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Abstract:

The jets in e^+e^--annihilation are identified with two back-to-back hadron-balls with a thermodynamic decay spectrum. The data on $\langle n_{ch} \rangle$, $s\, d\sigma/dx$, $d\sigma/dp_\perp$, $d\sigma/dy$ and on the angular coefficient α are reproduced nicely by the model. The data at 7.4 GeV shows a cigar-like shape displayed in a "jet-contour-diagram". Predictions for PETRA and PEP energies are made.

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Recent experiments on inclusive hadron production in e^+e^--annihilation at SPEAR[1] give clear evidence for a jet structure: hadrons with large momenta are emitted preferentially with a limited transverse momentum relative to a jet axis which has at the highest energy $W = 7.4$ GeV approximately a $1 + \cos^2\theta$ distribution.

In this note we show that the observed jet structure at large particle momentum finds a simple and most natural explanation in a model, where the jets are identified with two back-to-back hadron-balls*, which in their respective rest systems decay isotropically into the observed hadrons in a thermodynamic fashion.

In addition to the high momentum hadrons originating from the two jet-hadron-balls one observes copious production of low momentum particles in $e^+e^- \to$ hadrons. It has been argued in a previous publication[2] that these low momentum hadrons are the decay products of a single hadron-ball with an energy-independent average mass between 1 and 2 GeV which can be identified[3] in absolute magnitude with the typical hadron-ball produced at the ISR in the central region[4].

This leads us to a "minimal" model for $e^+e^- \to$ hadrons, viz. a three-hadron-ball model: one "central hadron-ball" at rest in the cm system of e^+e^- with fixed average mass M_o and two "jet-hadron-balls" with momenta $\vec{Q}$ and $-\vec{Q}$, respectively, and continuous mass M, $M_1 \leq M \leq E_J \equiv \sqrt{M^2 + \vec{Q}^2}$. From energy conservation: $E_J = \frac{1}{2}(W - M_o)$, W = total cm energy, $|\vec{Q}| = \lambda E_J$, $\lambda \equiv (1 - (M/E_J)^2)^{1/2}$.

Apart from their difference in mass the three hadron-balls are assumed to be identical with an invariant thermodynamic decay pattern

$$E \frac{d^3\sigma}{d^3p} \sim M e^{-\frac{E}{T}} = M e^{-\frac{pQ_\pm}{TM}} \quad . \tag{1}$$

(at rest) (boosted)

$Q_\pm = (M, \vec{0})$ $\quad Q_\pm = (E_J, \pm\vec{Q})$

* Hadron-balls are localised lumps of highly excited hadronic matter with well-defined properties as described below in this note.

$p = (E, \vec{p})$ is the 4-momentum of the detected hadron. The "temperature" T of the hadron-balls is constant and has been found in ref. 2) and 3) to be given by $T \simeq 200$ MeV **.

Since the probability for producting the three hadron-balls in the intermediate state, $e^+e^- \to 3$ hadron-balls $\to$ hadrons, is proportional to the total cross section for $e^+e^- \to$ hadrons, σ_{tot}, the single-particle inclusive cross-section is given by

$$E \frac{d^3\sigma}{d^3p} = C\,\sigma_{tot}\,\{M_o\, e^{-\frac{E}{T}} + \int \frac{d^3Q}{E_J}\, \Phi\, M\, [e^{-\frac{pQ_+}{TM}} + e^{-\frac{pQ_-}{TM}}]\} \tag{2}$$

Energy conservation (by means of the energy-sum rule) fixes the normalization constant C, $C = [4\pi m^2 T K_2(\frac{m}{T})]^{-1}$, where K_2 is the modified Bessel function of order 2. Here and in the following we assume for simplicity that the detected hadron is a pion, $E = \sqrt{m^2 + \vec{p}^2}$, m = pion mass.

$\Phi = \Phi(W, M, \theta_J)$*** is the probability for producing the pair of jet-hadron-balls with mass M and momenta $\pm\vec{Q}$ under an angle θ_J in the invariant volume $d^3Q/E_J = 2\pi\lambda$. $M\, dM\, d\cos\theta_J$ at a given total cm energy W. Φ is normalized as

$$\int \frac{d^3Q}{E_J} \Phi = 1 \; . \tag{3}$$

If we furthermore assume that the jet-hadron-balls behave effectively as two spin $\frac{1}{2}$ particles as e.g. suggested by the free quark model, we may write

$$\Phi(W, M, \theta_J) = \frac{3}{16\pi\lambda M} \frac{d\rho}{dM} (1 + \cos^2\theta_J)$$
$$\int_{M_1}^{E_J} dM \frac{d\rho}{dM} = 1 \; , \tag{4}$$

** Actually, in refs.2) and 3) we used exp(−2rp) instead of eq. (1), with $r = 2.5$ GeV^{-1}. Obviously, $T = 1/2r = 200$ MeV.

*** Since we discuss the case of unpolarized e^+e^--beams there is no azimuthal dependence.

where $\frac{d\rho}{dM}$ is the jet-mass distribution (with threshold M_1) at a given total energy W.

