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The Cyclical Advancement of Drastic Technologies

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Drastic technological changes are cyclical because basic R&D is carried on only at times when entrepreneurial profits for incremental technologies of the prevailing technological paradigm fall close to zero. The model is essentially an endogenous technological change framework. Varieties, input to the final good production, are composite goods. Each composite good is produced by a set of intermediaries, outgrowths of basic R&D and applied R&D. The basic intermediate, product of basic R&D, is modeled as in Romer (1990). Complementary intermediates, the outgrowths of applied R&D, do show the property of falling profits. The falling character of profits implies that basic R&D becomes more yielding than applied R&D at certain points in time. Research people switch back and forth between the applied and basic research sectors, creating (endogenous) cycles in the advancement of drastic technologies and economic activity.

Keywords: GPT; growth cycles; basic R&D; applied R&D; economic growth

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1 Introduction

It has been first debated by Kondratieff (1926) that capitalism has long waves, regular fluctuations in economic life with a wavelength of 45-60 years. Schumpeter (1939) proposed that the cause of long-run cycles might involve *discontinuities* in the process of drastic technical innovation. Historical evidence indeed indicates that neither production nor technological progress is a smooth process, and that major innovations *tend to* appear in clusters in certain periods (Olsson, 2001; Gordon, 2000; Mokyr 1990; Kleinknecht, 1987; van Duijn, 1983; Mensch, 1979).

Given the significant effect of technological change on economic growth (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992), a better understanding of the reasons behind the cyclical evolution of output and technology is important from a policy perspective. In particular, smoothing out the cyclical advancement may bring about improvement in the long-run performance of an economy.

Surprisingly, however, the clustered appearance of drastic technologies has not received much attention in the growth theory. Relatively recently, David (1990) and especially Bresnahan and Trajtenberg (1995) have made the term general-purpose technology (GPT) popular to the growth theory. The main aim of this literature is to emphasize the difference between drastic technologies and incremental technological changes in terms of their growth implications. Currently, the focus seems to be on whether an economy experiences a slowdown at the onset of a new technological change due to reallocation of resources from the old to the new sectors or not (see several chapters in Helpman, 1998). Hence, the focus is on the temporary cyclical effects that may be created by new technological paradigms at the onset of their introduction to the economy.

In this paper we take a different focus. The aim of this study is to show *why* drastic technological change tends to proceed in a cyclical fashion and *how* the long-run growth process is affected by this. We conjecture that the main factor behind observing that drastic technological changes appear in clusters is eventually exhausting profit opportunities in incremental technologies of the existing technological paradigm.

The model we employ to substantiate our claim is essentially an extension of Romer (1990). The model consists of two R&D-sectors, labeled basic and applied, which respectively generate basic innovations for basic intermediary sectors and applied innovations for complementary intermediate sectors. In particular, we suppose that each basic innovation (*i.e.*, drastic technology change) leads to the emergence of *one* basic intermediary good and n complementary intermediary goods. These $n+1$ intermediaries are used in the production of a composite good, which becomes a variety in the production of final good. Indeed, each new composite good pushes upward the production frontier of the final good. There are two types of inputs in the model, physical capital and labor. Labor is further divided into three types, namely unskilled labor, skilled labor, and research labor, each of which is demanded only in one sector: unskilled labor enhances final good production (together with composite good varieties), skilled labor is used in the production of complementary intermediate sectors, and research labor is employed in R&D sectors. Finally, capital is used in the production of the basic intermediary good in the form of foregone output.

A good example to the idea that we advance here is perhaps the computer. Suppose that the microprocessor represents the GPT (basic technology) innovation and hardware and software are the complementary applied technology innovations. Producers of intermediaries, each a monopolist, purchase patents of these technologies. The basic intermediate sector uses capital to produce microprocessors and the complementary intermediaries use skilled labor to produce the hardware and software. The computer, the outgrowth of assembling the microprocessor, the hardware, and the software, is a composite good and a variety (input) in Gross Domestic Production (GDP).

The crucial aspect of the model is that it generates declining profits among n "varieties" in the complementary sector. That is, each additional complementary innovation yields lower monopoly profits. The monopoly profits of intermediate sectors are transferred to R&D people in the form of wages (cf. Romer, 1990) and researchers will continue to exploit positive profit opportunities of a prevailing technological paradigm by making incremental, non-drastic innovations. As profit opportunities become exhausted, at a certain point, it becomes more yielding to invest in a new technological paradigm. Researchers then switch to work on the next drastic innovation (technological paradigm). Incremental innovation resumes within the new

paradigm and endures until profit opportunities fall close to zero again. Thus, drastic technological change and economic development proceeds in long waves.

The model contributes to the (growth) literature in several ways. First, it develops a formal model of a mechanism that creates endogenous long-run fluctuations in economic activity. Second, it introduces asymmetry in the intermediate market, which is rarely done in the literature.[1] This paper shows that asymmetric profit opportunities in the intermediate sector(s) are a lot more than a detail. Indeed, our paper shows that the falling character of these profits is the genuine source of economic fluctuations. Third, the model contributes to the literature on elaborating the causes of a possible slowdown at the onset of a new GPT. As such, our model generates insights in policy options to pursue when trying to overcome the temporary economic decline when new GPT's are introduced. Last but not least, our model elaborates on the role of basic and applied R&D mechanisms in the growth process. It shows that the impact of these two R&D sectors in the long-run growth process is significantly different.

The organization of the paper is as follows. Section 2 introduces the production structure of the model. Section 3 solves the model at the "GPT" equilibrium, the equilibrium point where the stock of basic technologies is given. An important finding of this section is that profit opportunities in the complementary sectors are falling towards zero across the varieties. In section 4 we look at the long-run equilibrium and the R&D switching generated in the model. This section shows that the exhausting profits in complementary intermediary-goods sectors are the source of fluctuations in economic activity. Section 5 analyzes the growth implications of long-run business cycles. Section 6 summarizes our findings and concludes.

2 The Production Structure

Consider an economy where the final good Y production technology is represented by

[1] To our knowledge, van Zon and Yetkiner (2003) is the only work studying asymmetric intermediate sectors in endogenous technological framework.

$$Y = L^{1-\beta} \sum_{i=1}^{B} z_i^{\beta} \qquad 0 < \beta < 1 \qquad (1)$$

with L representing unskilled labor that is solely used in the production of the final good (say, GDP) and with z_i being a *composite good* that reflects the use of all technological paradigms $i = 1,2,...,B$ that are available. The higher the i, the more recent the GPT that a composite good (or any other variable) is associated with. The final good sector is furthermore a perfectly competitive market and $1-\beta$ indicates the partial output elasticity of unskilled labor.

Each composite good, or technological paradigm, is produced by $n+1$ intermediaries. Unlike most endogenous technological change models, we thus use a vector of *composite* rather than *single* inputs in the production function of the final good Y. This is in line with the distinction we will make below between the basic R&D sector, which invents drastic innovations that have a potential to grow into new technological paradigms, and the applied R&D sector, which produces many complementary innovations to make that happen. As such, we will consider one of the $n+1$ intermediaries as the basic or core intermediary, whereas all the other intermediaries are dubbed applied intermediaries.

