A Cautionary Note on the Interpretation of EOFs

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Abstract

EOF and rotated EOF analyses are widely used tools in climate research. In recent years there have been several cases in which the EOF- or rotated EOF analyses were used to identify physical modes. These are the “tropical Atlantic” and the “tropical Indian Ocean SST dipole” modes and the different modes of the Northern Hemisphere winter surface air pressure variability. Here we would like to discus the problems in interpreting these statistically derived modes as physical modes. By constructing an artificial example we shall show that the patterns derived from EOF- or rotated EOF analysis can lead to misinterpretations. This study should be seen as a cautionary note to highlight the pitfalls which may occur when using the EOF or rotated EOF techniques.

1 Introduction

In recent years the EOF technique has been largely used to identify physical modes. The problems that may arise by using EOF or rotated EOFs is the subject of this note.

One source of uncertainty lies in the statistical uncertainty of the eigenvalues, which can lead to unstable EOF-patterns due to the so called degeneration of the EOF eigenvalues. This problem has been discussed in North et al. (1982) and von Storch and Hannoschöck (1985). North et al. (1982) present a rule of thumb for the stability of the EOF patterns. Related to the statistical uncertainty of the EOF analysis Richman (1986) discusses the problems in EOF analysis in a number of examples and points out some unpleasant characteristics of EOF patterns, which motivates him to introduce the rotation of the EOFs by the VARIMAX method.

Although Richman (1986) discusses some of the problems with the EOF patterns, his discussion is mostly focussed on the statistical uncertainty of the EOF analysis, which is one source of the problems with the EOF analysis. Here we would like to focus on those problems of the EOF and VARIMAX analyses which are more inherent to these methods and which are therefore not due to statistical uncertainties.

In order to derive the leading modes of variability in a multi-variate data set, the EOF and VARIMAX analyses work with some basic, subjective but well motivated, assumptions. One assumption made on which both the EOF and the VARIMAX analyses are based is that the time evolutions of the dominant modes are uncorrelated.

A second assumption is necessary to construct a basis of the multi-variate data set. For the EOF analysis it is assumed that the dominant modes are orthogonal in space and that the first pattern is the pattern which maximizes the explained variance over the total data set.

For the VARIMAX method, it is assumed that the basis of dominant modes is maximizing the 'simplicity', which leads to more localized patterns compared to the EOF analysis (Kaiser 1958).

It is often claimed that the VARIMAX method is more subjective than the EOF analysis, since there are more free parameters which have to be defined. However, within the context of this study the VARIMAX method is as objective as the EOF analysis, since we shall assume that the time evolutions of the dominant modes are uncorrelated, which eliminates one free parameter of the VARIMAX analysis. A second free parameter which has to be chosen for the VARIMAX method is the number of EOFs used for calculating the VARIMAX rotation. Again, this is not really a free parameter, since one simply has to take as much EOF patterns as necessary to present the dominant modes in the raw data set, which is the same number of EOFs that would be chosen to get rid of the noise.

Although the assumptions made by the EOF and VARIMAX methods seem to be well motivated, the discussion of the examples presented below will show that these assumptions are the source of the misinterpretation of the variability modes.

This paper is organized as follows: In the next section we shall present three examples of observed climate variability, in which the interpretation by EOF, VARIMAX and regression analyses has lead to conflicting results in recent publications. Section 3 deals with a simple low-dimensional example of multi-variate data analysis, which has by construction no statistical uncertainties in the determination of the EOF and VARIMAX patterns. We shall then discus the problems in interpreting the EOF, VARIMAX and regression patterns in the different examples and in general. This section will be the main focus of this note. We shall conclude this note by highlighting some caveats when interpreting the results of EOF or rotated EOF analyses.

2 Examples of EOF-analyses

In the following the dominant modes of variability in three different observed data sets are represented by EOF, VARIMAX and regression analyses. We would like to show here only the patterns obtained from the different analyses. We do not intend to present new evidence about the variability in three domains.

For the analysis of the sea surface temperature (SST) we have used monthly mean SST anomalies based on the data set from Reynolds (1994), which covers the period from 1958-98. For the sea level pressure (SLP) analysis we have taken the monthly mean anomalies from November to April from the NCEP data set covering the period from 1958-97 (Kalnay et al. 1996). For all these data sets the VARIMAX representation has been calculated on the basis of the first 10 EOFs with normalized principal components (PCs).

