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On higher-order flavour-singlet splitting and coefficient functions at large x*

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We discuss the large-x behaviour of the splitting functions $P_{\rm qg}$ and $P_{\rm gq}$ and of flavour-singlet coefficient functions, such as the gluon contributions $C_{2,\rm g}$ and $C_{L,\rm g}$ to the structure functions $F_{2,L}$, in massless perturbative QCD. These quantities are suppressed by one or two powers of $(1-x)$ with respect to the $(1-x)^{-1}$ terms which are the subject of the well-known threshold exponentiation. We show that the double-logarithmic contributions to $P_{\rm qg}$, $P_{\rm gq}$ and C_L at order $\alpha_{\rm s}^4$ can be predicted from known third-order results and present, as a first step towards a full all-order generalization, the leading-logarithmic large-x behaviour of $P_{\rm qg}$, $P_{\rm gq}$ and $C_{2,\rm g}$ at all orders in $\alpha_{\rm s}$.

1. Introduction

Inclusive deep-inelastic lepton-nucleon scattering (DIS) via the exchange of a colour-neutral (gauge) boson, for the basic kinematics see Fig. 1, is a benchmark process of perturbative QCD.

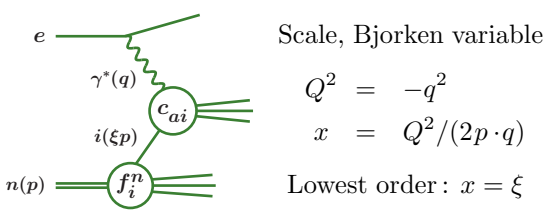

Figure 1. The kinematics and perturbative QCD factorization of photon-exchange DIS.

Disregarding contributions suppressed by powers of $1/Q^2$, the structure functions in electromagnetic DIS are given by

$$x^{-1}F_a^n(x,Q^2) = \left[\, C_{a,i}(\alpha_{\rm s}(Q^2)) \otimes f_i^{\,n}(Q^2)\right](x) \quad (1)$$

in terms of the coefficient functions $C_{a,i}$, $a = 2,\, L$, $i = {\rm q}, {\rm g}$, and the nucleon parton distributions $f_i^{\,n}$. Here $\otimes$ denotes the standard Mellin convolution, and the summation over i is understood. Without loss of information, we identify the renormalization and factorization scale with the physical scale Q^2 in Eq. (1) and throughout this article.

The scale dependence of the parton densities is

$$\frac{d\,f_i(\xi,\mu^2)}{d\ln Q^2} = \left[\, P_{ik}(\alpha_{\rm s}(Q^2)) \otimes f_k(Q^2)\right](\xi)\,. \quad (2)$$

The coefficient functions in Eq. (1) and the splitting functions P_{ik} can be expanded in powers of the strong coupling constant $a_{\rm s} \equiv \alpha_{\rm s}/(4\pi)$,

$$C_{a,i}(x,\alpha_{\rm s}) \;=\; \sum\nolimits_{l=0} a_{\rm s}^{\,l+l_a}\, c_{a,i}^{(l)}(x)\,, \quad (3)$$

$$P_{ik}(x,\alpha_{\rm s}) \;=\; \sum\nolimits_{l=0} a_{\rm s}^{\,l+1}\, P_{ik}^{(l)}(x) \quad (4)$$

with $l_a = 0$ for F_2 (and the Higgs-exchange structure function F_ϕ discussed below), and $l_a = 1$ for the longitudinal structure function F_L. In this notation, the N^{n}LO approximation includes the contributions with $l \le n$ in both Eqs. (3) and (4).

The above (spin-averaged) splitting functions are presently known to $n = 2$ [1,2], i.e., the next-to-next-to-leading order (NNLO $\equiv$ N^{2}LO). The coefficient functions for the most important structure functions (including F_3 for charge-averaged W-exchange) have also been fully computed to order $\alpha_{\rm s}^3$ [3,4,5], while the less important charge-asymmetry W-cases are available only through a couple of low-integer Mellin-N moments [6,7]. The frontier in the present massless case are now the $\alpha_{\rm s}^4$ corrections, for which first results have been obtained at the lowest value of N [8,9]. See Ref. [10] for the status of the third-order computation of the heavy-quark contributions to DIS.

