quad a_2 \geq \frac{c}{2},\quad b_1 \geq \frac{d}{2},\quad b_2 \geq \frac{d}{2} \qquad \text{(A.10)}$$

There obviously exists a finite range where (A.10) are satisfied. The potential retains its most general form and the renormalizibility condition is satisfied.

We can extend the same calculation to include the third multiplet L, which transforms as symmetric second rank tensor under SU(4) and so on.

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Figure Captions

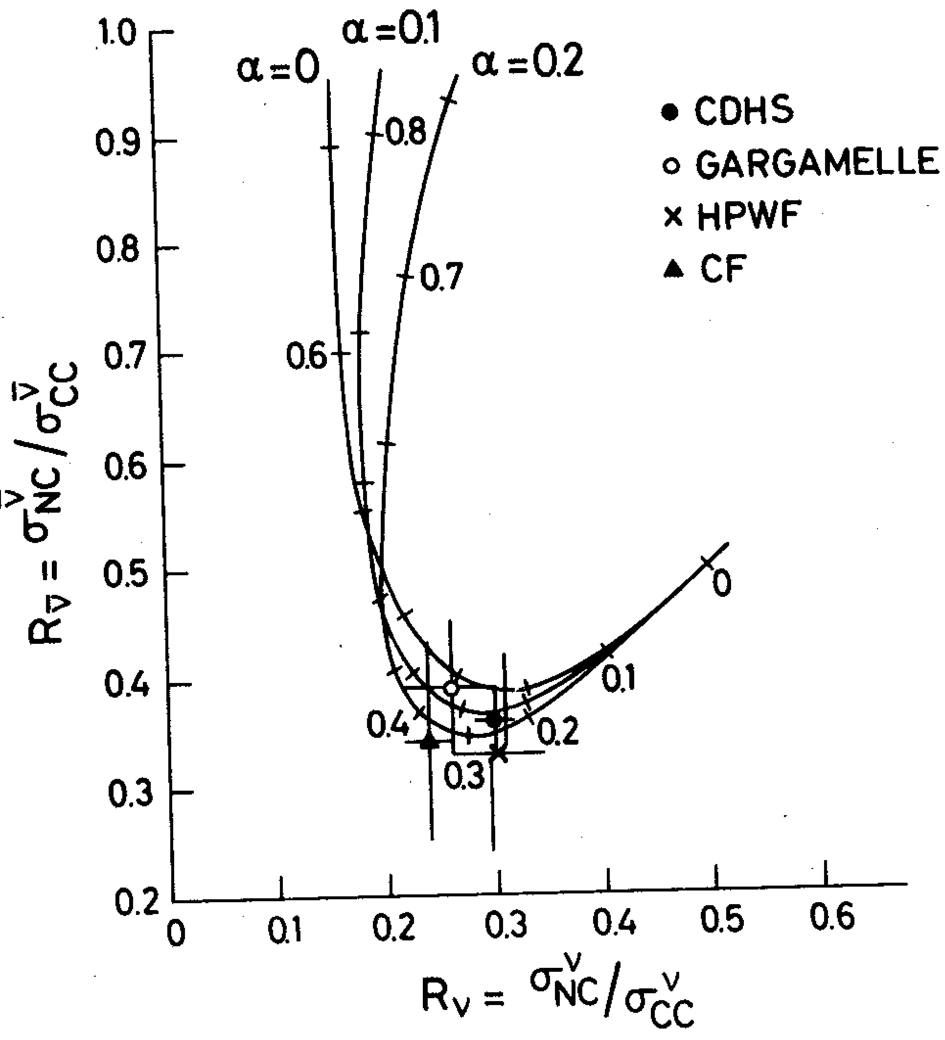
Fig. 1 The ratios of neutral current cross sections versus charged current cross sections for neutrino and anti-neutrino reactions as functions of $\sin^2\theta_W$ (labeled beside the curves). The difference between this model prediction and the Weinberg-Salam model is negligibly small for $\sin^2\theta_W$ of interest. α is the ratio of the anti-quark versus quark content inside the isoscalar target. The data are from ref. 29.

Fig. 2 The elastic $\nu_\mu e$, $\bar{\nu}_\mu e$ cross sections as functions of $\sin^2\theta_W$ (labeled beside the curves). The data are from ref. 30.