2.1 SST in the tropical Atlantic

In Fig. 1 the first two EOFs and VARIMAX patterns of the tropical Atlantic SST anomalies are shown. The EOF-2 is an inter-hemispheric dipole pattern. Two regression patterns between the box averaged SST of the centers of the dipole pattern and the SST field are shown for comparison in Fig. 1.

The inter-hemispheric dipole pattern in the EOF-2 has attained a lot of interest in recent publications, in terms of whether this pattern represents a physical mode of SST variability on decadal time scales (Chang et al. 1997), or whether this pattern is only an artifact of the EOF analysis (Houghton and Tourre 1992, Enfield et al. 1999, Dommenget and Latif 2000). Dommenget and Latif (2000) basically argue on the basis of coupled model results and observations that the dipole in the tropical Atlantic does not represent a physical mode.

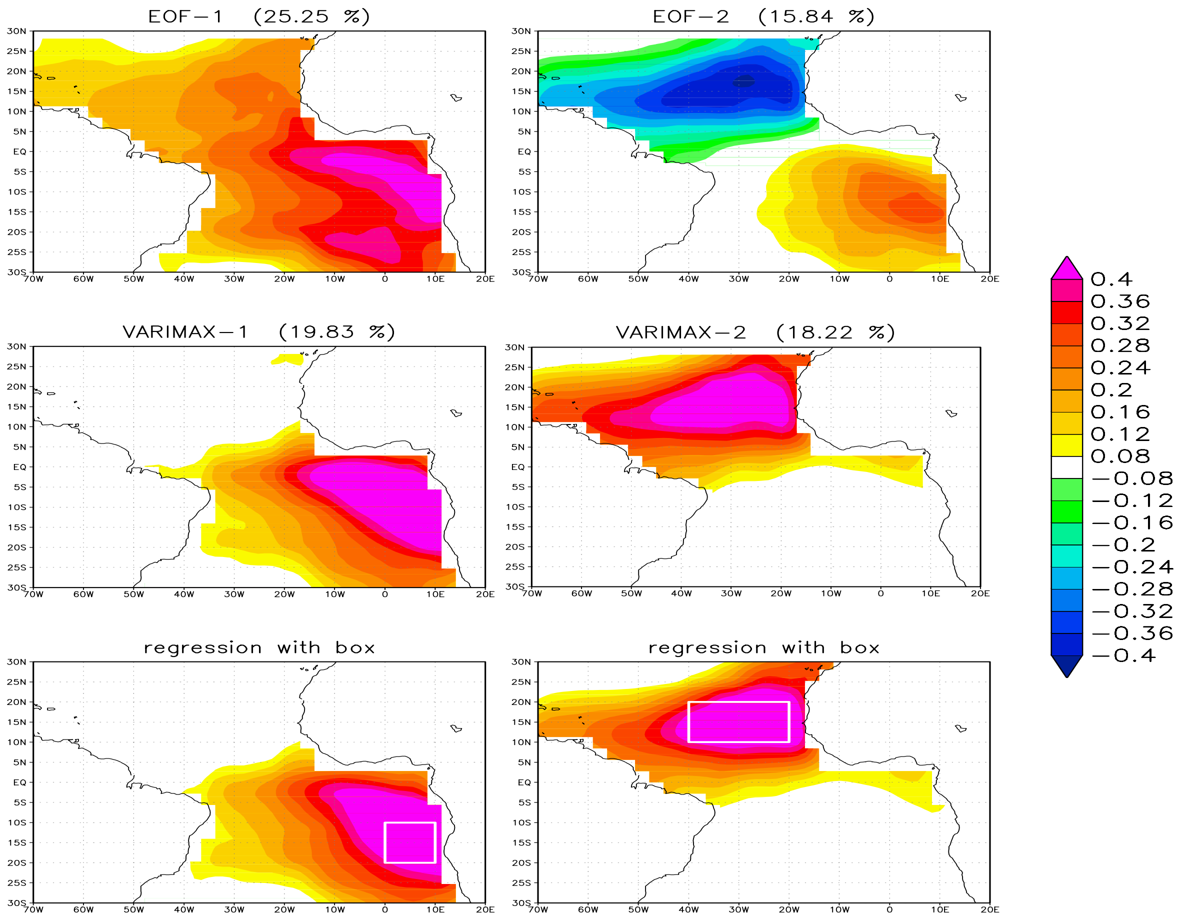

Figure 1: The EOFs, VARIMAX patterns and regressions of box averaged monthly mean sst in the tropical Atlantic Ocean.