High momentum particles are produced preferentially by high momentum hadron-balls which implies that the typical jet-hadron-ball has to be much lighter than the highest possible mass $E_J = \frac{1}{2}(W - M_o)$. Thus the mass distribution $\frac{d\rho}{dM}$ has to decrease with increasing M, the decrease being uniquelly fixed, $\frac{d\rho}{dM} \sim M^{-2}$, by the requirement of scaling. We are therefore lead to the simple ansatz

$$\frac{d\rho}{dM} = \frac{M_1 E_J}{E_J - M_1} \frac{1}{M^2}, \qquad M_1 \leq M \leq E_J, \tag{5}$$

where the energy dependence is completely determined by the normalization (3,4).

Eqs. (4) and (5) give the jet-production probability Φ which substituted into eq. (2) completes our model for inclusive hadron production in e^+e^-. The model applies above the energy threshold $W_{th} = M_o + 2 M_1$.

Identifying the "central hadron-ball" with average mass M_o with the typical hadron-balls produced in hadron-hadron scattering[3], we infer from a recent comprehensive study performed by Arneodo and Plaut[4], $M_o = 1.46$ GeV. Since the temperature is already known to be $T = 200$ MeV[2)3)], we conclude that only M_1 is left as a free parameter which is determined by comparison of our model with the SPEAR data*.

Up to now the inclusive spectra have been measured at the highest energies only for charged particles. Under the assumption that the sum over all charged particles is a momentum independent fraction, f_{ch}, of the total distribution, we obtain via the energy-sum rule

$$\left(E\frac{d^3\sigma}{d^3p}\right)_{ch} = f_{ch} \cdot E\frac{d^3\sigma}{d^3p}$$
$$f_{ch}(W) = \left\langle \frac{E_{ch}}{W} \right\rangle \tag{6}$$

* Since the jet mass distribution (5) is peaked at threshold, $M_{th} = M_1$, $(d\rho/dM)_{th} \sim 1/M_1$ for $W >> W_{th}$, M_1 is a direct measure for the strength of the jets, the smaller M_1 the more jet is produced. This is clearly seen in eq. (7) and in the exponential slope of eq. (9).

with f_{ch} being the fraction of cm energy appearing in charged particles**.

As a first check of our model we calculate the mean charged multiplicity, $\langle n_{ch}\rangle$, from the multiplicity-sum rule

$$\langle n_{ch}\rangle = f_{ch}\, A\left\{\frac{M_o}{T} + 2\,\frac{\langle M\rangle}{T}\right\}.$$

$$\langle M\rangle \equiv \int_{M_1}^{E_J} dM\, M\,\frac{d\rho}{dM} = \frac{M_1 E_J}{E_J - M_1}\ln\left(\frac{E_J}{M_1}\right) \tag{7}$$

$$A \equiv \tfrac{1}{2}[1 - K_o\left(\tfrac{m}{T}\right)/K_2\left(\tfrac{m}{T}\right)].$$

The mean charged multiplicity has a constant contribution $\sim M_o/T$ due to the central hadron-ball and a logarithmically increasing contribution $\sim 2\langle M\rangle/T \sim (2M_1/T)\ln W$, which is proportional to the average jet mass $\langle M\rangle$ of the jet-hadron-balls. A numerical evaluation*** of eq. (7) shows excellent agreement with the results of SPEAR[1], giving for example a value of 3.9 at $W = 4.2$ GeV and $\langle n_{ch}\rangle = 4.9$ at the highest energy $W = 7.4$ GeV. At the highest PETRA energy, $W = 38$ GeV, we predict $\langle n_{ch}\rangle = 7.8$ (for $f_{ch} = \frac{1}{2}$).

For the inclusive quantity $s\, d\sigma/dx$, $x \equiv 2p/W$, $s \equiv W^2$, we obtain from eq. (2)

$$s\frac{d\sigma_{ch}}{dx} = 2\pi f_{ch}\, C\, \sigma_{tot}\left(\frac{W^3 p^2}{E}\right)\left\{M_o\, e^{-\frac{E}{T}} + \frac{2\,T\,M_1}{p(E_J - M_1)}\int_{M_1}^{E_J}\frac{dM}{\lambda}\, e^{-\frac{EE_J}{MT}}\sinh\left(\frac{\lambda p E_J}{MT}\right)\right\}. \tag{8}$$

** $f_{ch}(W)$ has been measured at SPEAR[1] and turns out to vary between 0.55 and 0.48 in the energy range $4.0 \leq W \leq 7.4$ GeV, thus it is approximately constant, $f_{ch} \simeq \frac{1}{2}$.

*** Here we have chosen $M_1 = 1$ GeV.

In Fig. 1 we compare eq. (8) with the data from SPEAR[1] * at energies $W = 4.1$, 4.8, 6.2 and 7.4 GeV. Here we have chosen $M_1 = 1$ GeV, which leads to excellent agreement with the data.

In Fig. 2 we show our prediction for the cross-section at the highest PETRA energy, $W = 38$ GeV, assuming $R \equiv \sigma_{tot}/\sigma_{\mu\mu} = 5$ and $f_{ch} = \frac{1}{2}$.