Fig. 3 The elastic $\nu_\mu p$ cross section as a function of $\sin^2\theta_W$ The data are from ref. 31.

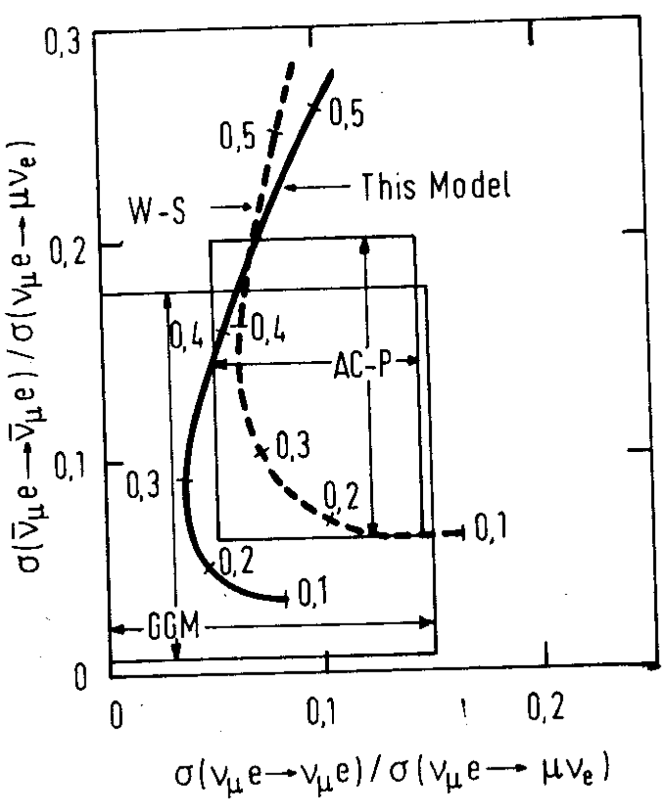
Fig. 4 The ratio of elastic $\bar{\nu}_\mu p$ versus $\nu_\mu p$ cross sections as a function of $\sin^2\theta_W$. The data are from ref. 31.

Fig. 5 The Feynman diagrams for the leptonic decays of τ lepton.



α=0
α=0.1
α=0.2
● CDHS
○ GARGAMELLE
× HPWF
▲ CF
0.6
0.8
0.7
0.4
0.3
0.2
0.1
0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
$R_{\bar\nu} = \sigma_{NC}^{\bar\nu}/\sigma_{CC}^{\bar\nu}$
$R_\nu = \sigma_{NC}^{\nu}/\sigma_{CC}^{\nu}$

Fig.1



0,5
0,5
W-S
This Model
0,4
0,4
AC-P
0,3
0,3
0,2
0,2
0,1
0,1
GGM
0,3
0,2
0,1
0
0
0,1
0,2
$\sigma(\bar\nu_\mu e \to \bar\nu_\mu e)/\sigma(\nu_\mu e \to \mu\nu_e)$
$\sigma(\nu_\mu e \to \nu_\mu e)/\sigma(\nu_\mu e \to \mu\nu_e)$