2.2 SST in the tropical Indian Ocean

A similar analysis is now repeated for the tropical Indian Ocean. In Fig. 2 the first two EOF and VARIMAX patterns and two regression patterns between box averaged SST and the SST field are shown.

Again, the EOF-2 of the SST variability is characterized by a dipole. However, there are some significant differences compared to the tropical Atlantic. First the EOF-1 explains much more variance than the EOF-1 of the tropical Atlantic and second, the EOF-1 explains also much more variance than the EOF-2 of the tropical Indian Ocean. Furthermore, the VARIMAX patterns do not pick up the two centers of EOF-2. Actually the eastern center of the dipole does not show up in any of the four most dominant VARIMAX patterns (patterns 3 to 4 are not shown).

The first two EOF patterns have been interpreted in terms of physical processes by Saji et al. (1999). They point out that the EOF-1 has a strong correlation with the El Niño in the tropical Pacific and can therefore be interpreted as the Indian Ocean response to El Niño (see also Venzke et al. 2000). Since the EOF-2 has an orthogonal time evolution to EOF-1,

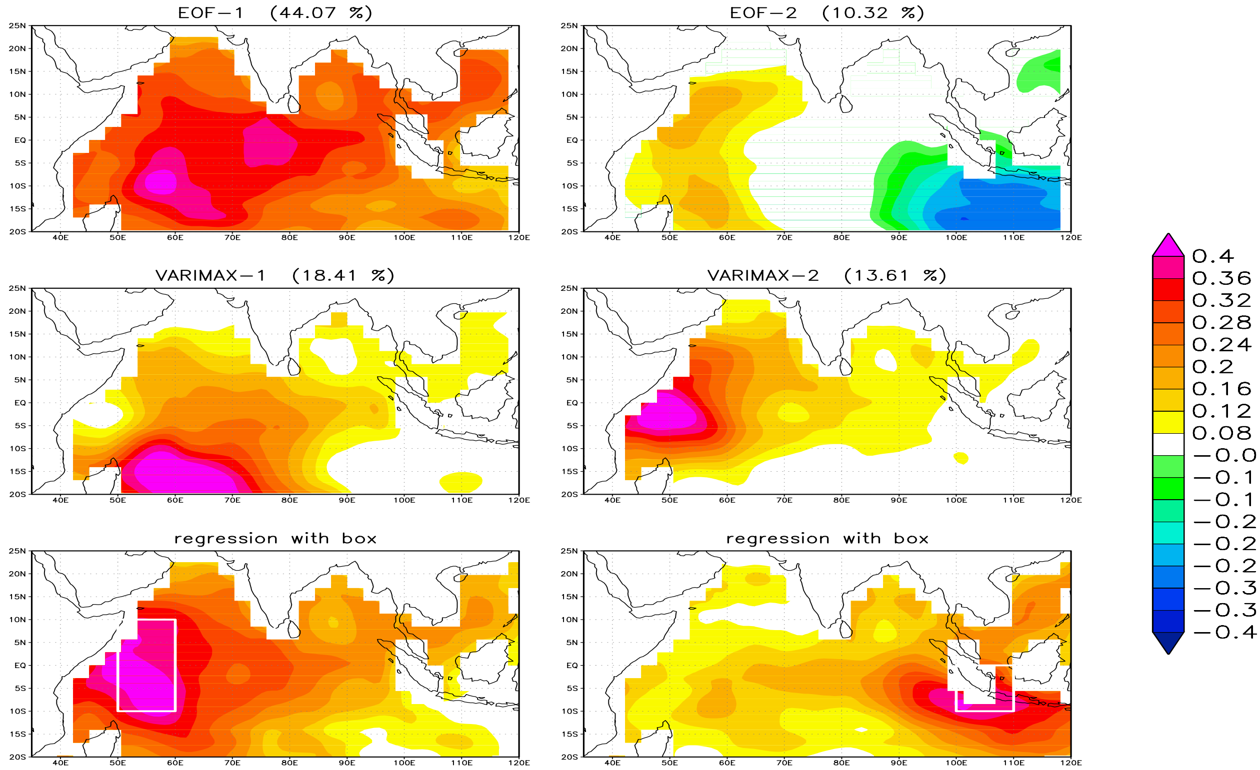

Figure 2: The EOFs, VARIMAX patterns and regressions of box averaged monthly mean sst in the tropical Indian Ocean.

they argue that the EOF-2 can be interpreted as an El Niño independent mode of variability, which is unique to the tropical Indian Ocean. However, the VARIMAX representation and the regressions provide no indication for the existence of a dipole mode, as suggested by EOF-2.