The production function of composite goods is Cobb-Douglas. Hence, we assume:

$$z_i = \prod_{j=0}^{n} \left(x_{ij}\right)^{\alpha_j} \qquad \forall i = 1,2,...,B;\ \forall j = 0,1,...,n;\ \sum_{j=0}^{n} \alpha_j = 1,\ \alpha_j > 0 \qquad (2)$$

where x_{ij} is the jth intermediary used in the production of the ith composite input, and α_j indicates the *relative share* of jth input in the total product of composite good z_i. Equation (2) assumes implicitly that $\alpha_{ij} = \alpha_{i'j}$ for $\forall i, i' \in 1,2,..,B$. We need this assumption for a tractable solution. We will show that this assumption does not cause any symmetry within a GPT and across GPTs for complementary intermediaries and therefore is not as 'harmful' as it might be thought at first instance. We associate subscript 0 with the basic intermediary good and $1,2,...,n$ with complementary

intermediaries.[2] Consequently, α_0 and $\alpha_1,...,\alpha_n$ are interpreted as the respective relative shares of the basic and applied intermediary goods in the total production of a composite good. From now on, we shall use *0* and *j* to designate the core intermediary and complementary intermediary related variables and parameters, unless otherwise stated.

The number of complementary intermediaries, *n*, is a large positive integer, which is *constant* and *identical* across the composite goods. Hence, the model is 'forced' to generate the same number of intermediaries along GPTs. This is another assumption that we need in order to guarantee a tractable model. Given that *n* is a very large number, this assumption is by no means restrictive though. Also regarding the value of α_j, we make several assumptions. First, we assume that *complementary* intermediates are ranked such that $\alpha_j > \alpha_{j'}$ if $j < j'$, $\forall j, j' \in 1,2,..,n$. This assumption is not restrictive since it is a matter of reordering in a Cobb-Douglas technology. It is noteworthy in this respect that we do not impose any condition on the ordinal value of α_0. Moreover, the assumption contains that $\alpha_j \neq \alpha_{j'}$; that is, none of any pair of $(\alpha_j, \alpha_{j'})$ is alike. Second, we assume that α_n is at the *neighborhood* of zero, which is a reasonable assumption, given that (i) n is a large number, (ii) α_j are in descending order, and (iii) the sum of α_j is one. The intuition behind this reasoning will be clear as we progress.

The blueprints that are needed to be able to produce intermediary goods are forwarded by the R&D sector. We assume that each innovation, whether basic or applied, is the result of innovative activities from labor in that sector (labeled *R*). This labor is endowed with the frontier knowledge that is required to do research and can be engaged in basic or applied research. The determination of the specific activity the research labor engages in depends on the relative profitability of both types of innovations, which, in turn, depends on the profitability of adding applied intermediate goods to an already existing technological paradigm (for which the basic intermediary already exists) versus creating a completely new paradigm (for which a new basic intermediary is required). As we will show, the profitability of applied intermediaries falls the later it is introduced, so that there is a certain point at which

[2] Complementary intermediaries can be associated with "innovational complementarity" character of GPTs as advanced by Bresnahan and Trajtenberg (1995).

pursuing basic innovations are more profitable than doing applied research, and research labor shifts from doing applied research to doing basic research. Due to the stylistic nature of the model, there will be only corner solutions. This implies that R labor is either engaged in basic research or in applied research. The right 'down-to-earth' interpretation of this result is that "the intensity of research must be switching between applied and basic R&D".

We stylistically assume that blueprints of basic and applied intermediaries accumulate according to the following technologies:

$$B_{t+1} - B_t = \delta_B R_B B_t \qquad\qquad n_{\omega+1} - n_\omega = \delta_A R_A B_t \tag{3}$$

In these equations, t and ω represent time (see below for explanation), B_t is the stock of basic innovations at time t, δ_B and δ_A represent the productivity of the blueprint generation process for, respectively, the basic and applied innovations, and where R_B and R_A is the amount of research people used in generating blueprints either for the basic sector or for the applied sector (for the most recent GPT). The critical difference between the two blueprint accumulation functions is that the accumulation over time of the applied innovations is not a function of previous applied R&D efforts, irrespective of the 'age' of the paradigm, while the stock of basic technology is a positive externality for both accumulation functions. The motivation for this is that the outcome of applied research is assumed to be too specific to be *directly* useful for other applied research. A deeper reason behind this assumption is our perception that basic knowledge is the true engine of increasing productivity in an economy (this is accounted for in the applied R&D blueprint accumulation function by linking applied R&D productivity to B_t, the aggregate stock of knowledge in society).

Whatever the specific engagement of R&D labor, the output of research labor is always an innovation, which we assume is patented and which serves as an input to the production of intermediary goods. The costs of producing intermediate goods, therefore, include the costs of getting hold of the patent. Next to that we assume that the production of complementary intermediaries takes high-skilled labor (H), whereas the production of the basic intermediate good requires capital in the form of forgone output. Hence, the total costs of intermediate production can be portrayed as:

$$TC(x_{i0}) = P_{i,0} + r\eta x_{i0} \text{ and } TC(x_{ij}) = P_{i,j} + w_h h_{ij} \quad (4)$$

In these equations, $P_{i,j} \quad j = 0,1,...,n$ is the price of the patent of the ith GPT, r and w_h respectively denote the cost of capital and high-skilled labor, h_{ij} is the amount of *skilled labor* used in the production of applied intermediate x_{ij}, and ηx_{i0} stands for the units of resources in terms of foregone output that is required to produce x_{i0}.[3] Our motivation behind modeling the input use of the complementary intermediary sector different than of the basic intermediary sector is our perception that the production of a basic intermediary requires "something more fundamental" than the production of a complementary intermediary. We capture this difference by differentiating their input needs. If we continue with our computer example, the production of the processor (i.e., the basic intermediate) requires immense investment in resources that currently only two firms, under the big dominance of one, can operate globally. On the other hand, we observe many firms are able to produce a complementary intermediary, which indicates the 'easiness' of its production in terms of resources required.

This concludes the description of the production side of the economy. To sum up, we have three distinct types of labor that one way or the other all contribute to the economy's final good production. In a way, final goods production starts with research labor *R*, which is engaged in either basic or applied innovative activities. This generates patented ideas for drastic or applied intermediary goods, which are produced at a certain capital cost (basic intermediaries) or by means of high-skilled labor *H* (complementary intermediaries). Any basic intermediary, along with its outgrowth of applied intermediaries, serves as a distinct, composite input –labeled a technological paradigm– for the production of final goods. Finally, low-skilled labor *L* is needed to transform all technological paradigms into final goods.