In the scaling limit, $s \to \infty$, x fixed, the central hadron-ball contribution in eq. (8) vanishes exponentially, whereas the jet contribution scales (if $f_{ch} \to f_\infty = \text{constant} \neq 0$ for $s \to \infty$)

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_{ch}}{dx} \xrightarrow[\text{scaling limit}]{} (8\pi f_\infty C M_1 T^2) \frac{e^{-\frac{M_1}{2T}x}}{x} \tag{9}$$

Thus our hadron-ball model leads to a scaling cross-section which has the typical $1/x$ singularity at $x = 0$ needed for a logarithmic multiplicity, and vanishes exponentially for $x \to 1$.

Keeping the angular dependence in eq. (2) we obtain

$$s \frac{d\sigma}{dx\, d\cos\theta} \sim (1 + \alpha \cos^2\theta), \tag{10}$$

where θ is the polar angle of the detected hadron measured with respect to the incident e^+-direction. The angular coefficient α is given by

$$\alpha = \frac{3B}{\{\ldots\} - B}$$

$$B = \frac{M_1 E_J}{2(E_J - M_1)} \int_{M_1}^{E_J} \frac{dM}{M}\, e^{-\frac{EE_J}{MT}} \sqrt{\frac{\pi}{2Z}}\, I_{5/2}(Z), \qquad Z = \frac{\lambda p E_J}{M T}, \tag{11}$$

* In this note we do not discuss the recent data from DORIS[5], since these data are at lower energies, $W \leq 5$ GeV, where no clear evidence is found for the production of jets. The lower cross-sections at large x-values reported by the DASP and PLUTO groups[5] can be reproduced in our model, if one chooses a somewhat bigger value for M_1 as one expects from the M_1-dependence of the exponential slope in eq. (9).

where $\{\ldots\}$ denotes the bracket in eq. (8) and $I_{5/2}$ is a modified Bessel function of the first kind. In Fig. 3 we compare our result (11) for the angular coefficient α at fixed energy W as a function of x with the SPEAR data[1]. Again we obtain good agreement within the large experimental uncertainties.

Next we discuss the $p_\perp$-distribution, where $p_\perp$ is the component of the detected particle momentum perpendicular to the jet axis. Our model gives the following simple formula

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_{ch}}{dp_\perp} = \frac{2\pi C}{A} T \langle n_{ch}\rangle p_\perp K_o\left(\frac{m_\perp}{T}\right), \tag{12}$$

where $m_\perp \equiv \sqrt{m^2 + p_\perp^2}$ is the transverse mass and K_o the modified Bessel function order zero. We predict a universal $p_\perp$-distribution, with the energy dependence completely given by the multiplicity $\langle n_{ch}\rangle$, and with an exponential cut-off in $p_\perp$, $\frac{1}{\sigma_{tot}} \frac{d\sigma}{dp_\perp} \sim \sqrt{p_\perp}\, e^{-p_\perp/T}$ for $p_\perp \to \infty$. For the average $p_\perp$ we obtain the energy independent result $\langle p_\perp \rangle = 360$ MeV *, in nice agreement with the result at SPEAR[1], where $\langle p_\perp \rangle$ "was found to be in the range 325 to 360 MeV with no particular energy dependence". The comparison with the data is shown in Fig. 4.

As a last test of our model we study the rapidity dependence, where the rapidity y is defined by $E = m_\perp \cosh y$, $p_\parallel = m_\perp \sinh y$, $y = \frac{1}{2} \ln\left(\frac{E + p_\parallel}{E - p_\parallel}\right)$, and $p_\parallel$ is the component of the detected particle momentum parallel to the jet axis. The result is

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_{ch}}{dy} = 2\pi f_{ch} C T^2 \Big\{ M_o \frac{e^{-\frac{m}{T}\cosh y}}{(\cosh y)^2} \left(1 + \frac{m}{T}\cosh y\right) + \frac{M_1}{E_J(E_J - M_1)} \int_{M_1}^{E_J} dM\, M \left[\frac{e^{-\frac{mEJ}{MT} U_+}}{U_+^2} \left(1 + \frac{m E_J}{MT} U_+\right) + (U_+ \to U_-)\right] \Big\} \tag{13}$$

* In the zero mass limit one obtains $\langle p_\perp \rangle_{m=o} = \frac{\pi}{2} T = 314$ MeV, whereas for heavy particles, $m \gg T$, we obtain $\langle p_\perp \rangle \sim \sqrt{(\pi/2) T m}$, giving for nucleons a value of about 540 MeV.

$U_{\pm} = \cosh y \pm\lambda \sinh y .$

In Fig. 5 eq. (13) is compared with the SPEAR data[1] at $W = 4.8$ and 7.4 GeV. The agreement is excellent.

The jet structure exhibited by our model is most clearly visible in Fig.6, which shows lines of constant cross-section $(4\pi E/\sigma_{tot})\cdot d^3\sigma/d^3p)$ in a $(p_{\parallel},p_{\perp})$-plot, $p_{\parallel}$ and $p_{\perp}$ defined with respect to the jet axis as before. At the lower energy, $W = 3.5$ GeV, one sees a sphere-like object, whereas at 7.4 GeV a very nice cigar has developped illustrating in an unambiguous way that a pair of jets has been produced. Since the relation between the fashionable sphericity distribution[1] and the existence of jets is much less evident, we propose to display future data in a "jet-contour-diagram" as shown in Fig. 6.