2.3 SLP variability in the northern hemisphere

We shall now analyse the Northern Hemisphere winter SLP variability. The following example is different in many aspects compared to the ones described above. In contrast to SST anomalies, SLP anomalies in one region are usually compensated by SLP anomalies of opposite sign in a nearby region at the same time. Therefore the patterns of SLP have in general a dipole or multipole structure.

In Fig. 3, the first two EOF and VARIMAX patterns and two regression patterns are shown. Again, the different methods of representing the SLP variability in the Northern Hemisphere give quite different results, with respect to the teleconnections. This may be one of the reasons why there is a scientific debate about which of these patterns describe best the dominant modes of SLP variability. For an overview of this controversy see Ambaum et al. (2000), Thompson and Wallace (2000) and Wallace (2000).

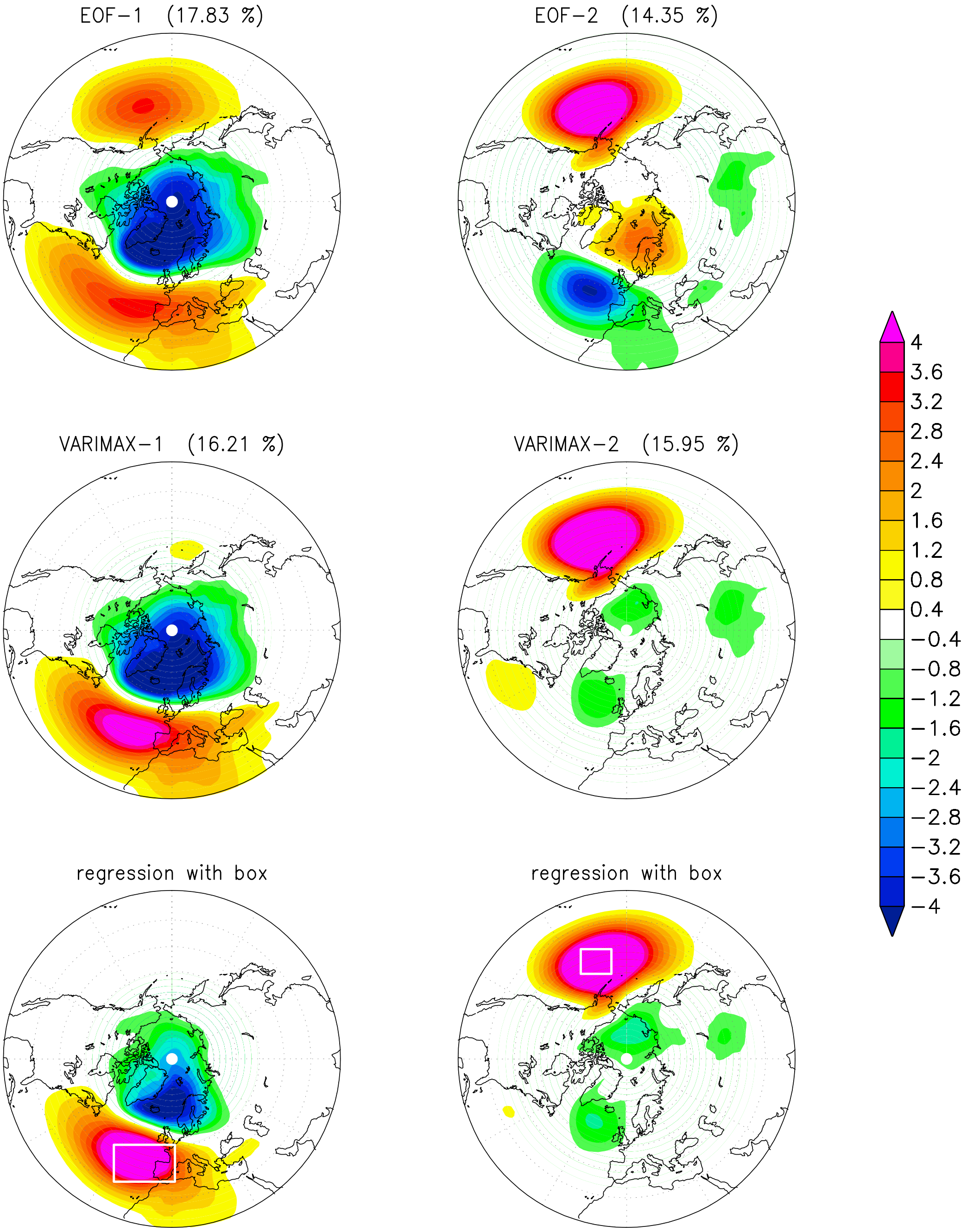

Figure 3: The leading EOFs, VARIMAX patterns and regressions of box averaged monthly mean winter time (november to april) SLP in the Northern Hemisphere.

3 A simple low-dimensional example

A simple 3-dimensional example might help to understand the difficulty in interpreting the patterns of the former examples. The advantage of the following artificial example compared to the ones described above is that we discuss a low-dimensional problem which is well-defined and in which statistical uncertainties do not exist.

We assume that our domain can be divided into three regions. We then define three different modes of variability, which are shown in the upper panel of Fig. 4. We therefore have one mode which only acts in the left region, one only in the right region and one which covers all three regions. The explained variance of each mode is also shown in the titles of each plot in Fig. 4. We also assume that the time evolutions of theses modes are uncorrelated and that the standard deviation of all time series of these modes amounts to unity.