Before we proceed, it is instrumental to discuss how we perceive time in our model. This is important since in our set-up we have basic innovations that need time to grow into paradigms by means of having applied intermediaries (*i.e.*, the evolvement over time of *n*), whereas the model also features discrete growth steps

[3] We will change the notation of $P_{i,j}$ slightly in Section 4. For presentational purposes, we denote in that way at this introductory level.

when new paradigms evolve (the process by which B changes over time). In our discussion we will therefore consider three concepts of time. First, there is real time or calendar time, denoted by s, which is continuous and is used in usual way to, for instance, assess the evolvement of GDP over time. Second, we index the time points at which the model-economy realizes jumps in the drastic technology stocks by t and call it *GPT-time*. The difference between t and $t+1$ is therefore the real time needed to complete a new paradigm; that is, to get from B_t to B_{t+1}.[4] Third, we use the concept of *applied R&D time*, to be denoted by ω. These are time points on the real time line between t and $t+1$ that index the evolvement of applied innovations. As we show, basic R&D and applied R&D do not take place simultaneously but follow another under an endogenous switching mechanism. This is illustrated heuristically in figure 1 below

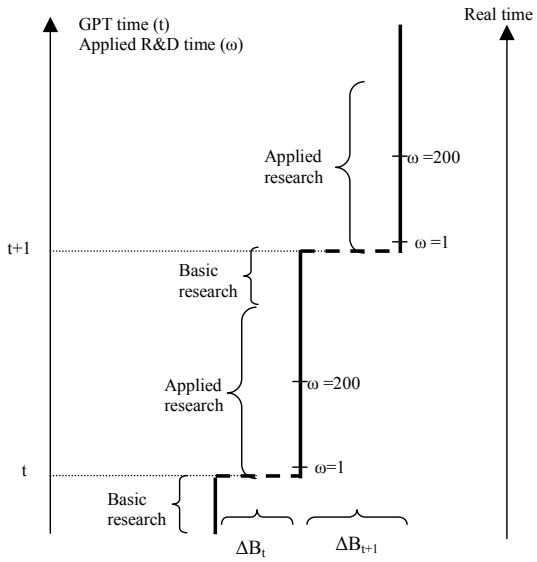

Figure 1. Associating inventions with real time

For discussing equilibria in our model, this implies that we may distinguish between types of equilibria as well. First, we can distinguish production equilibrium, which gives all relations between the endogenous variables that should hold at any

[4] Note that each GPT time-block includes the invention of blueprints first for complementary intermediaries and next for the basic intermediary. We find this timing more useful as the inclusion of the next-generation basic technology does not change the interpretation.

GPT time, given wages and given the cost of patents. These are typically the conditions that result from profit maximization in final goods and intermediate goods production. Next, we may distinguish a 'market equilibrium', which also determines the wages that should hold, but still ignores the evolvement over time of technological paradigms. Together, we call these two equilibria *GPT equilibrium*. This equilibrium will yield a specification for the final output of the economy as a function of factor endowments, the rental cost of capital and the number of technological paradigms. Third, and most interesting, we can consider the long-run equilibrium. This is the equilibrium that also incorporates the progress of B over time, thus identifying the real time evolvement of Y as a function of exogenous variables only. Finally, we may differentiate the equilibria under intertemporally optimized preferences and under exogenously determined consumption assumptions. In the latter case, the interest rate r is constant and identical, which is consistent with the stylized facts of growth, at least in the long-run equilibria.

3 The GPT Equilibrium

To determine the GPT equilibrium, we first identify the economic relations that should hold between the alternative phases of final goods production that we have dubbed production equilibrium. A representative firm's profits are

$$\Pi_Y = L^{1-\beta}\sum_i z_i^{\beta} - \sum_i p_i z_i - w_L L \tag{5}$$

where we have normalized the price of Y to one and where p_i and w_L respectively denote the user cost (price) of the composite input z_i and unskilled labor L. First order conditions with respect to z_i and L are

$$p_i = \beta L^{1-\beta} z_i^{\beta-1} \tag{6}$$

$$w_L = (1-\beta)L^{-\beta}\sum_i z_i^{\beta} \tag{7}$$

These equations can be used to determine the inverse input demand function for any intermediate product by linking them to profit maximization in composite good production. To do that let us suppose that the intermediary-good prices are denoted by $(q_{i0}, q_{i1}, ..., q_{in})$, in which the first price is associated with the core sector, x_{i0}, and others are associated by the complementary sector, $(x_{i1}, ..., x_{in})$. Then, total cost corresponding to the composite good i is $C_i = \sum_{j=0}^{n} q_{ij} x_{ij}$. Minimizing total costs subject to equation (2) yields

$$q_{ij} \cdot x_{ij} = \lambda_i \alpha_j z_i \qquad j \in 0,1,..,n \qquad i \in 1,2,..,B \tag{8}$$

The summation of equation (8) over j gives $C_i = \lambda_i z_i$. That is, the cost of producing the composite intermediate z_i is the shadow price of composite input times quantity. Hence, λ_i works also as a unit-price p_i of composite input i.

Substituting the optimum condition for the jth intermediary of the ith GPT, x_{ij}, from equation (8) into equation (2) gives

$$\lambda_i = \prod_{j=0}^{n} \left(\frac{q_{ij}}{\alpha_j} \right)^{\alpha_j} . \tag{9}$$

This shows that the shadow price of the ith composite input, λ_i, is a kind of geometric average of intermediate-good prices weighted by their respective input shares. Note that equation (9) is a straightforward extension of a two-input cost minimization problem under Cobb-Douglas technology.

Using equations (6) and (9) in equation (8) gives the inverse input-demand function for any intermediate good

$$x_{ij} = \left(\frac{\alpha_j}{q_{ij}} \right) \beta^{\sigma} L \prod_{k=0}^{n} \left(\frac{\alpha_k}{q_{ik}} \right)^{\alpha_k (\sigma - 1)} \tag{10}$$

where $\sigma = 1/(1-\beta) > 1$ is the inverse of partial output elasticity of unskilled labor.

Profit maximization in the intermediary sector is handled à la Romer (1990). Let us first consider the core sector, indexed by 0. The derived demand function of the core sector, x_{i0}, by using equation (10), is

$$x_{i0} = \left(\frac{\alpha_0}{q_{i0}}\right)^{1+(\sigma-1)\alpha_0} \beta^{\sigma} L \prod_{k=1}^{n} \left(\frac{\alpha_k}{q_{ik}}\right)^{\alpha_k(\sigma-1)} \tag{11}$$

As equation (11) indicates, x_{i0} is inversely related with its own price. Throughout this study, we assume that prices of other intermediary goods (complementary goods in this case) do not have any (cross) price effect.

Following Romer (1990, pp. S85-S88.) we assume there is a monopolist holding patent rights of the basic intermediary associated with a GPT. Given the cost structure of intermediate good production (cf. equation 4), the profit equation of any intermediary firm in the core sector is

$$\pi_{i0} = q_{i0}x_{i0} - r\eta x_{i0} \tag{12}$$

where we recall our assumption that each unit of production uses ηx_{i0} units of resources in terms of foregone output. Profit maximization leads to the well-known markup over unit cost pricing condition:

$$q_0 = r\eta \frac{\varepsilon_0}{1+\varepsilon_0} = r\eta\phi_0 \tag{13}$$

In (13), $|\varepsilon_0| = 1 + (\sigma-1)\alpha_0 > 1$ is the own price elasticity of input x_{i0}, and markup rate $\phi_0 = 1 + \frac{1}{(\sigma-1)\alpha_0}$ is greater than one ($\phi_0 > 1$). It must be noted that the price of the core-intermediary is symmetric along 'generations' only if the rental cost of capital r is identical along the generations. Finally, we note that there is an inverse hyperbolic relationship between ϕ_0 and α_0 such that ϕ_0 is monotonically declining in α_0, i.e., $\partial\phi_0 / \partial\alpha_0 < 0$.

