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Pion transition form factor in k_T factorization

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It has been pointed out that the recent BaBar data on the $\pi\gamma^* \to \gamma$ transition form factor $F_{\pi\gamma}(Q^2)$ at low (high) momentum transfer squared Q^2 indicate an asymptotic (flat) pion distribution amplitude. These seemingly contradictory observations can be reconciled in the k_T factorization theorem: the increase of the measured $Q^2 F_{\pi\gamma}(Q^2)$ for $Q^2 > 10$ GeV^2 is explained by convoluting a k_T dependent hard kernel with a flat pion distribution amplitude, k_T being a parton transverse momentum. The low Q^2 data are accommodated by including the resummation of $\alpha_s \ln^2 x$, x being a parton momentum fraction, which provides a stronger suppression at the endpoints of x. The next-to-leading-order correction to the pion transition form factor is found to be less than 20% in the considered range of Q^2.

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The asymptotic and soft behaviors of the pion transition form factor $F_{\pi\gamma}(Q^2)$ involved in the process $\pi\gamma^* \to \gamma$ have been derived [1]:

$$\lim_{Q^2\to\infty} Q^2 F_{\pi\gamma}(Q^2) \;=\; \sqrt{2} f_\pi \;=\; 0.185\,, \tag{1}$$

$$\lim_{Q^2\to 0} F_{\pi\gamma}(Q^2) \;=\; \frac{\sqrt{2}}{4\pi^2 f_\pi}\,, \tag{2}$$

Q^2 being the momentum transfer squared carried by the virtual photon, and $f_\pi = 0.131$ GeV the pion decay constant. The former is predicted by perturbative QCD (PQCD) in the collinear factorization theorem [2, 3, 4, 5, 6, 7], while the latter is determined from the axial anomaly in the chiral limit. However, the recent BaBar data on $F_{\pi\gamma}(Q^2)$ exhibits an intriguing dependence on Q^2 [8]: $Q^2 F_{\pi\gamma}(Q^2)$ exceeds Eq. (1) for $Q^2 > 10$ GeV^2, and continues to grow up to $Q^2 \approx 40$ GeV^2 as shown in Fig. 1. It has been commented [9] that this behavior cannot be explained by perturbative effects, like higher-order or higher-twist contributions.

It is known that one can extract nonperturbative information on the shape of the leading-twist pion distribution amplitude (DA) from the measurement of $F_{\pi\gamma}(Q^2)$ [2]. Inspired by the BaBar data, a model for the pion DA, differing from those investigated in [9], has been proposed [10],

$$\phi_\pi(x,\mu_0) = N + (1-N)6x(1-x), \tag{3}$$

with N being a free constant. This model remains finite at the endpoints of the momentum fraction $x \to 0, 1$ for the normalization point $\mu_0 = 0.6 \sim 0.8$ GeV. It was argued [10] that the pion DAs from the instanton theory of QCD vacuum [11, 12], from the the Nambu-Jona-Lasinio model [13], from the chiral quark model [14], and from the large-N_c Regge model [15] are expected to be rather flat. At the same time, the hard kernel, proportional to the internal quark propagator, was modified by introducing an infrared regulator m^2 [10]. Note that an endpoint singularity would be developed for the model in Eq. (3) without the infrared regulator. Then the factorization formula

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx\, \frac{\phi_\pi(x,Q)}{x + m^2/Q^2}\,, \tag{4}$$

leading to a logarithmic increase $\ln Q^2$, explains the growth of $Q^2 F_{\pi\gamma}(Q^2)$ observed by BaBar with the tuned parameters $N = 1.3 \pm 0.2$ and $m = 0.65 \pm 0.05$ GeV. The regulator m^2 was interpreted as the inverse instanton size, which sets the scale for nonperturbative effects in a quark propagator [10].