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2. The general large-x behaviour

We are interested in the leading contributions, in terms of powers in $(1-x)$, to Eqs. (3) and (4). The form of the diagonal splitting functions is stable under higher-order corrections in the $\overline{\rm MS}$ scheme, viz [11]

$$P_{ii}^{(l)} = A_i^{(l)}(1-x)_+^{-1} + B_i^{(l)}\delta(1-x) \,+\, \ldots \,. \qquad (5)$$

The off-diagonal quantities, however, receive a double-logarithmic higher-order enhancement,

$$P_{i\neq j}^{(l)} \,=\sum_{a=0}^{2l} A_{ij,a}^{(l)} \ln^{2l-a}(1-x) \,+\, \ldots \,, \qquad (6)$$

where $A_{ij,a}^{(l)} \propto (C_A - C_F)^{\,l-a}$ for $a < l$ for (at least) $l \leq 2$ [2], i.e., all double logarithms vanish for $C_F = C_A$, which is part of the colour-factor choice leading to an $\mathcal{N}=1$ supersymmetric theory.

The leading large-x parts of 'diagonal' coefficient functions, e.g., $C_{2,\rm q}$ and $C_{\phi,\rm g}$, are given by

$$c_{\rm diag}^{(l)} \,=\, \sum_{a=0}^{2l-1} D_{i,a}^{(l)} \left[\frac{\ln^{2l-1-a}(1-x)}{1-x}\right]_+^{-1} + \,\ldots \,. \qquad (7)$$

These terms are resummed by the soft-gluon exponentiation [12]. For DIS structure functions (and some other semi-leptonic processes) this resummation is known at the next-to-next-to-next-to-leading logarithmic accuracy, i.e., the highest six logs are completely known to all orders [13].

No resummation has been derived so far for the off-diagonal (flavour-singlet) coefficient functions such as $C_{2,\rm g}$ and $C_{\phi,\rm q}$ which are of the form

$$c_{\rm off-d}^{(l)} \,=\sum_{a=0}^{2l-1} O_{i,a}^{(l)} \ln^{2l-1-a}(1-x) \,+\, \ldots \,. \qquad (8)$$

The coefficient functions for F_L are suppressed by one power in $(1-x)$ with respect to those of F_2,

$$c_{L,i}^{(l)} \,=\, \sum_{a=0}^{2l} L_{L,i}^{(l)} (1-x)^{\delta_{ig}} \ln^{2l-a}(1-x) \,+\, \ldots \,, \qquad (9)$$

recall our notation with $l_L = 1$ in Eq. (3). The double-log contributions to $C_{L,\rm q}$ (and the $C_F = 0$ part of $C_{L,\rm g}$) have been resummed in Ref. [14], i.e., the respective highest three logarithms ($a =$ 0, 1 and 2 in Eq. (9)) are known to all orders.

Our aim is to derive corresponding predictions for all quantities in Eqs. (6), (8) and (9). The present contribution is a brief status report of this programme, which has not been finished so far.

3. Physical evolution kernel for (F_2, F_ϕ)

The results of Ref. [14] and their extension to the non-leading corrections for $C_{2,\rm q}$ and other quantities at all orders in $(1-x)$ [15], see Ref. [16] for a brief summary, have been obtained by studying the non-singlet physical evolution kernels for the respective observables. It is thus natural to study also flavour-singlet physical kernels.

The most natural complement to the standard quantity F_2 with $c_{2,i}^{(0)} = \delta_{iq}\,\delta(1-x)$ is a structure function for a probe which directly interacts only with gluons, such as a scalar ϕ with a $\phi\, G^{\mu\nu} G_{\mu\nu}$ coupling to the gluon field [17]. In the Standard Model this interaction is realized for the Higgs boson in the limit of a heavy top-quark [18,19]. The coefficient functions $C_{\phi,i}$ have been determined recently in Refs. [20] and [21] to the second and third order in $\alpha_{\rm s}$, respectively.

We thus consider the 2-vector singlet structure function and 2×2 coefficient-function matrix

$$F \,=\, \begin{pmatrix} F_2 \\ F_\phi \end{pmatrix} \,, \quad C \,=\, \begin{pmatrix} C_{2,\rm q} & C_{2,\rm g} \\ C_{\phi,\rm q} & C_{\phi,\rm g} \end{pmatrix} \,. \qquad (10)$$

With P denoting the matrix of the splitting functions (7) and (8), the evolution kernel for F reads

$$\begin{aligned} \frac{dF}{d\ln Q^2} \,&=\, \frac{d\,C}{d\ln Q^2}\, f \,+\, CPf \\ &=\, \Big(\beta(a_{\rm s})\,\frac{d\,C}{da_{\rm s}}\,C^{-1} + CPC^{-1}\Big)F \qquad (11) \\ &=\, KF \quad \text{with} \quad K \,=\, \begin{pmatrix} K_{22} & K_{2\phi} \\ K_{\phi 2} & K_{\phi\phi} \end{pmatrix} \,. \end{aligned}$$

$\beta(a_{\rm s}) = -\beta_0 a_{\rm s}^2 + \ldots$ with $\beta_0 = 11\,C_A/3 - 2\,n_f/3$ is the standard beta function of QCD. All products of x-dependent quantities have to be read as convolutions (or products of their Mellin transforms).