For the complementary sector, indexed by $1,2,...,n$, the results of profit maximization are to a large extent similar. When a GPT and the basic intermediate of that drastic technology appear in the market, the idea but the patent is a public good. If profit opportunities in the intermediate market are sufficiently high, then blueprints of complementary goods will be developed by the applied R&D sector. Using these blueprints, monopolists of the intermediate sector produce complementary intermediaries.

We recall that the main input in the production of complementary intermediaries is skilled labor. We assume one unit of skilled labor produces one unit of complementary-intermediate, $x_{ij} = h_{ij}$, $\forall j \in 1,2,...,n$, where h_{ij} is the amount of *skilled labor* used in the production of intermediate good x_{ij}. Perfect factor mobility across complementary sectors within each GPT and across GPT sectors implies a single factor price w_h in the complementary sector, and the profit maximization leads to:

$$q_{ij} = w_h \phi_j \qquad j = 1,....,n \tag{14}$$

where $\phi_j = 1 + \frac{1}{(\sigma - 1)\alpha_j} > 1$. As before, the inverse hyperbolic relationship holds between ϕ_j and α_j. Given our assumptions on α_j (*cf.* Section 2), this implies that there is an inverse relationship between the "order of appearance" of complementary intermediates in the market and the markup rate. That is, the later a complementary input enters the market, the higher its mark-up will be. To see this intuitively, recall that $\alpha_j > \alpha_{j'}$ if $j < j'$ and that α_n is at the neighborhood of zero. Consider now x_n. Its relative input share in total product of composite input is at the neighborhood of zero but it is marginally the most critical input in the sense that the production of the composite good is impossible without it, though all other core and complementary inputs could have been produced. In other words, relatively speaking, x_n has the highest importance among all complementary intermediates in the production of the composite good. Therefore, the markup over unit cost is the highest, though it is the last in the order of appearance. Economically, this also makes sense, since later complementary sectors have lower input shares in the total product of the composite

good and therefore face lower price elasticities. Therefore, relatively speaking, they can charge higher prices for their intermediaries to exploit the positive profit opportunities of their product.

Using (14) in (10) gives[5]

$$x_{ij} = \left(\frac{\alpha_j}{\phi_j}\right)\beta^{\sigma} L\left(\frac{\alpha_0}{r\eta\phi_0}\right)^{\alpha_0\beta\sigma}\left(w_h\right)^{-[1+(1-\alpha_0)\beta\sigma]}\prod_{k=1}^{n}\left(\frac{\alpha_k}{\phi_k}\right)^{\alpha_k\beta\sigma} \tag{15}$$

Equation (15) shows the inverse relationship between demand for any intermediate good and the rental costs of inputs in the production of intermediaries.

This finalizes the production equilibrium relations that should hold in our economy. To get to market equilibrium (*i.e.*, GPT equilibrium), recall that we assumed the use of skilled labor is limited to complementary sector. Under this assumption, for given supply, it is straightforward to calculate 'sector-specific' rental price of skilled labor w_h.

Let us suppose that we are at GPT equilibrium, the state that a cluster of new composite goods (a cluster of basic intermediaries together with their complementary inputs) has been just added to the production frontier. Then, the demand-supply equilibrium of skilled labor in the complementary sector would be

$$H = \sum_{i=1}^{B}\sum_{j=1}^{n} h_{ij} = \sum_{i=1}^{B}\sum_{j=1}^{n} x_{ij} \tag{16}$$

Using (15) in (16) gives the equilibrium wage rate for skilled labor for given H, L, and r:

$$w_h = \beta^{\sigma\chi} L^{\chi}\left(\frac{\alpha_0}{r\eta\phi_0}\right)^{\alpha_0\beta\sigma\chi}\left(\frac{G_2 B}{H}\right)^{\chi} G_1^{\chi} \tag{17}$$

[5] Note that $\sigma - 1 = \beta\sigma$.

where $\chi = \frac{1-\beta}{1-\alpha_0\beta}$, $G_1 = \prod_{k=1}^{n}\left(\frac{\alpha_k}{\phi_k}\right)^{\alpha_k\beta\sigma}$, and $G_2 = \sum_{k=1}^{n}\left(\frac{\alpha_k}{\phi_k}\right)$. Note that (i) $0 < \chi < 1$ due to the fact that $\alpha_0\beta < \beta$, (ii) G_1 and G_2 are constants due to our assumptions that n is constant and identical across GPTs and that $\alpha_{ij} = \alpha_{i'j}$ for $\forall i, i' \in 1,2,...,B$, and (iii) $G_1 < 1$ and $G_2 < 1$.[6]

From equation (17) we infer that skilled labor wages increase as the stock of GPTs rises for given L, H, and r. This is a 'normal' result in the sense that, as new GPTs are introduced, more intermediaries use the same (given) resource. Moreover, an increase in H or a decrease in L will lower skilled wages. An (exogenous) increase in the supply of skilled labor will certainly have a direct impact on its own price. The latter is the result of a rather indirect mechanism. A decrease in L lowers the 'demand for composite inputs' due to lower final good production. Consequently, the demand for complementary inputs is undercut and hence wages for skilled labor decreases.

The equilibrium price of a complementary product q_j mimics the skilled labor wage (cf. equation (14) and (17)). However, we recall that complementary-goods prices are "asymmetric" along varieties within a GPT because q_j is a function of input-share parameters. Thus, 'later' complementary intermediates charge higher prices. As we explained before, this makes intuitively sense since an intermediary that enters the market later becomes pivotal in finishing the composite good, which is captured in the model by an increase in monopoly power.

The equilibrium value of each complementary intermediate can be calculated by using equations (15) and (17):

$$x_j = \left(\frac{\alpha_j}{\phi_j}\right)\frac{H}{BG_2} \tag{18}$$

[6] To see this, note that $\sum_{j=1}^{n}\alpha_j = (1-\alpha_0)$, by definition. Then, given the fact that $\left(\alpha_j/\phi_j\right) < \alpha_j$, necessarily, $\sum_j(\alpha_j/\phi_j) < (1-\alpha_0) < 1$ and $\prod_{j=1}^{n_i}\left(\frac{\alpha_j}{\phi_j}\right)^{\alpha_j} < 1$. Similarly, given the fact that

Three characteristics of equation (18) are in order. First, equilibrium values of intermediaries are dissimilar within a GPT (but identical along GPTs). The first term in the parenthesis on the right hand side of the equation is the source of asymmetry across complementary goods. Second, the equilibrium value rises with α_j. It is straightforward to see this result by checking $\frac{\partial(\alpha_j / \phi_j)}{\partial \alpha_j}$, which is positive. In other words, the earlier the intermediate appears in the market, the higher its equilibrium output level. Note that this is inline with our earlier intuition that earlier entrants have less monopoly power. Third, the output of complementary intermediaries increases with *H/B*. This is plausible, given that (i) we assumed the number of complementary intermediaries per GPT constant, and (ii) complementary intermediaries only use high skilled labor. As a consequence, *H/B* is a direct proxy of the output level of individual firms in the complementary intermediate goods sector.