A similar conclusion on the shape of the leading-twist pion DA has been drawn from the BaBar data in [16], but with the difference in the formalism that the parton transverse momentum k_T was taken into account. The hard kernel is the same as in the collinear factorization, and the $\ln Q^2$ dependence comes from the integration of the pion wave function over the intrinsic parton k_T up to xQ [16]:

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{\sqrt{2} f_\pi}{6\pi} \int_0^1 \frac{dx}{x} \int_0^{xQ} k_T dk_T \, \psi_\pi(x, k_T) \, , \tag{5}$$

with the model

$$\psi_\pi(x, k_T) = \frac{2\pi\phi_\pi(x)}{x(1-x)\sigma} \exp\left[-\frac{k_T^2}{2\sigma x(1-x)}\right] . \tag{6}$$

The cutoff of k_T at xQ guarantees that the integration over x down to 0 is finite even for a flat distribution $\phi_\pi(x)$, with which the BaBar data are accommodated by choosing the parameter $\sigma = 0.53$ GeV^2. A concern was raised for a broad pion DA in the conventional collinear factorization [16]: the next-to-leading-order (NLO) correction to the hard kernel [17] is huge for the renormalization scale $\mu \sim Q$, or μ has to be much smaller than the scale $\Lambda_{\rm QCD}$ in order to diminish the NLO correction. For the calculation of the next-to-next-to-leading-order correction to the pion transition form factor in the collinear factorization, refer to [18].

The BaBar data are consistent with those of CELLO [19] and CLEO [20] below $Q^2 \approx 9$ GeV^2, from which an endpoint suppressed shape of the pion DA has been extracted [21, 22]. Taking into account high-power corrections, the asymptotic pion DA can also explain the CLEO data [23]. The QCD sum rule analyses of the pion DA and of the pion transition form factor led to an endpoint suppressed model [9, 21, 24, 25, 26, 27, 28]. It has been noticed [29] that the CLEO data favor the asymptotic form rather than the Chernyak-Zhitnitsky one [5], which emphasizes the endpoint region. The E791 di-jets data support the nearly asymptotic pion DA above $M_J^2 = 10$ GeV^2 [30], M_J^2 being the mass squared of the di-jets. A review on the determination of the leading-twist pion DA can be found in [31]. It seems that the measurements of the pion transition form factor at low and high Q^2 imply different shapes of the pion DA, which are contradictory to each other.

In this letter we shall study the process $\pi\gamma^* \to \gamma$ using the k_T factorization theorem [32, 33, 34, 35, 36, 37]. If a flat pion DA is favored, it will enhance the contribution from the region with small momentum fraction x. Once the small x region is important, the parton transverse momentum squared k_T^2, being of the same order as xQ^2 in the internal quark propagator, is not negligible. The regulator m^2 in Eq. (4) can be interpreted as k_T^2 at LO, whose inclusion smears the endpoint singularity from $x \to 0$. Besides, the loop correction to the virtual photon vertex generates the large double logarithms $\alpha_s \ln^2(Q^2/k_T^2)$ and $\alpha_s \ln^2 x$ in the small x region [38]. The former is absorbed into the k_T dependent wave function, and organized to all orders by the k_T resummation [35, 39, 40]. The latter is absorbed into a jet function, and organized to all orders by the threshold resummation [41, 42, 43]. With the above resummation factors, a flat pion DA does not develop an endpoint singularity.

The NLO correction to the hard kernel of the pion transition form factor has been computed in the k_T factorization [38]. Fourier transforming Eq. (44) in [38] into the impact parameter b_T space, we derive

$$\begin{aligned} F_{\pi\gamma}(Q^2) &= \frac{\sqrt{2} f_\pi}{3} \int_0^1 dx \int_0^\infty b_T db_T \, \phi_\pi(x) \exp[-S(x, b_T, Q, \mu)] \, S_t(x, Q) K_0\left(\sqrt{x} Q b_T\right) \\ &\quad \times \left[1 - \frac{\alpha_s(\mu)}{4\pi} C_F \left(\ln \frac{\mu^2 b_T}{2\sqrt{x} Q} + \gamma_E + 2\ln x + 3 - \frac{\pi^2}{3}\right)\right] , \end{aligned} \tag{7}$$

where $C_F = 4/3$ is a color factor and γ_E the Euler constant. The NLO term in the square brackets implies that the renormalization scale μ should be chosen as $\mu^2 \sim O(\sqrt{x} Q/b_T)$ in order to minimize the logarithm. However, we have always set $\mu = \max(\sqrt{x} Q, 1/b_T)$ in our previous analysis of exclusive processes [36], so we shall continue to adopt this choice here. It has been confirmed that the above two choices of μ produce almost identical results. The Sudakov exponent from the k_T resummation is given by

$$S(x, b_T, Q, \mu) = s\left(x\frac{Q}{\sqrt{2}}, b_T\right) + s\left((1-x)\frac{Q}{\sqrt{2}}, b_T\right) + 2\int_{1/b_T}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \tag{8}$$

where the explicit expression of the function s and the quark anomalous dimension γ_q can be found in [44, 45].