After expanding in $\alpha_{\rm s}$, the first term in the second line of Eq. (11) receives double-logarithmic contributions from the non-singlet and singlet coefficient functions (7) and (8). The second term, absent in the non-singlet cases of Refs. [14,15], includes also the double-log terms of Eq. (6).

The crucial observation, proven by available three-loop calculations to order $\alpha_{\rm s}^4$ for the non-singlet parts (thanks to Eq. (5)) and to order $\alpha_{\rm s}^3$

for the singlet contribution, is that the physical kernel K is only single-log enhanced [21], i.e.,

$$K_{ab}^{(l)} = \sum_{\eta=0}^{l} A_{ab,\eta}^{(l)} (1-x)^{-\delta_{ab}} \ln^{l-\eta}(1-x) + \ldots \quad (12)$$

where the expansion coefficients $K_{ab}^{(l)}$ are defined as in Eq. (4) for the splitting functions above.

We conjecture that also the flavour singlet part remains single-log enhanced at the fourth order. This implies a cancellation between the double-logarithmic contributions to the, so far unknown, off-diagonal $l = 3$ splitting functions (6) and the known [4,21] coefficient functions to order α_s^3 from which the former can be deduced. The results are

$$\begin{aligned} P_{\rm qg}^{(3)}/n_f &= \ln^6(1-x) \cdot 0 \\ &+ \ln^5(1-x)\Big[\frac{22}{27} C_{AF}^3 - \frac{14}{27} C_{AF}^2 C_F + \frac{4}{27} C_{AF}^2 n_f\Big] \\ &+ \ln^4(1-x)\Big[\Big(\frac{293}{27} - \frac{80}{9}\zeta_2\Big) C_{AF}^3 - \frac{116}{81} C_{AF}^2 n_f \\ &\quad + \Big(\frac{4477}{16} - 8\zeta_2\Big) C_{AF}^2 C_F - \frac{13}{81} C_{AF} C_F^2 \\ &\quad + \frac{17}{81} C_{AF} C_F n_f - \frac{4}{81} C_{AF} n_f^2\Big] \\ &+ \mathcal{O}\left(\ln^3(1-x)\right) \quad , \end{aligned} \quad (13)$$

$$\begin{aligned} P_{\rm gq}^{(3)}/C_F &= \ln^6(1-x) \cdot 0 \\ &+ \ln^5(1-x)\Big[\frac{70}{27} C_{AF}^3 - \frac{14}{27} C_{AF}^2 C_F - \frac{4}{27} C_{AF}^2 n_f\Big] \\ &+ \ln^4(1-x)\Big[\Big(\frac{3280}{81} + \frac{16}{9}\zeta_2\Big) C_{AF}^3 - \frac{256}{27} C_{AF}^2 n_f \\ &\quad + \Big(\frac{637}{18} - 8\zeta_2\Big) C_{AF}^2 C_F - \frac{49}{81} C_{AF} C_F^2 \\ &\quad + \frac{17}{81} C_{AF} C_F n_f + \frac{32}{81} C_{AF} n_f^2\Big] \\ &+ \mathcal{O}\left(\ln^3(1-x)\right) \end{aligned} \quad (14)$$

with $C_{AF} \equiv C_A - C_F$. The vanishing of the leading $\ln^6(1-x)$ contributions is due to a cancellation of contributions. Below we will address the question whether this cancellation is accidental or a structural feature. Eqs. (13) and (14) show the colour-factor pattern already noted for $l \leq 2$ below Eq. (6). The feature is not an obvious consequence of our derivation and can thus be viewed as a non-trivial check of the above conjecture.

The extension of the above results to all powers of $(1-x)$ can be found in Ref. [21].