The profits of the jth firm in the ith GPT (in the complementary intermediaries) is found by substituting the respective values of w_h and x_j from (17) and (18) in profit equation $\pi_j = (\phi_j - 1) \cdot w_h \cdot x_j$:

$$\pi_j = \left(\frac{1}{\sigma - 1}\right)\left(\frac{1}{\phi_j}\right) L^{\chi} \beta^{\sigma\chi} \left(\frac{\alpha_0}{r\eta\phi_0}\right)^{\alpha_0 \beta\sigma\chi} \left(\frac{H}{BG_2}\right)^{1-\chi} (G_1)^{\chi} \tag{19}$$

The most obvious characteristic of profits in equation (19) is its falling nature in input shares. Recall that we assumed α_j are ranked in a descending order. Thus, *the later the intermediate appears, the less the profit it earns*, according to equation (19). Accordingly, whereas prices are higher for firms that enter later, the equilibrium level of output is also lower, such that lower profits result. The reasoning for this goes back to the inverse relationship between the order of appearance of an intermediate and its relative importance in finishing the composite output. As we know, this leads to a higher mark-up over marginal cost, but also to lower (monopoly) output levels. In

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$\left(\frac{\alpha_j}{\phi_j}\right)^{\alpha_j} < \left(\frac{\alpha_j}{\phi_j}\right)$ for any α_j, it is then always true that $G_1 < G_2$.

addition, the output share by itself is lower for later entrants. As a consequence, it is not surprising that profits decline.

What is the importance of this finding? Under perfect foresight assumption, entrepreneurs in the complementary intermediate market would be aware of the *profit opportunities* of all intermediaries 1 to n. Then, a monopolist would prefer to produce the intermediate that promises the highest profit opportunity among n varieties. Hence, the order of appearance of intermediaries is function of the order of size of input shares. The assumption we made initially that input shares were ordered in a descending form therefore reflects the declining market opportunities in the complementary sector.

Finally, we can calculate x_0. Using equations (11) and (17), x_0 is[7]

$$x_0 = \beta^{\sigma\chi} \cdot L^{\chi} \cdot \left(\frac{\alpha_0}{r\eta\phi_0}\right)^{\sigma\chi} \cdot \left(\frac{H}{BG_2}\right)^{1-\chi} G_1^{\chi} \tag{20}$$

This is the equilibrium of x_0. Note that x_0 implies the following equilibrium profit for the basic intermediate (cf. equation (12)):

$$\pi_0 = r\eta \cdot (\phi_0 - 1) \cdot \beta^{\sigma\chi} \cdot L^{\chi} \cdot \left(\frac{\alpha_0}{r\eta\phi_0}\right)^{\sigma\chi} \cdot \left(\frac{H}{BG_2}\right)^{1-\chi} (G_1)^{\chi} \tag{21}$$

Following x_0, π_0 are similar across the core sectors (*i.e.*, along the GPTs).

As we now have all information concerning the composite good, we can proceed to find the equilibrium values of 'aggregate variables' for the GPT equilibrium. Employing (18) and (20) in equation (2) gives us z_i. Using this value in (1), we can show that final output Y is[8]

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[7] It is helpful to see (i) $-(1-\alpha_0)\beta\sigma = -((1-\chi)/\chi)$, (ii) $1+\alpha_0\beta\sigma\chi = \sigma\chi$, and (iii) $1-\alpha_0\chi = (1-\alpha_0)\sigma\chi$.

[8] We can calculate aggregate capital and check if the ratio of the two is constant, fitting to stylized facts. Aggregate capital is obtained by summing x_0 along the GPTs, $K = \eta \cdot \sum_i x_0$. It is

$$Y = \beta^{\alpha_0 \beta \sigma \chi} L^{\chi} \left(\frac{\alpha_0}{r \eta \phi_0} \right)^{\alpha_0 \beta \sigma \chi} \left(\frac{H}{G_2} \right)^{1-\chi} (G_1)^{\chi} B^{\chi} \tag{22}$$

Equation (22) is not very much different than any reduced form final output production function but is richer. First, the "technological variety" variable B is the source of endogenous growth in the model, very much like the "love of variety" variable in Romer (1990). The basic difference is that B pushes the output frontier forward cyclically (that we will show in the next section). The fundamental similarity with the existing literature is that the growth rate of B is function of level of R&D people employed in the basic R&D. We will pursue this point further in the next section. Second, unskilled labor and skilled labor are (exogenous) sources of growth of output, supposing these variables are allowed to grow over time. Third, though applied R&D plays a critical role in terms of producing new composite varieties, it does not play any explicit role in the advancement of output growth. Hence, our model suggests that we need to reach a better understanding of the way several elements (R&D, H, L) contribute to growth and development, which is indeed we will turn to now.

4 Long-run Equilibrium and R&D Switching

The sequence of long-run equilibrium points is generated by the R&D sectors in our model. Recall that we assumed that basic and applied research sectors use *research labor*, a special type of labor endowed with frontier knowledge, in generating blueprints. The two R&D sectors compete for the 'scarce' research labor in the model. In this section we will show that this competition is linked to falling profit opportunities in the intermediate market, and that therefore drastic technologies are advanced in clusters.

With respect to the generation of blueprints in the basic intermediate sector, we recall that they accumulate according to the following difference function:

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straightforward to show that $K / Y = \alpha_0 \beta / r\phi_0$. The ratio is constant for a constant r, which must be true, at least at long-run.

$$B_{t+1} - B_t = \delta_B R_B B_t \tag{23}$$

where as before B_t denotes the stock of basic innovations at time t and δ_B is a productivity measure of research people employed in the basic sector (R_B). Recall that the time index t identifies the moments in real time when basic innovations emerge. The way we defined the GPT generation mechanism is a simple difference equation and its solution is $B_t = (1+\delta_B R_B)^t$. The mechanism generates (discrete) perpetual growth. In particular, the stock of GPTs increase at increasing rates at equal time distances. This result can be rationalized by the public good character of ideas (cf. Romer (1990)).

The dynamics of the applied R&D sector are substantially different from that in the basic sector, even though the blueprint accumulation function resembles the accumulation function of the blueprints for the basic intermediary goods. We recall that the development over time of applied innovations does not depend on applied R&D efforts of previous GPTs, nor on the current applied R&D activities. The outcome of the applied R&D research is too specific to be *directly* useful for other applied research, though it is indirectly via the knowledge spill-overs that are reflected in B_t. We conjecture the applied R&D accumulation function as follows:

$$n_{j,\omega+1} - n_{j,\omega} = \begin{Bmatrix} \delta_A R_A B_t & if & n_j \le B_{t+1} - B_t \\ 0 & & otherwise \end{Bmatrix} \qquad j = 1,2,... \tag{24}$$

where n_j denotes the stock of applied blueprints for the latest GPT bundle, δ_A represents the productivity of research labor employed (R_A). Note that the time index we use is now ω (applied R&D time), which identifies the moments in real time when applied innovations materialize. Equation (24) says that the innovation process for the jth blueprint will stop when each GPT (in the new bundle) gets one.