The structures of the physical modes are motivated by the analyses of the SST in the tropical Atlantic and Indian Oceans. For the SLP in the Northern Hemisphere for instance, Mode-1 could be interpreted as the North Atlantic Oscillation (NAO) of the Atlantic-European region (similar to VARIMAX-1 in Fig. 3), Mode-2 as the Pacific North America pattern (PNA) (similar to VARIMAX-2 in Fig. 3) and the Mode-3 would be an annular mode (similar to EOF-1 in Fig. 3, but much weaker and more zonal). The three regions of the simple example would then be interpreted as the Atlantic-European region (the left region in Fig. 4), the Pacific domain (the right region) and the rest of the Northern Hemisphere (the central region).

However, to keep the problem as simple as possible we represent each region by one point only. The values at these points are printed on top of the mode (see Fig. 4). We can therefore interpret each physical mode as a three dimensional vector, where each component of this vector represents the variability of one region. Or differently stated the modes are nothing but a colorful representation of the vectors (compare the values of the vectors with the color shading of the modes in Fig.4).

The construction of our example allows us to calculate the covariance matrix exactly, since our example has been constructed such that the characteristics of the physical modes are known exactly. Therefore all structures that appear in the following statistical analysis are well-defined.

The square root of the covariance matrix yields the regressions of one coordinate of the vector space (one region) with all coordinates (regions) of the vector space. The regression patterns and values are shown in the lower panel of Fig. 4.

Based on the covariance matrix we can also calculate the EOF vectors exactly. We therefore do not have to consider the sampling error problem, which can lead to unstable estimations of the EOF vectors (North et al. 1982). The EOFs are also shown in Fig. 4. The EOF vectors are not degenerated, since all eigenvalues of the covariance matrix are different (see explained variances of the EOFs in Fig. 4).

The time evolution of the EOFs is given by the principle components (PCs), which can be presented by a linear combination of the time evolutions of the initial basis modes:

$$PC_i(t) = A_1 P_1(t) + A_2 P_2(t) + A_3 P_3(t) \quad (1)$$

The coefficients $A_{1,2,3}$ are listed in Table 1. The coefficients $A_{1,2,3}$ quantify the relative influence of the three basis modes to the time evolution of the EOF patterns. For example, it can easily be seen (in Fig. 4) that the EOF-2 includes the Mode-2 with positive loadings and the Mode-1 with slightly smaller negative values (Table 1). Please note that the EOF-2