Q (GeV)	2	3	4	5	6
c	1.7	0.75	0.47	0.32	0.25

TABLE I: The power c in the threshold resummation at different Q^2.

The power $c \approx 0.3$ in the parametrization $S_t(x,Q)$ for the threshold resummation [46]

$$S_t(x,Q) = \frac{2^{1+2c}\,\Gamma(3/2+c)}{\sqrt{\pi}\,\Gamma(1+c)}[x(1-x)]^c, \tag{9}$$

was derived at the scale of the B meson mass $m_B = 5.28$ GeV. The power-law behavior should be modified as Q^2 runs within a large range in the present case. We determine the Q^2 dependence of the parameter c by repeating the procedure in Appendix D of [46], which involves the best fit of Eq. (9) to the exact resummation formula in the Mellin space N

$$S_t(N,Q) = \exp\left[\frac{1}{2}\int_0^{1-1/N}\frac{dz}{1-z}\int_{(1-z)}^{(1-z)^2}\frac{d\lambda}{\lambda}\gamma_K\left(\alpha_s\left(\sqrt{\lambda Q^2/2}\right)\right)\right], \tag{10}$$

with the anomalous dimension

$$\gamma_K = \frac{\alpha_s}{\pi}C_F + \left(\frac{\alpha_s}{\pi}\right)^2 C_F\left[C_A\left(\frac{67}{36}-\frac{\pi^2}{12}\right)-\frac{5}{18}n_f\right], \tag{11}$$

n_f being the number of quark flavors and $C_A = 3$ a color factor. The outcome is listed in Table I, where the rapid increase of c with decreasing Q arises from the exponentiated radiative correction proportional to $\alpha_s(Q^2)$. It becomes more difficult to determine c for $Q < 3$ GeV, because the allowed interval of N shrinks. Therefore, we freeze c at $c = 1$, when it exceeds unity. To simplify the analysis, we propose a parabolic parametrization

$$c \to c(Q^2) = 0.04Q^2 - 0.51Q + 1.87\,, \tag{12}$$

if $c \leq 1$. A large value $c \sim 1$, resulting in a quick falloff as $x \to 0$, will improve the agreement of our prediction with the low Q^2 data, when the flat model is adopted. That is, a careful treatment of the threshold resummation effect with a running $c(Q^2)$ is crucial for accommodating the BaBar data in both the low and high Q^2 regions.

We adopt Eq. (6) for the pion wave function, which is written as

$$\phi_\pi(x,b_T) = \phi_\pi(x)\exp\left[-\frac{1}{2}\sigma x(1-x)b_T^2\right] \tag{13}$$

in the impact parameter space. The Gaussian form in b_T^2 is the same as proposed in [29], and consistent with that from [47]. For the asymptotic model, $\phi_\pi(x) = 6x(1-x)$, the parameter σ can be fixed by the normalization condition [48]

$$\int_0^1 dx\int d^2b_T\,\phi_\pi(x,b_T) = \frac{3}{\pi f_\pi^2}\,, \tag{14}$$

giving $\sigma = 4\pi^2 f_\pi^2 = 0.677$ GeV^2. For the flat model, $\phi_\pi(x) = 1$, the above normalization condition does not apply, and the parameter σ will be tuned to fit the BaBar data. In the analysis below we choose $\sigma = 2.5$ GeV^2. Note that the renormalization-group evolution of the Gegenbauer expansion starting with the asymptotic form has been known. However, the evolution starting with the flat form has not yet been studied. Hence, the models adopted here should be regarded as being defined at a low normalization point μ_0.