In conclusion we emphasize that the excellent agreement of our hadron-ball model with the SPEAR data gives strong support to the idea that balls of hadronic matter are produced, objects which have been used extensively in the past in theoretical models but have never been isolated experimentally. The production of jets in e^+e^- and the excellent agreement of our model with the data provides us for the first time with clear evidence about the existence of hadron-balls with properties as analyzed in this note.

References

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3) F. Elvekjaer and F. Steiner, DESY 76/15, to be published in Nuovo Cimento.

4) A. Arneodo and G. Plaut, Nucl. Phys. B107 (1976)262, and Nucl. Phys. B113 (1977) 156.

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U. Timm, Talk at the European Conference on Particle Physics, Budapest (1977).

Figure Captions

Fig. 1 Single-particle cross-sections, $s\, d\sigma/dx$, for inclusive production of charged particles VS. x for $W = 4.1$, 4.8, 6.2 and 7.4 GeV. The curves show the predictions of the hadron-ball model, eq. (8), for $M_1 = 1$ GeV. Data from SPEAR[1].

Fig. 2 Prediction for the inclusive charged particle cross-section, $s\, d\sigma/dx$, at the highest PETRA energy, $W = 38$ GeV, for $M_1 = 1$ GeV, $f_{ch} = \frac{1}{2}$.

Fig. 3 Inclusive angular coefficient α VS. x at $W = 4.1$, 4.8, 6.2 and 7.4 GeV. The curves show the predictions of the hadron-ball model, eq. (11). Preliminary data from fits of eq. (10) for $|\cos\theta| \leq 0.6$ from SPEAR[1].

Fig. 4 Transverse momentum distribution, $(1/\sigma_{tot})d\sigma/dp_{\perp}$, for W = 4.8, 6.2 and 7.4 GeV. The curve shows the prediction of the hadron-ball model, eq. (12), for W = 7.4 GeV. Data from SPEAR[1)] for events with $x_{max} > 0.5$.

Fig. 5 Rapidity distribution, $(1/\sigma_{tot})\, d\sigma/dy$, for W = 4.8 and 7.4 GeV. The curves show the predictions of the hadron-ball model, eq. (13). Data from SPEAR[1)] for events with $x_{max} > 0.5$; pion masses assumed.

Fig. 6 Jet-contour-diagram. Lines of constant invariant cross section, $(4\pi E/\sigma_{tot})\, d^3\sigma/d^3p$, are shown in a $(p_{\parallel}, p_{\perp})$-plot at W = 3.5 and 7.4 GeV.