The blueprint generation mechanism in equation (24) is a simple difference equation and its solution is

$$n_{j,\omega} = \begin{Bmatrix} \omega \cdot \xi \cdot R_A \cdot B_t & if & n_j \le B_{t+1} - B_t \\ 0 & & otherwise \end{Bmatrix} \qquad j = 1,2,.... \qquad (25)$$

given that $n_{j,0}$ is zero for all j. According to equation (25), blueprints accumulate as a linear positive function of the amount of research labor used as long as it is less than number of GPTs produced in the most recent basic R&D activity. For clarity, we would like to illustrate equation (25) with an example. Suppose that the economy has just produced nine new GPT blueprints. Then, according to equations (24) and (25), the applied R&D has to produce nine units of blueprints for the first ($j = 1$) complementary intermediary, nine units of blueprints for the second complementary intermediary ($j = 2$), and so on. Furthermore, suppose that the applied R&D sector can produce three units of blueprint per applied R&D time ω in accordance with equation (25). Then, the graphical illustration of equation (25) would be as in Figure 2:

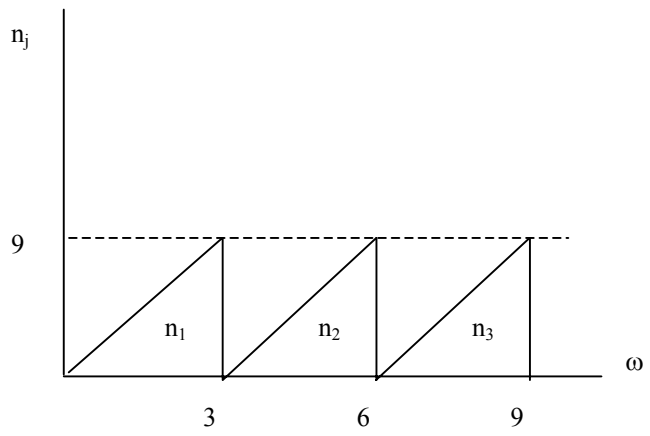

Figure 2. Blueprint accumulation in Applied R&D (an example)

As both types of blueprint generation mechanisms require R&D labor in order to generate new blueprints, they are in competition with each other in attracting *R*. As usual, the proceeds of blueprints are paid as wages in the relevant R&D sector whenever R&D is undertaken. This implies that the wages paid in both sub-sectors of

the R&D sector will play a decisive role in the distribution of R over both R&D activities. These, in turn, depend on the profitability of each sub-sector.

Suppose that B_t has already been invented (thus given). The profits for the *next* basic R&D activity $\pi_{t+1,0}$ would be

$$\pi_{t+1,0} = P_{t+1,0}(\delta_B R_B B_t) - w_{t+1,0} R_B \tag{26}$$

where $P_{t+1,0}$ is the price of a basic design in the next bundle, and $w_{t+1,0}$ represents the wage rate of R&D labor in the basic R&D sector for the next generation of GPTs. Subscript *zero* indicates that the variable is related to basic R&D, and subscript $t+1$ shows that drastic inventions are made between times $t+1$ and t. The equilibrium process yields

$$w_{t+1,0} = P_{t+1,0}\delta_B B_t, \tag{27}$$

a condition that must be satisfied when research staff is ever to be employed in the basic R&D. Note that the stock of B_t is taken as given as anyone engaging in basic research can freely take advantage of the entire existing stock of GPT blueprints.

Suppose again that B_t has already been invented. The profits of the jth design will be $\pi_j = P_{t,j}\delta_A B_t R_A - w_{t,j} R_A$ and the equilibrium process produces

$$w_{t,j} = P_{t,j}\delta_A B_t. \qquad j = 1,2,.... \tag{27}$$

where $P_{t,j}$ is the price of the jth complementary-good design, $w_{t,j}$ is the rental rate of R&D labor in the jth design, and t indexes the prevailing GPT bundle generated at times t and $t-1$. Equation (28) gives the wage rate w_j that the applied R&D sector must pay in order to undertake research in the sector for activity j.

The comparison of wages between both types of R&D activities, therefore highly depends on the prices that new blueprints yield. In our set-up, the unit value of a new blueprint is equal to the present discounted value of profit stream generated in the intermediary sector, given that R&D sectors operate under perfect competition. The

intuition is simple. Because the market for designs is competitive, the price for designs will be bid up until it is equal to the present value of the profit stream that a monopolist can extract. Hence, the price of each innovation, whether basic or applied, is equal to present discounted value of profit stream of the respective intermediary producer (cf. Romer (1990)).

It is easy to calculate profit streams of intermediate sectors by using equations (19) and (21). Suppose that the basic R&D associated with $t+1$ has already been invented. The present value of profits of the basic sector for any GPT in the *next* GPT-cluster would be[9]

$$PV_{t+2,0} = \sum_{s=\tau}^{\infty} (1+r)^{-(s-\tau)} \pi_{t+2,0} \Rightarrow$$

$$PV_{t+2,0} = r\eta \cdot (\phi_0 - 1) \cdot \beta^{\sigma\chi} \cdot \left(\frac{\alpha_0}{r\eta\phi_0} \right)^{\sigma\chi} \frac{G_1^{\chi}}{G_2^{1-\chi}} \sum_{s=\tau}^{\infty} (1+r)^{-(s-\tau)} \frac{L^{\chi} \cdot H^{1-\chi}}{(B_{t+1})^{1-\chi}} \qquad (29)$$

In Equation (29), s denotes the real time and τ indicates the present. We assume that the growth dynamics of L and H are known to the system (this assumption includes constant L and H). It is critical to note that equation (29) is derived at equilibrium, meaning that the present value of profits received by the basic-intermediate producer is calculated under the assumption that n complementary goods for each GPT in the new cluster will have been produced. In other words, agents calculate the present value of profits as if they have already attained the next equilibrium point. Under perfect foresight assumption, this is not unrealistic at all, because it is easy to imagine that all GPT times here to infinity is the relevant comparison measure for making a decision for any agent what to invent next.

Similarly, the present value of the jth complementary-good at time τ, where the latest GPT stock available at that time is B_{t+1}, will be

$$PV_{t+1,j} = \frac{1}{\sigma - 1} \cdot \frac{1}{\phi_j} \cdot \beta^{\sigma\chi} \cdot \left(\frac{\alpha_0}{r\eta\phi_0} \right)^{\alpha_0\beta\sigma\chi} \frac{G_1^{\chi}}{G_2^{1-\chi}} \sum_{s=\tau}^{\infty} (1+r)^{-(s-\tau)} \frac{L^{\chi} \cdot H^{1-\chi}}{(B_t)^{1-\chi}} \qquad (30)$$

[9] For expositional purposes, we assume that r is constant and L and H are growing at exogenously given rates in equations (29) and (30). The switching mechanism is not dependent on this assumption.

The most interesting property of discounted profit streams in (30) is its falling nature. In particular, $PV_{t+1,n}$ must be at the neighborhood of zero, given our assumption that the very last input share α_n is at the neighborhood of zero (i.e., $PV_{t+1,n+1} = 0$). Evidently, R&D people will stop working on the prevailing technological paradigm after producing the nth blueprint under perfect foresight assumption.