The LO and NLO predictions for the pion transition form factor from the k_T factorization are presented in Fig. 1. It is observed that the curves for $Q^2F_{\pi\gamma}(Q^2)$ from the asymptotic pion DA saturates quickly for $Q^2 > 5$ GeV^2. That is, it is difficult to explain the continuous growth of $Q^2F_{\pi\gamma}(Q^2)$ above $Q^2 \approx 10$ GeV^2 using the asymptotic model. The overshoot in the low Q^2 region is expected, since this portion of data favors an endpoint suppressed shape [21, 22]. Note that the overshoot can be resolved within the theoretical uncertainty from

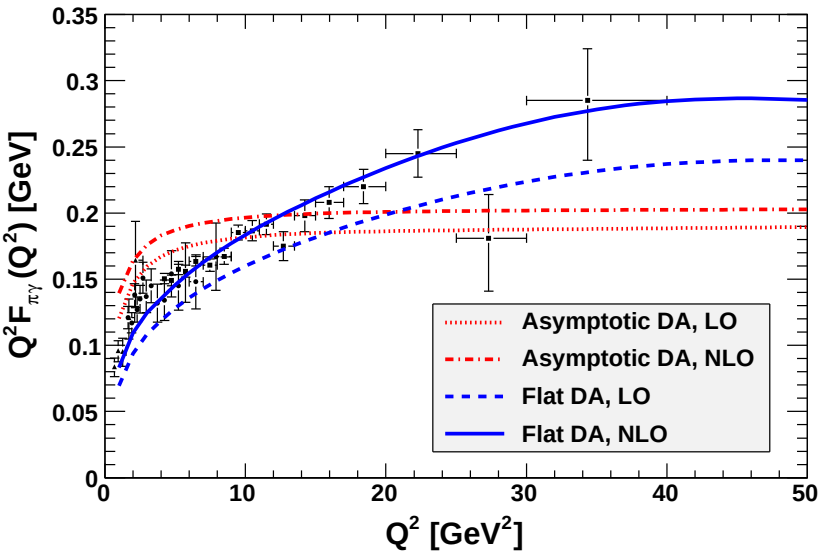


FIG. 1: Q^2 dependence of $Q^2F_{\pi\gamma}(Q^2)$ up to LO and NLO from the k_T factorization for the asymptotic (red, dotted and dot-dashed lines, respectively) and flat (blue, dashed and solid lines, respectively) pion DAs. The points with errors are the data from CELLO [19], CLEO [20], and BaBar [8].