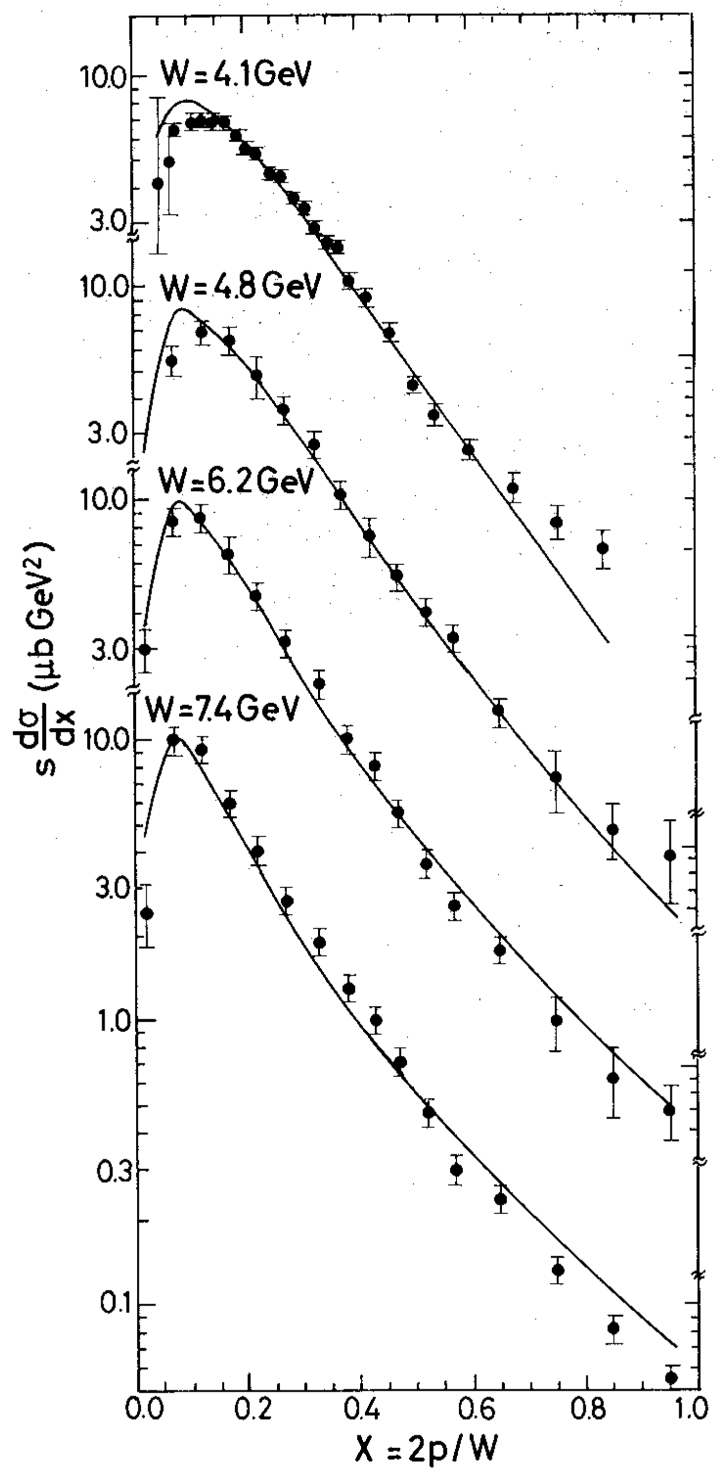
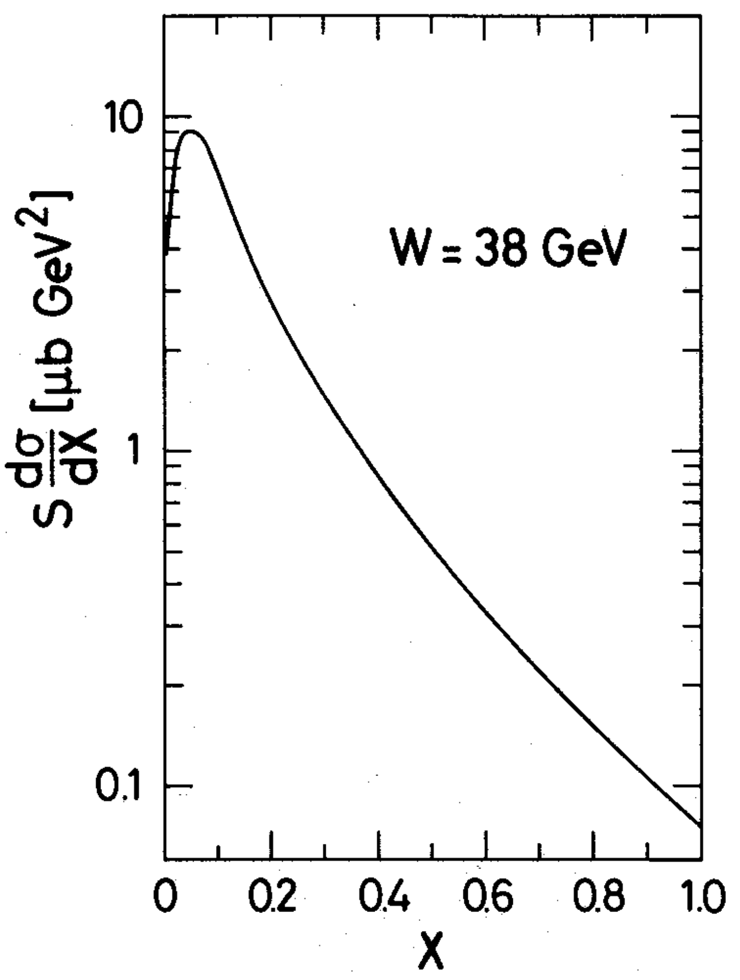
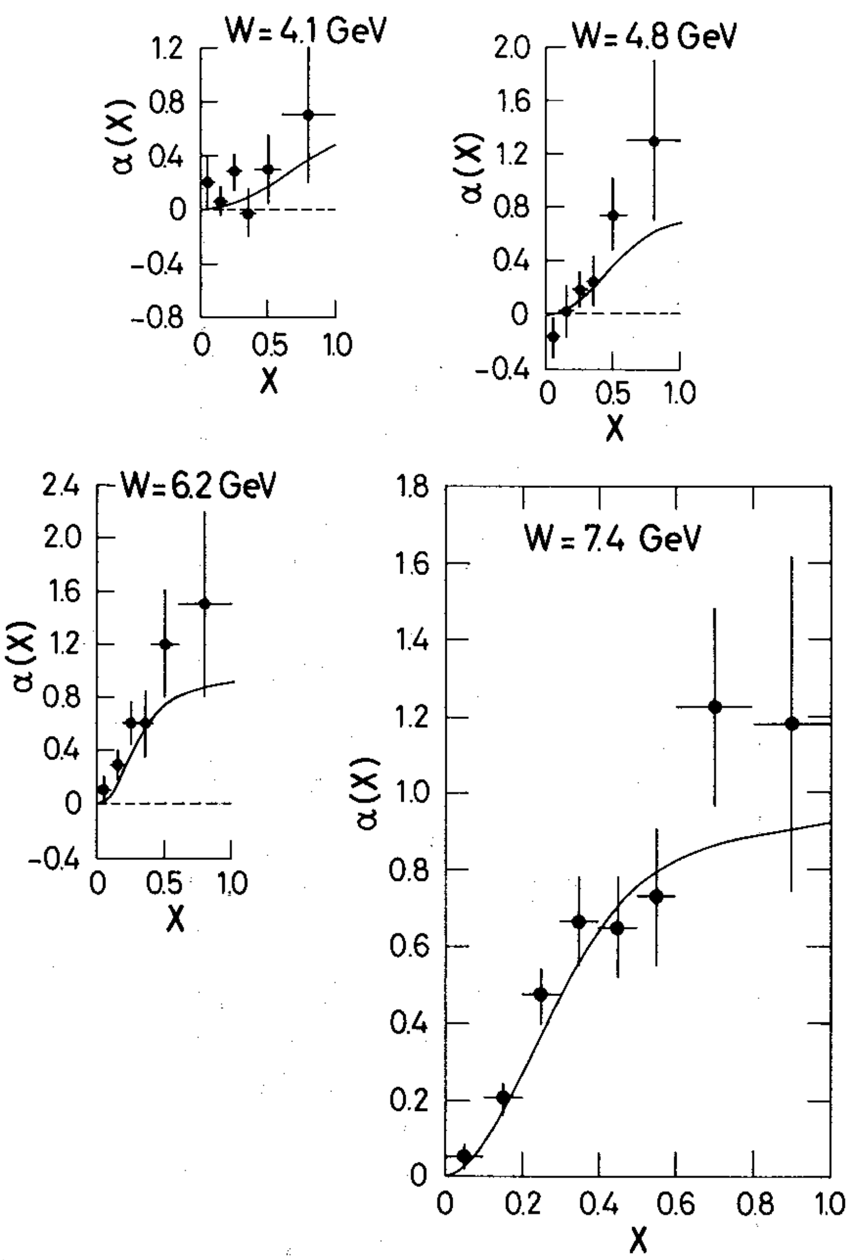