Recall that profit streams are captured by R&D people, independent of whether they are employed at basic R&D sector or at applied R&D sector (cf., equations (27) and (28)). Then, the falling nature of profit streams must be also reflected in the wages of R&D people employed in the applied sector. In particular, wages received by the R&D people working in the applied R&D must be falling as new blueprints for intermediaries are produced. This characteristic of our model is indeed the heart of cyclical advancement of technologies and long-run business cycles.

To see this in more detail, we note that research labor decides on the use of their labor by comparing the real wages offered by the two research-sectors at any time. Given the linear blueprint production functions, all R&D people will be employed in only one sector, that is, only corner solutions are viable in the model (clearly, linearity is only for stylistic purposes). Suppose now that the basic R&D sector has just used the whole research staff. In particular, suppose that we have just produced the blueprint bundle for the basic intermediate of GPT $t+1$. The question is whether they would switch to produce complementary intermediaries for this GPT or switch to work on a new GPT bundle. In our set up, the following conditions must hold in order to make the switch to applied R&D work viable:

$$\begin{aligned} &w_{t+1,1} > w_{t+2,0} \\ &w_{t+1,2} > w_{t+2,0} \\ &\quad (\cdots) \\ &w_{t+1,n} > w_{t+2,0} \end{aligned} \tag{31}$$

Equation (31) indicates that in order for a GPT bundle, say $B_{t+1} - B_t$, to be viable, then first and foremost the real wage offered by the applied R&D sector for the first complementary good must be higher than the wage rate offered by the basic R&D sector of the next GPT cluster, *i.e.*, $B_{t+2} - B_{t+1}$. If this condition holds, then the entire research people will shift to applied research. The same condition must also hold for

blueprints of intermediaries $2,3,\ldots$ Nonetheless, there is always an end to this process. Our assumption that α_n is at the neighborhood of zero implies that $w_{t+1,n}$ is also at the neighborhood of zero and hence the condition for switching back to basic research for the next GPT technology is always secured. It might be argued that the assumption that α_n is at the neighborhood of zero is too strong. However, it must be noted that the genuine generator of the switching mechanism is not that assumption but the fact that profits in the complementary sector have a falling nature. Assuming that α_n is at the neighborhood of zero only secures the constancy of number of varieties in the model, which is hardly restrictive for n being a large number.

It is worth to mention that the wages in the R&D sector also experiences cycles. From equation (31) above, we know that $w_{t+1,1} > w_{t+2,0}$ but wages decline (towards zero) as new intermediaries are produced. When the model-economy starts to produce the next generation GPT bundle, first research people's wages experience $w_{t+2,0}$, which must satisfy, $w_{t+2,0} > 0$ and next a jump to $w_{t+2,1}$, where the latter can be substantially greater than the former. Then, it starts to fall again. This mechanism creates cycles in R&D wages, and none of these cycles necessarily produce similar wage rates *as* the stock of GPTs increase over time and *if* skilled labor and unskilled labor changes.

5 Output Dynamics in the Long-run

To look at the evolvement of long-run equilibria over time, we have to close the model with a demand side. There are two ways to close the model. First and foremost, in order to not further complicate the model, we may presume that consumption is determined by an exogenous saving rate. Interestingly, an exogenous saving rate assumption does *not* mean that the rate is assigned arbitrarily in our context. Quite the opposite, due to the way the model is constructed, the exogenous saving rate is determined from the model. To see this, first note that each GPT time refers also to a period, in which a group of basic intermediates are produced, that is, physical investment is made. For example, time $t+1$ refers also to the investment made in the

amount $x_{t+1,0} \cdot (B_{t+1} - B_t)$. Savings must balance this investment, instantaneously. In that respect, the saving rate is *ex ante* defined in our model. Details are as follows. Suppose that we denote the saving rate by, say ψ. We conjecture that the saving rate would be identical and constant at GPT equilibrium points. To see this, using (22) and the equation of capital (not shown in the text), it is sufficient to calculate, say, $\frac{K_t - K_{t-1}}{Y_{t-1}}$ and $\frac{K_{t+1} - K_t}{Y_t}$. If L and H are constant, then, $\psi = \frac{\alpha_0 \beta}{r\phi_0}[(1+\delta_B R_B)^{\chi} - 1]$. If L and H grow at exogenously given rates, say, g_L and g_H, respectively, then $\psi = \frac{\alpha_0 \beta}{r\phi_0}[(1+g_L)^{\chi}(1+g_H)^{1-\chi}(1+\delta_B R_B)^{\chi} - 1]$. Hence, under the small-country assumption, it is possible to close the model by

$$C_t = (1-\psi)Y_t \tag{32}$$

where C_t is consumption. Naturally, the fact that investment in the model is made only at GPT times does not mean that savings have to be also made only at these instances. Indeed, assuming a particular distribution of savings, which sums up exactly to the investment needs at GPT times is sufficient.

Second, in accordance with intertemporal optimization of consumption assumption, r can be determined endogenously. In particular,

$$Max \;\; U_s = \sum_{s=0}^{\infty} \mu^s u(C_s)$$

$$s.t. \;\; W_{s+1} - W_s = r_s W_s + LaborIncome - C_s$$

where $\mu = \frac{1}{1+\rho}$ is the discount factor, ρ is the subjective rate of time preference, $u(C) = \frac{C^{1-\theta} - 1}{1-\theta}$ is the utility, $1/\theta$ is the intertemporal elasticity of consumption, W is the asset stock of the society, and s is the calendar time. The maximization yields $\frac{C_{s+1}}{C_s} = \mu^{\theta}(1+r_s)^{\theta}$. The critical characteristic of our modeling approach that has to be recalled is the fact that the r is determined in such a way that also assures

consumption is equal to output minus net investment (*i.e.*, in Romer-based setups, the way r is determined assures also that the macroeconomic budget constraint is always satisfied).

Since our model does not allow us to look at transitional analysis in-between GPT times (see also the discussion below on the broader concept of output), we can only look at the determination of r at GPT times. A balanced growth implies that

$$\frac{C_{t+1}-C_t}{C_t}=\frac{K_{t+1}-K_t}{K_t}$$

If L and H are constants, then the right-hand side is $[\left(\frac{r_t}{r_{t-1}}\right)^{-\sigma\chi}(1+\delta_B R_B)^{\chi}-1]$. If L and H are growing, then the right-hand side becomes $[(1+g_L)^{\chi}(1+g_H)^{1-\chi}\left(\frac{r_t}{r_{t-1}}\right)^{-\sigma\chi}(1+\delta_B R_B)^{\chi}-1]$. The left hand side is $\mu^{\theta}(1+r_t)^{\theta}$. The implicit solution of this difference equation defines the time path of r at GPT times.

Finally, we may discuss the time profile of output and the growth rate of the economy. We focus on those results that assumes constant r, as it allows for a tractable solution. Firstly, from equation (22), we observe that GPT equilibrium experiences 'jumps' at GPT times due to the fact that the number of composite input varieties increases. If L and H are constants, then the growth rate of output is $g=[(1+\delta_B R_B)^{\chi}-1]$ for a constant r. If L and H are growing, then the growth rate of output is $g=[(1+g_L)^{\chi}(1+g_H)^{1-\chi}(1+\delta_B R_B)^{\chi}-1]$.