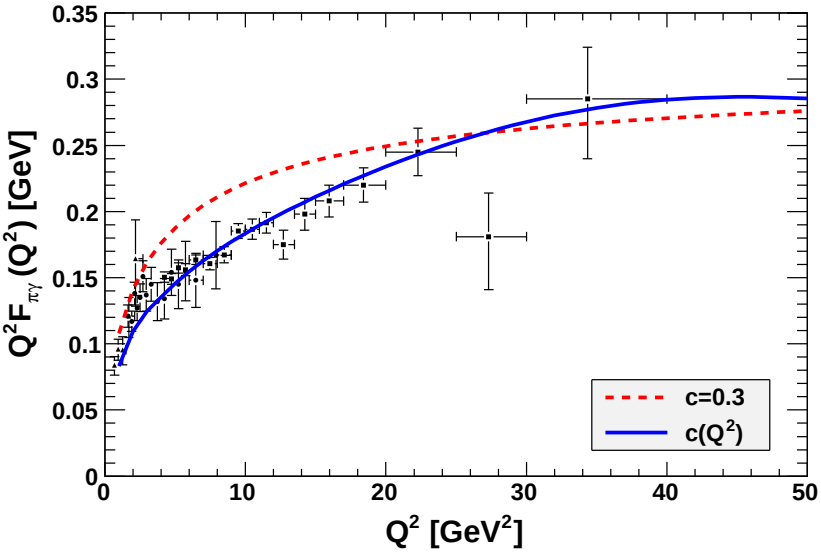
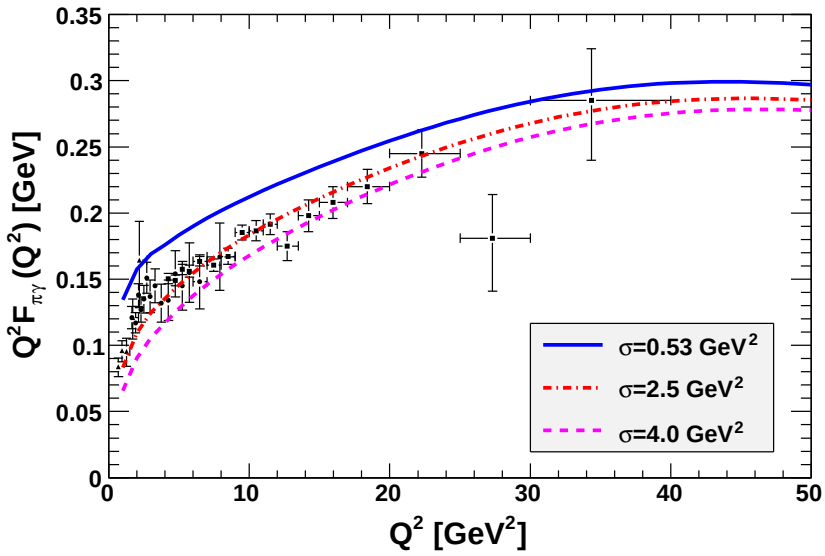
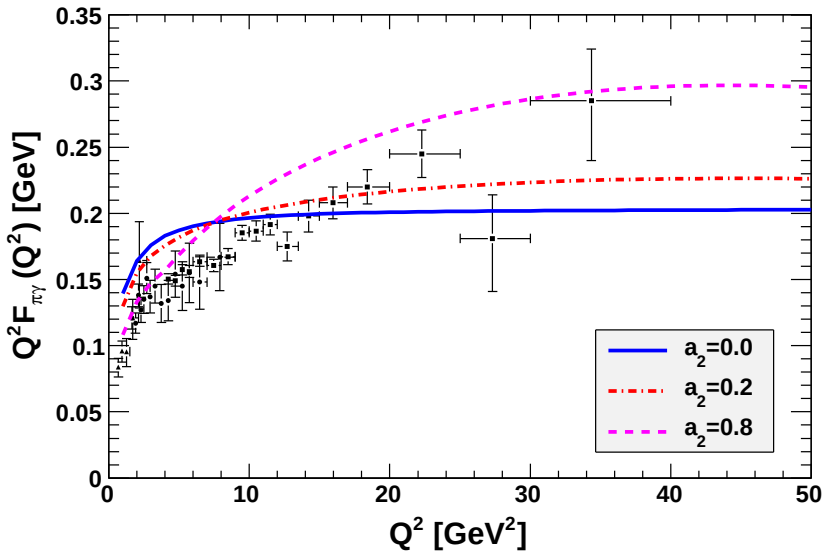


FIG. 2: Q^2 dependence of $Q^2F_{\pi\gamma}(Q^2)$ up to NLO with the power $c = 0.3$ (red, dashed line) and $c(Q^2)$ (blue, solid line) in Eq. (12).

varying parameters in our formalism. For example, multiplying the arguments $xQ/\sqrt{2}$ and $(1-x)Q/\sqrt{2}$ by a factor 2 in Eq. (8) [35] leads to stronger suppression at $x \to 0$, and better agreement with the data at low Q^2. The curves from the flat pion DA show a good fit to the BaBar data, whose logarithmic increase with Q^2 is a combined consequence of the inclusion of the parton k_T and the employment of the flat model. We stress that the fit would deteriorate in the low Q^2 region without a careful treatment of the threshold resummation effect. To highlight this effect, we compare the results from the flat model with a constant power $c = 0.3$ and with a



(a)



(b)

FIG. 3: Q^2 dependence of $Q^2F_{\pi\gamma}(Q^2)$ from (a) the flat model with different σ. and (b) Eq. (15) with different a_2.

running $c(Q^2)$ in Fig. 2. It is found that the former exceeds the data around $Q^2 \approx 10$ GeV^2. The results from the asymptotic model is almost independent of the running of $c(Q^2)$, because the asymptotic model decreases quickly enough at small x by itself. The NLO correction, amounting up to 20% for the flat pion DA as shown in Fig. 1, is under control in the k_T factorization. This observation is opposite to that postulated in [10], and to that obtained in the collinear factorization [16].