Fig. 1



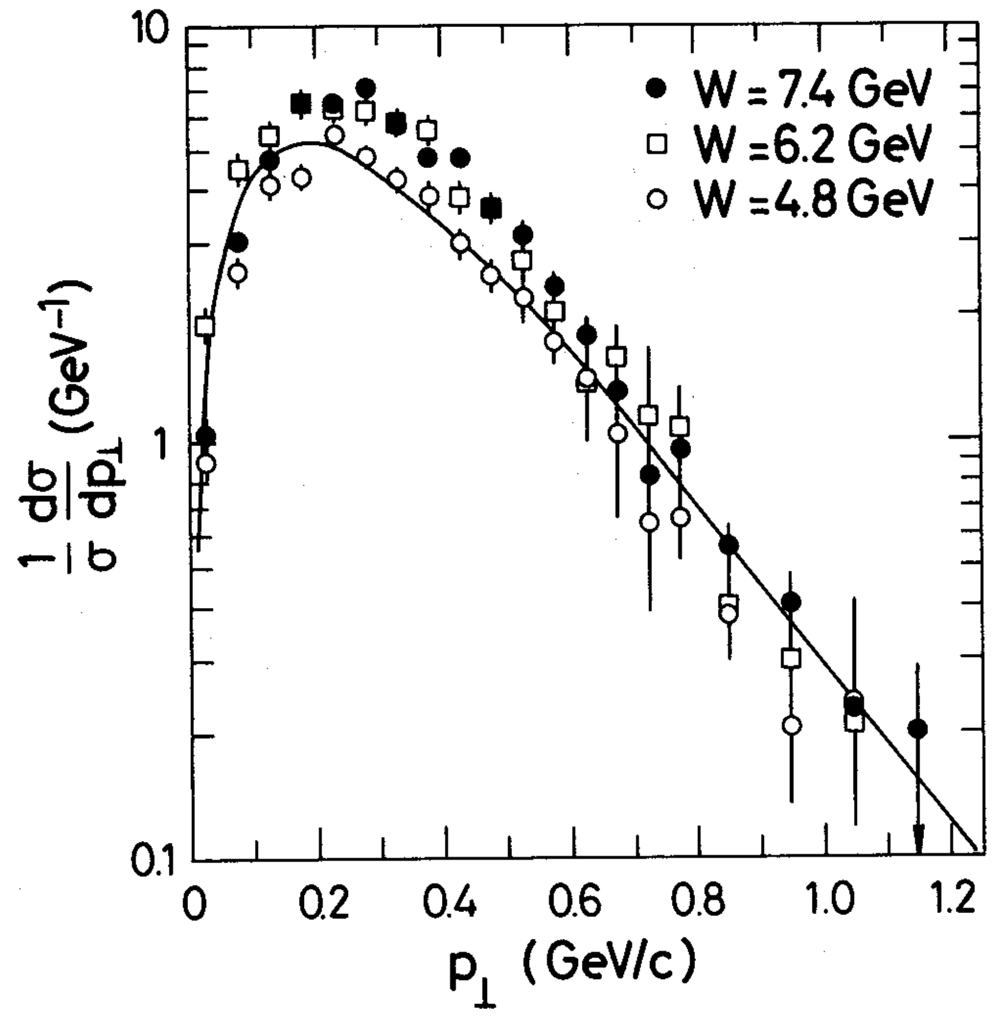
W = 38 GeV
S dσ/dX [μb GeV²]
10
1
0.1
0
0.2
0.4
0.6
0.8
1.0
X