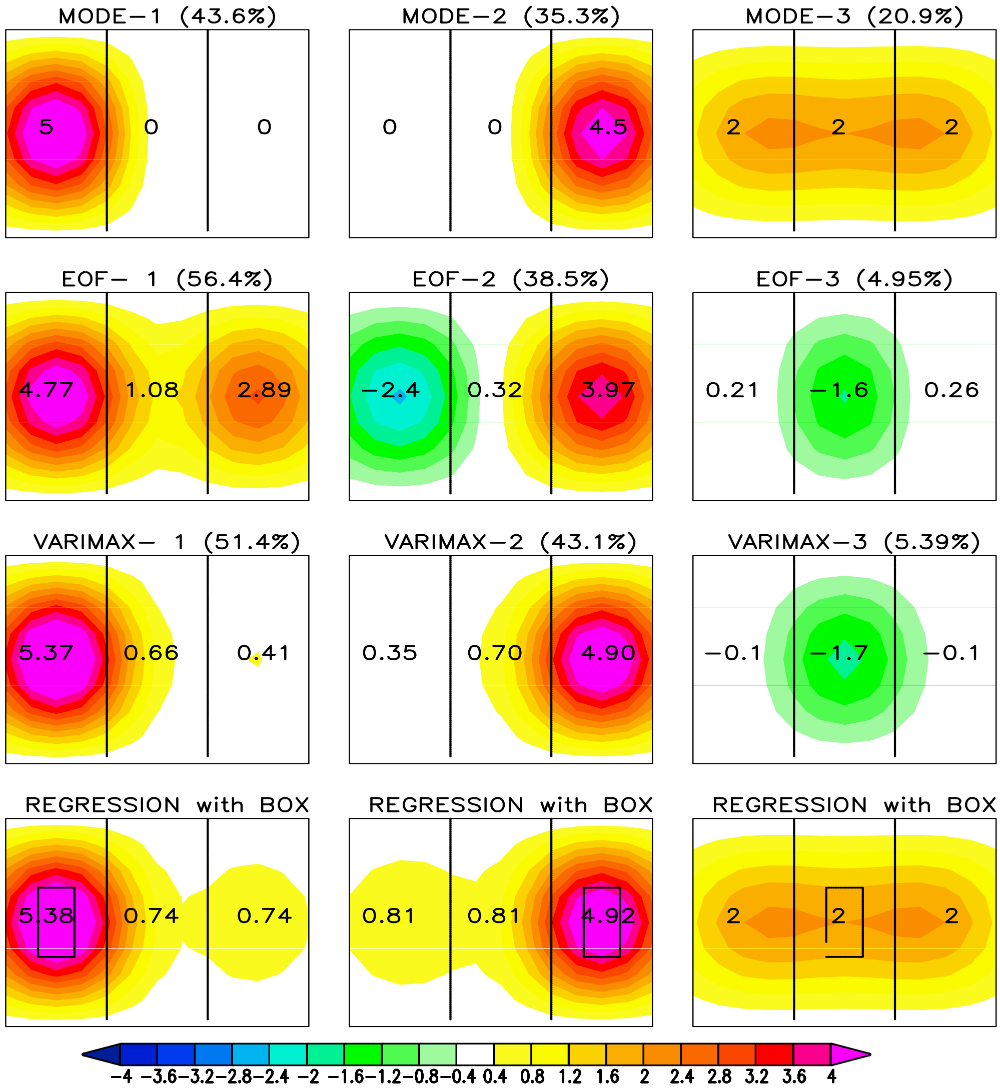

Figure 4: The physical modes (1st panel from top), EOF (2nd panel from top), VARIMAX (3rd panel from top) pattern and the regressions patterns of each coordinate with all coordinates (bottom panel) of the simple low-dimensional example are shown. The values plotted on top of the patterns represent the associated vectors and are indentical to the amplitudes of the patterns in the respective region.

represents a pattern which does not really exist in our simple example, so that it is completely artificial.