We should keep in mind that the BaBar data suffer from a large uncertainty in the high Q^2 region. In particular, the data point for $Q^2 = 27.31$ GeV^2 drops, and is consistent with the asymptotic limit in Eq. (1). Hence, we examine the dependence of our prediction on the parameter σ associated with the intrinsic b_T dependence in Eq. (13). The results are displayed in Fig. 3(a), indicating that the slope of the curves is

insensitive to the variation of σ especially in the high Q^2 region. If including the higher Gegenbauer term into the asymptotic model

$$\phi_\pi(x) = 6x(1-x)\left[1 + a_2\, C_2^{3/2}(2x-1)\right], \tag{15}$$

with the Gegenbauer polynomial $C_2^{3/2}(t) = (3/2)(5t^2-1)$, the slope of the curves changes significantly with a_2 as shown in Fig. 3(b). The prediction is enhanced by 10% for $a_2 = 0.2$ in the high Q^2 region, and saturates around $Q^2 \approx 10$ GeV^2. Only with an extremely large $a_2 \approx 0.8$, which also emphasizes the endpoint regions, the prediction becomes close to that from the flat pion DA. It has been mentioned [10] that a very broad pion DA, vanishing at the endpoints, but with a rapid increase ($\phi'_\pi(0)/6 \gg 1$), serves the purpose of accommodating the BaBar data. However, the big value $a_2 \approx 0.8$ may render the Gegenbauer expansion starting with the asymptotic model questionable, as claimed in [10]. In summary, the precise measurement of the pion transition form factor in the high Q^2 region will settle down the issue on the shape of the leading-twist pion DA: if the growth with Q^2 observed currently persists, the flat model is favored. If the growth becomes milder in the future, the asymptotic model with nonvanishing higher Gegenbauer terms will be still allowed.

Finally, we comment on the k_T factorization formalism developed in [42], which involves on-shell partons in the calculation of hard kernels. The usual procedure of factorizing a k_T dependent wave function is to neglect the minus component k^- in a hard kernel, and then to integrate out k^- in the general wave function that depends on the four components $k^+ = xp^+$, k^-, and k_T,

$$\psi(x, k_T) \equiv \int dk^-\, \Psi(x, k^-, k_T). \tag{16}$$

This is the reason why a parton, participating in the hard scattering, carries an off-shell momentum $(k^+, 0, k_T)$ in our formalism [49]. The prescription for calculating a k_T dependent hard kernel, the definition for a k_T dependent wave function, and the gauge invariance of the k_T factorization have been carefully addressed in [38, 49], which confront the criticism from [50, 51][1]. The transverse momentum squared k_T^2 appears in the hard kernel for the pion transition form factor through the internal quark invariant mass, $(k - P_\gamma)^2 = -xQ^2 - k_T^2$, for the final-state photon momentum $P_\gamma = (0, P_\gamma^-, 0_T)$ and $k^2 = -k_T^2$. Assuming on-shell partons, the hard kernel is independent of k_T^2 because of $(k - P_\gamma)^2 = -xQ^2$ for $k^2 = 0$. Therefore, it will be difficult to explain the BaBar data using the formalism of [42].

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[1] The authors of [51] rebutted our comments addressed in [49] on [50]. They claimed that the k_T factorization theorem for exclusive processes violates gauge invariance for the reasons: 1) Our method for the one-loop calculation of Fig. 2(b) in [49] leads to a wave function nonvanishing for $x > 1$, and violates the translation invariance, since the contribution from $q^+ < 0$ is non-zero, q being the loop momentum. 2) Our method depends on the contour chosen in the q^- integration. We disagree with both of them. In our method, the $q^+ < 0$ contribution comes only from the limit $q^+ \to 0$ from the left, which corresponds to $x \to 1$, and does not violate the translational invariance. As for the latter, the authors of [51] missed the point of including the semicircles, whose purpose is to avoid the ambiguity from $q^+ \to 0$, when the double pole $q^- = (q_\perp^2 - i\varepsilon)/(2q^+)$ moves to the infinity. If choosing the lower semicircle, the poles will cross the semicircle as $q^+ \to 0$, and the ambiguity comes back. Besides, the authors of [51] relied on the claim that all contributions from the semicircles in the q^- contour integration vanish in the limit of the semicircle radius $R \to \infty$, but it is incorrect. As explained in [49], these contributions do not vanish as $q^+ \to 0$. Note that the same problem has been discussed in literature, *e.g.* in [52]. Moreover, the reply in [51] is selective; *e.g.* there was no answer to our question raised in [49]: *why is the singularity from a highly off-shell gluon with $q^2 \to \infty$ a light-cone singularity?*

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