Fig.2

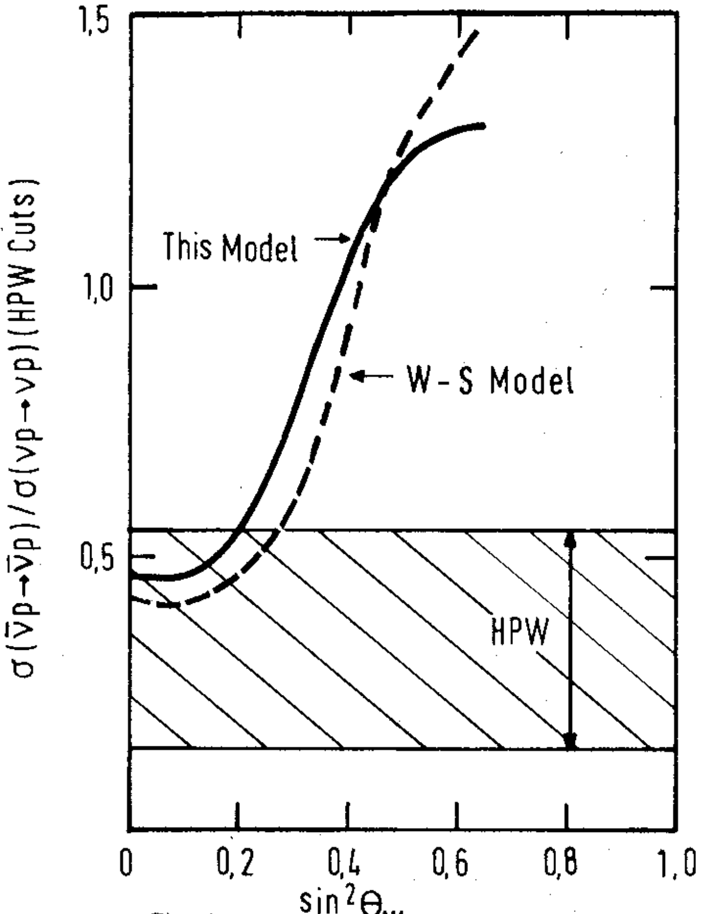


Fig. 4

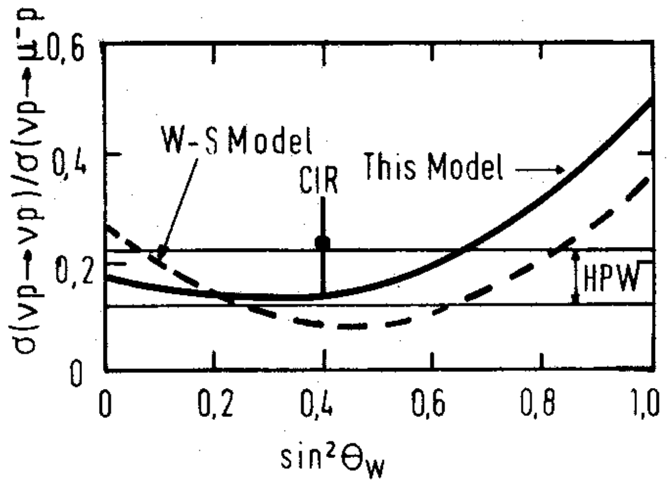


Fig. 3

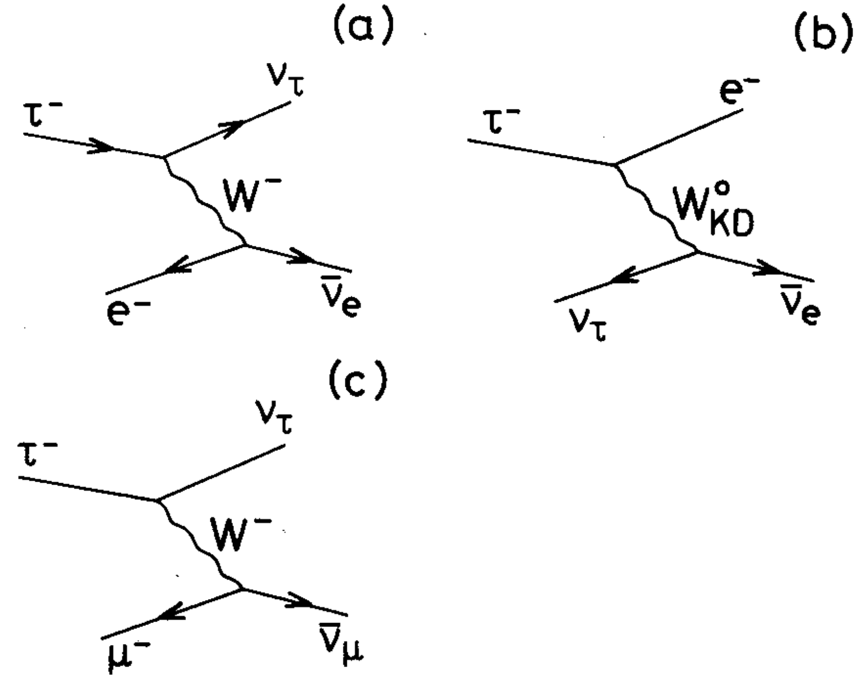


Fig. 5