Secondly, analogous to our previous analysis, it is not possible to determine the complete time profile of output at in-between GPT times in our model. This is because our model actually features two types of output: *equilibrium* (GPT) output Y and the *transitional* output or *the broader concept of output* Q.[10] In our context, the former is associated with GPT equilibrium points of final goods production, whereas the latter refers to values of output in-between GPT equilibrium points. To see why it is not easy to calculate transitional output, let us illustrate it by an example. Suppose that the model economy has just realized Y_t. Say at the next real time, $s+1$, the

economy will generate the next GPT bundle, $B_{t+1} - B_t$, and hence ΔB_{t+1} units of core intermediaries $x_{t+1,0}$ are associated with the recent bundle. Clearly, the broader concept of output is *not* $Q_{s+1} = Y_t + x_{t+1,0} \cdot (B_{t+1} - B_t)$ because (i) all production activities of existing GPTs are affected inversely by addition of a new intermediary and (ii) skilled and unskilled labor may be growing. Hence, the exact time profile of the broader concept of output is function of B, L and H. To see this, suppose first that L and H are constant. Then, from t to $t+1$, more intermediaries will use the fixed supply of H and therefore output per complementary intermediary will fall. On the other hand, production of new intermediaries, $x_{t+1,j} \cdot (B_{t+1} - B_t)$, implies an increase in the transitional output. This suggests that that the transitional output may fall or rise, depending on the 'cost' of producing new complementary intermediaries in terms of the reduction in the volume of all composite intermediaries. When L and H are growing during that period, the transitional output is enhanced directly by higher L and indirectly by higher H (as the volume of composite intermediaries may increase). Hence, our model indicates that output may decline or rise at the onset of a new GPT, depending on B, L and H.

The policy implication of this finding is that the time profile of output will (almost certainly) be positive in the transitional period, if L and H are growing. This finding suggests that if the policy maker is able to match the additional demand to skilled labor at times that there is a growing demand for them, then it is possible to smooth out long run business cycles, which will both lead to higher levels of output and growth rates during the transitional period in addition to jumps at equilibrium points.

6 Conclusion

This study showed that exhausting profits in the incremental technologies with the existing technological paradigm could be the source of long-run business cycles. New technological paradigms are advanced cyclically because R&D activities focus on the existing technological paradigm as long as there remain positive profit opportunities

[10] See Chapter 5 of Barro and Sala-i-Martin (1995) on the broader concept of output. We exploit this term in our paper for presentational purposes. Otherwise, the use has no analogy.

on it. Focus returns to basic R&D whenever the profit opportunities of the next bundle of drastic technologies are higher than that of the existing paradigm. Switching between the basic and applied technologies creates long-run cycles in the economy. The paper showed also that temporary falls in growth at the onset of a new technological paradigm might be because the pace of growth of inputs was not meeting the additional resource needs created by the new paradigm.

This paper has many possible extensions. First and foremost, we did not look into the policy implications of Kondratieff cycles analytically. We showed that these cycles are generated endogenously due to market opportunities. If the market opportunities arising with new technological paradigms do not quickly indulge the input markets to enhance the arising demand, there is room for policy maneuvers in the sense that the growth of these inputs can be induced by necessary policy actions. We left this question analytically untreated in this study. Secondly, we assume that the forthcoming technological paradigms have no impact on the 'performance' of the existing technological paradigms. This may be true in some cases but not necessarily in all instances. This argument is true for both basic and applied technologies. A recent invention in basic technology or in applied technology may find its place also in improving productivity of a previous applied technology. We ignored these backward linkages in this study. Thirdly, for matter of tractability, we imposed a constant number of intermediaries within each paradigm. A demanding extension is to endogenize the number of varieties within each composite input.

References

Aghion, P., and P. Howitt (1992), A Model of Growth through Creative Destruction, Econometrica 60, 323-351.

Bresnahan, T., and Trajtenberg, M. (1995). "General Purpose Technologies", Journal of Econometrics, Vol. 65, pp.83-108.

David, P., (1990). "The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox", American Economic Review Papers and Proceedings, Vol. 80(2), pp.355-361.

van Duijn, J.J. (1983), The Long Wave in Economic Life, London: George Allen&Unwin.

Gordon, R.J., (2000). "Does the 'New Economy' Measure up to the Great Inventions of the Past?", NBER WP No.7833.

Grossman, G.M., and E. Helpman (1991), *Innovation and Growth in the Global Economy* (Cambridge, MIT Press).

Helpman, E. (1998) (ed.). General Purpose Technologies and Economic Growth, Mass: MIT Press

Kleinknecht, Alfred (1987), *Innovation Patterns in Crisis and Prosperity: Schumpeter's Long Cycle Reconsidered* (London, Macmillan Press).

Kontratieff, N.D. (1926). "Die Langen Wellen der Konjunktur", Archiv fur Sozialwissenschaft und Sozialpolitik, Tubingen, Vol.56, 573ff., (English translation in: Review of Economic Statistics, Vol.17, 105ff., reprinted in: Lloyds Bank Review, No.129, July 1978).

Mensch, G., (1979). *Stalemate in Technology: Innovations Overcome Depression, Cambridge*, Mass:Ballinger Publishing Company.

Mokyr, J., (1990). *The Lever of Riches*, Oxford University Press.

Olsson, O. (2001). "Why Does Technology Advance in Cycles?", Available at: http://swopec.hhs.se/gunwpe/abs/gunwpe0038.htm.

Romer, P. (1990) "Endogenous Technological Change", Journal of Political Economy, Vol. 98, pp.S71-102.

Schumpeter, J.A., (1939). *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*, 2 Vols, New York, McGraw-Hill.

van Zon, Adriaan and I. Hakan Yetkiner (2003), "An Endogenous Growth Model á la Romer with Embodied Energy-Saving Technological Change", Resource and Energy Economics, Vol. 25 (1), pp.81-103.

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Kemfert, C. and R.S.J. Tol (2001), *Equity, International Trade and Climate Policy*, **FNU-5** (*International Environmental Agreements*, **2**, 23-48).

Tol, R.S.J., Downing T.E., Fankhauser S., Richels R.G. and Smith J.B. (2001), *Progress in Estimating the Marginal Costs of Greenhouse Gas Emissions*, **FNU-4**. (*Pollution Atmosphérique – Numéro Spécial: Combien Vaut l'Air Propre?*, 155-179).

Tol, R.S.J. (2000), *How Large is the Uncertainty about Climate Change?*, **FNU-3** (*Climatic Change*, **56** (3), 265-289).

Tol, R.S.J., S. Fankhauser, R.G. Richels and J.B. Smith (2000), *How Much Damage Will Climate Change Do? Recent Estimates*, **FNU-2 (***World Economics,* **1** (4), 179-206)

Lise, W. and R.S.J. Tol (2000), *Impact of Climate on Tourism Demand*, **FNU-1** (*Climatic Change*, **55** (4), 429-449).