Fig. 2



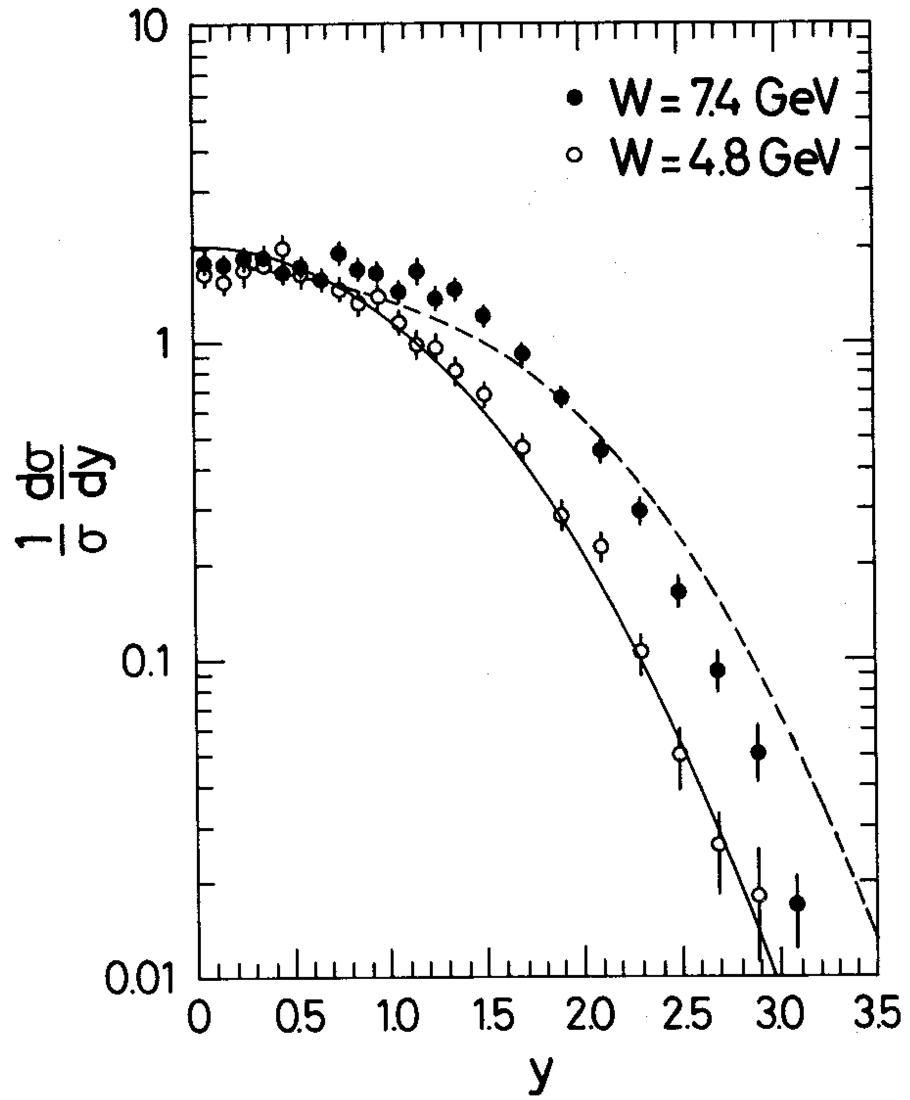
W = 4.1 GeV
W = 4.8 GeV
W = 6.2 GeV
W = 7.4 GeV
α (X)
X