Usually the VARIMAX representation is calculated by using the EOF-patterns. Here we can directly calculate the VARIMAX representation from our basis vectors, since the basis vectors are already given with uncorrelated time evolutions, which is usually not the case in climatological data sets. Therefore the VARIMAX vectors are well-defined. The VARIMAX patterns and their explained variances are also shown in Fig. 4 and the coefficients $A_{1,2,3}$ for the PCs of the VARIMAX vectors are listed in Table 2.

principle component	A1	A2	A3
PC-1	0.74	0.40	0.54
PC-2	-0.56	0.81	0.6
PC-3	0.37	0.43	-0.82

Table 1: The coefficients $A_{1,2,3}$, by which the PCs of the EOFs vector are constructed.

principle component	A1	A2	A3
VPC-1	0.94	-0.06	0.33
VPC-2	-0.07	0.93	0.35
VPC-3	0.33	0.36	-0.87

Table 2: The coefficients $A_{1,2,3}$, by which the PCs of the VARIMAX vectors are constructed.

4 Discussion

We used three different statistical methods (EOF, VARIMAX and regression analysis) to identify the different variability modes in different multi-variate datasets.

In the following discussion we compare the results from the simple low-dimensional example with those from the other three examples using observed data. In the simple low-dimensional example we have three variability modes. The three modes can be interpreted as the 'real physical modes' of the domain. From a mathematical point of view all four representations of the simple low-dimensional example are equally valid, but from a physical point of view we would like to find the representation, which is most clearly pointing towards the 'physical modes' of the problem.

We constructed the simple low-dimensional example by two local and spatially orthogonal modes, which should represent some simple internal modes (see Fig. 4). The third mode in this example represents a domain-wide mode, which may be regarded, for instance, as the response of the domain to some kind of external influence. The third mode is not orthogonal in space with the other two, which will be important in the following discussion. By construction the simple low-dimensional example does not contain any statistical uncertainties, which allows us to determine the EOF and VARIMAX patterns exactly.

Although the Mode-3 is the weakest one in the simple low-dimensional example, the EOF-1 is very similar to it (see Fig. 4). Despite the fact that it captures some features of the two other basis modes, it may be interpreted as the domain response to some kind of external influence, similar to how Saji et al. (1999) have interpreted the EOF-1 of the tropical Indian Ocean. Although the EOF-1 in the simple low-dimensional example is very similar to the Mode-3, the PC-1 is a superposition of all three basis modes and the Mode-3 explains only 25% of total variance of the PC-1 (see Table 1). It can therefore be assumed that the EOF-1 of most domains exhibiting some weak domain-wide mode will look like this weak mode. The EOF-1, however, will be a superposition of many different modes.

In the tropical Indian and Atlantic Ocean SSTs this kind of weak external influence may be the ENSO response or a greenhouse warming trend, as expressed by the leading EOFs (see Fig. 1 and Fig. 2). In the Northern Hemisphere SLP such an external influence might manifest itself as an annular mode like the EOF-1 of Northern Hemisphere SLP (see Fig. 3).

On the other hand we would like to clarify that the EOF-1 does not need to be a superposition of many modes due to some weak domain-wide mode. If we would have chosen the Mode-3 as the most dominant mode in our simple example then the patterns of the EOF and VARIMAX analyses would not look much different compared to the one shown in Fig. 4, but the PC-1 would clearly be dominated by the Mode-3. It is, for example, well known that the EOF-1 of the tropical Pacific SST is really representing the El Niño mode.

The orthogonality constraint in space forces the EOF-2 of the simple low-dimensional example to be a domain-wide dipole, although the two centers of the dipole are not anti-correlated by construction (see Fig. 4). It can therefore be concluded that in a domain which has an EOF-1 pattern with a shape of a domain-wide monopole must have a dipole in the EOF-2. The dipole, however, is totally an artifact of the orthogonality constraint.

The EOF-3 and VARIMAX-3 patterns are interesting, since they indicate a kind of central mode which does not really exist. Interestingly, the time evolution of this mode is a superposition of all three basis modes. This leads to the fact that the PC-3 includes variability from the basis Mode-1 and Mode-2 which actually are not influencing this region at all (see Table 1 and 2).

In all examples presented here the regression patterns seem to be most instructive in representing the dominant modes of variability. However, the disadvantage of the regression analysis is that the choice of the index region is highly subjective and it is much easier to choose an index that is not instructive at all, than to choose the right index. For the SLP in the Northern Hemisphere, for instance, we could have chosen an index region over the north pole and the regression would look very much like the EOF-1 (the regression pattern is not shown, but see Fig. 3 for the EOF-1). Thus, the disadvantage of the regression analysis is its subjectivity so that one always needs to argue why a certain index has been chosen.