Fig. 3



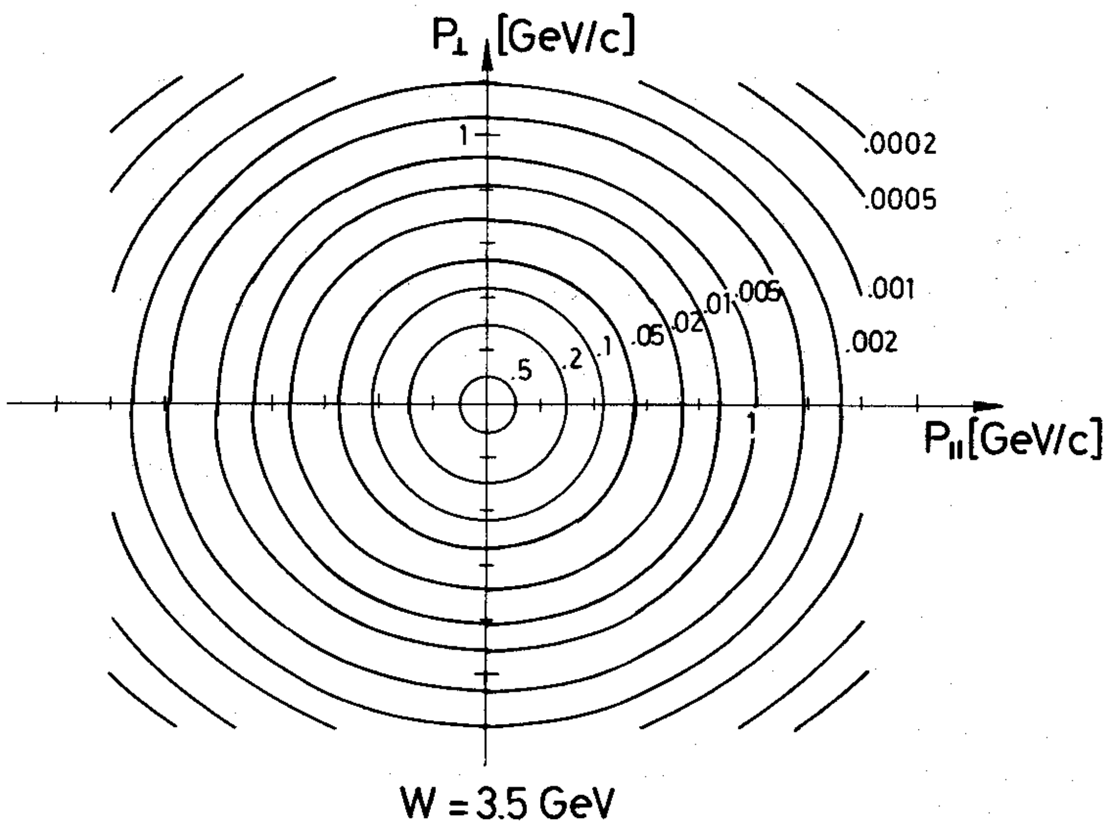
W = 7.4 GeV
W = 6.2 GeV
W = 4.8 GeV
$\frac{1}{\sigma}\frac{d\sigma}{dp_\perp}$ (GeV^{-1})
$p_\perp$ (GeV/c)
10
1
0.1
0
0.2
0.4
0.6
0.8
1.0
1.2

Fig. 4



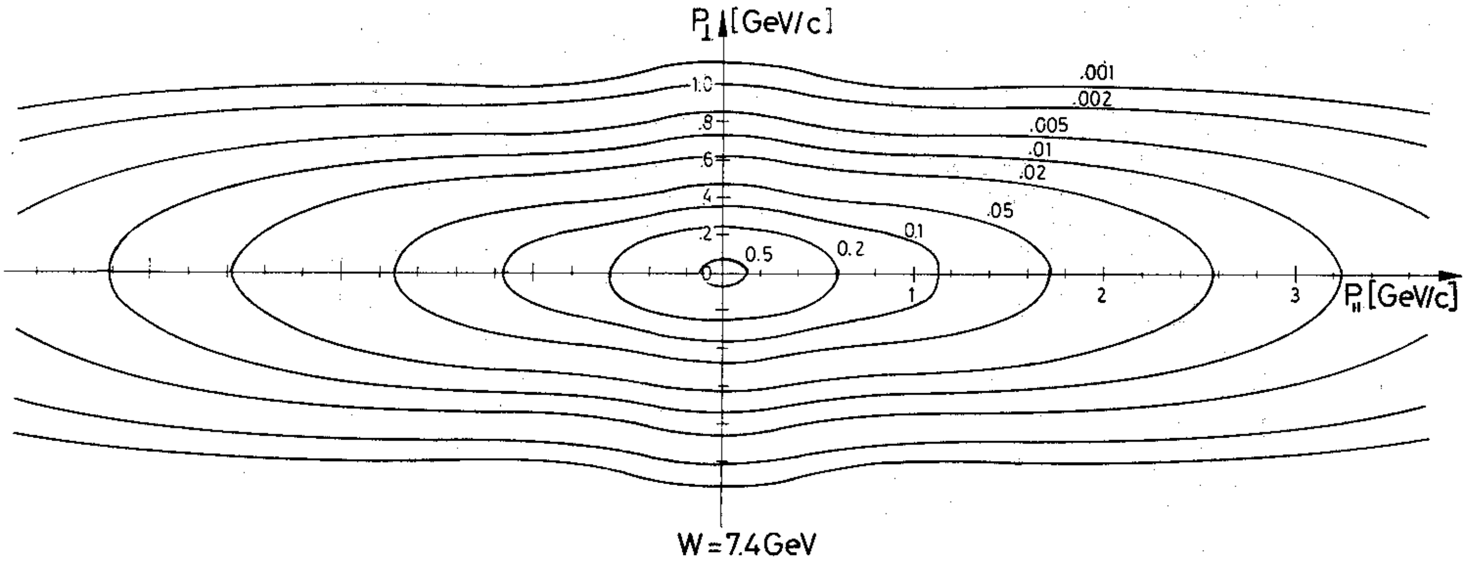
W = 7.4 GeV
W = 4.8 GeV
$\frac{1}{\sigma}\frac{d\sigma}{dy}$
y
10
1
0.1
0.01
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

Fig. 5



P⊥ [GeV/c]
1
.0002
.0005
.001
.002
.005
.01
.02
.05
.1
.2
.5
1
P∥ [GeV/c]
W = 3.5 GeV

Fig.6a



P⊥ [GeV/c]
1.0
.8
.6
.4
.2
0
.001
.002
.005
.01
.02
.05
0.1
0.2
0.5
1
2
3
P∥ [GeV/c]
W = 7.4 GeV

Fig.6b