5 Conclusions

We have shown that EOF and rotated EOF analyses have problems in identifying the dominant centers of action or the teleconnections between these centers of action in multi-variate data sets. We therefore have to be very careful in interpreting the EOF or rotated EOF modes as physical modes.

The problems in interpreting the patterns derived from EOF and VARIMAX analyses arise from the basic assumptions that are made by these statistical methods, which are not identical to the assumptions that we make to derive the so called 'physical modes' of the problem. The EOF analysis always represents modes of variability that are orthogonal in time and space. The constraint of the orthogonality in space is often not consistent with the real nature of the problem, like in the simple example, in which the basis modes are not orthogonal in space (see Fig. 4). The VARIMAX analysis is looking for localized modes, which is also not instructive for representing the simple example, since the Mode-3 is highly non-local.

We would like to conclude our discussion with the following caveats for the interpretation of the results of the EOF or VARIMAX methods:

- The teleconnection patterns derived from the orthogonal analysis cannot be necessarily interpreted as teleconnections which are associated with a physical process (e.g. the dipole pattern Fig. 4).
- The centers of action derived from the EOF or VARIMAX methods do not need to be the centers of action of the real physical modes (see EOF-3 or VARIMAX-3 in Fig. 4).
- The PCs of the dominant patterns are often a superposition of many different modes, which are uncorrelated in time and which are often modes of remote regions that have no influence on the region in which the pattern of this PC has its center of action.

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References

Ambaum, M. H. P., B. J. Hoskins and D. B. Stephenson, 2000: Arctic Oscillation or North Atlantic Oscillation ? draft.

Chang, P Ji, L Li, H , 1997: A decadal climate variation in the tropical Atlantic Ocean from thermodynamic air-sea interactions . *Nature*,**385**,516-518.

Dommenget, D and Latif, M , 2000: Interannual to decadal variability in the tropical Atlantic . *J. Clim.*,**13**, 777-792.

Enfield, DB Mestas-Nunez, AM Mayer, DA Cid-Serrano, L , 1999: How ubiquitous is the dipole relationship in tropical Atlantic sea surface temperatures. *J. Geophys. Res.-Oceans*,**104**, 7841-7848.

Houghton, R. W. and Y.M. Tourre, 1992: characteristics of low-frequency sea surface temperature fluctuations in the tropical Atlantic. *J. Climate*, **5**, 765-771.

Kaiser, H. F., 1958: The Varimax criterion for analytic rotations in factor analysis. *Psychometrika*, **23**, 187.

Kalnay, E. Kanamitsu, M. Kistler, R. Collins, W. Deaven, D. Gandin, L. Iredell, M. Saha, S. White, G. Woollen, J. Zhu, Y. Chelliah, M. Ebisuzaki, W. Higgins, W. Janowiak, J. Mo, K. C.Ropelewski, C. Wang, J. Leetmaa, A. Reynolds, R. Jenne, Roy Joseph, Dennis, 1996: The NCEP/NCAR 40-year reanalysis project. *B AM METEOROL SOC* **77: (3)**, 437-471.

Reynolds, R.W., T.M. Smith, 1994: Improved global sea surface temperature analyses using optimum interpolation. *J. Clim.*,**7**, 929-948.

Richman, M. B.,1986: Review article: Rotation of principal components. *J. Climatology*, **6**, 293-335.

Saji, N. H.; B. N. Goswami; P. N. Vinayachandran; T. Yamagata; 1999: A dipole mode in the tropical Indian Ocean. *Nature*, Vol. 401, p. 360-363

Thompson, DWJ Wallace, JM , 2000: Annular modes in the extratropical circulation. Part I: Month- to-month variability . *J. Clim.*,**13**, 1000-1016.

Venzke, S Latif, M Villwock, A , 2000: The coupled GCM ECHO-2. Part II: Indian ocean response to ENSO . *J. Clim.*,**13**, 1371-1383.

von Storch, H. and G. Hannoschöck, (1985): Statistical aspects of estimated principal vectors (EOFs) based on small sample sizes. - *J. Climate Appl. Met*, **24**, 716-724.

Wallace, J. M. 2000: North Atlantic Oscillation/annular mode: Two paradigms - one phenomenon. *Q. J. Roy. Met. Soc.*, Vol. 126, pp. 791-805.