4. Physical evolution kernel for (F_2, F_L)

The system of standard DIS structure functions

$$F = \begin{pmatrix} F_2 \\ \widehat{F}_L \end{pmatrix} \quad , \quad \widehat{F}_L = F_L / \left(a_s\, c_{L,\rm q}^{(0)}\right) \quad , \quad (15)$$

studied before in Refs. [22,23], can be analyzed in complete analogy to the previous section. Our normalization of $\widehat{F}_L$ (of course Eq. (15) involves a simple division only in Mellin-N space) leads to

$$C = \begin{pmatrix} \delta(1-x) & 0 \\ \delta(1-x) & \hat{c}_{L,\rm g}^{(0)} \end{pmatrix} + \sum_{l=1} a_s^l \begin{pmatrix} c_{2,\rm q}^{(l)} & c_{2,\rm g}^{(l)} \\ \hat{c}_{L,\rm q}^{(l)} & \hat{c}_{L,\rm g}^{(l)} \end{pmatrix} . \quad (16)$$

The resulting elements of the physical kernel

$$K = \begin{pmatrix} K_{22} & K_{2\rm L} \\ K_{\rm L2} & K_{\rm LL} \end{pmatrix} \quad (17)$$

are again single-log enhanced at large x and read

$$K_{ab}^{(l)} = \sum_{\eta=0}^{l} \widehat{A}_{ab,\eta}^{(l)} (1-x)^{-1} \ln^{l-\eta}(1-x) + \ldots \quad (18)$$

at, at least, $l \leq 3$ for the upper row of Eq. (17), with $\widehat{A}_{2\rm L,0}^{(l)} = 0$, and at $l \leq 2$ for the lower row.

Conjecturing that this behaviour holds at $l = 3$ also for K_{L2} and K_{LL}, the three-loop results of Refs. [1,2,3,4] together with Eq. (13) yield

$$\begin{aligned} c_{L,\rm q}^{(3)}/C_F &= \ln^6(1-x)\, \frac{16}{3} C_F^3 \\ &+ \ln^5(1-x)\Big[(72 - 64\,\zeta_2)\, C_F^3 + \frac{80}{9} C_F^2 n_f \\ &\qquad - \Big(\frac{728}{9} - 32\,\zeta_2\Big) C_F^2 C_A\Big] \\ &+ \ln^4(1-x) \cdot \big[\text{known coefficients}\big] \\ &+ \mathcal{O}\left(\ln^3(1-x)\right) \quad , \end{aligned} \quad (19)$$

$$\begin{aligned} c_{L,\rm g}^{(3)}/n_f &= (1-x)\ln^6(1-x)\, \frac{32}{3} C_A^3 \\ &+ (1-x)\ln^5(1-x)\Big[-\frac{2080}{9} C_A^3 + \frac{64}{9} C_A^2 n_f \\ &\quad + \frac{104}{3} C_A^2 C_F + \frac{40}{3} C_F^3\Big] \\ &+ (1-x)\ln^4(1-x)\Big[\Big(\frac{70760}{27} - 352\,\zeta_2\Big) C_A^3 \\ &\quad - \Big(\frac{25306}{27} - \frac{320}{3}\zeta_2\Big) C_A^2 C_F - \frac{4192}{27} C_A^2 n_f \end{aligned}$$

$$+\Big(\frac{1600}{27}+32\,\zeta_2\Big)C_A C_F^2+\frac{556}{27}\,C_A C_F n_f$$
$$+\frac{32}{27}\,C_A n_f^2+\Big(38-\frac{320}{3}\,\zeta_2\Big)C_F^3+\frac{308}{27}\,C_F^2 n_f\Big]$$
$$+\mathcal{O}\left((1-x)\ln^3(1-x)\right)\ , \qquad (20)$$

where the coefficient of $\ln^4(1-x)$ in Eq. (19) has been suppressed for brevity. The complete form of this equation has been given in Ref. [14], where it was derived in another manner which did not involve the off-diagonal splitting functions. Consequently the consistency of the two derivations provides another confirmation of the correctness of the above result for $P_{\rm qg}^{(3)}$. The non-C_F parts of Eq. (20) – here, as App. C of Ref. [21], given for W-exchange, i.e., without the fl_{11}^g contribution for the photon case [4] – have also been derived, but not explicitly written down, in Ref. [14].

5. Unfactorized off-diagonal amplitudes

The single-log enhancement of the above physical kernels suggests an iterative structure of the unfactorized structure functions (forward amplitudes), from which the splitting and coefficient functions are obtained in Mellin-N space via the mass-factorization relations

$$T_{a,j}\ =\ \widetilde{C}_{a,i}\,Z_{ij}\ ,\ \ -\gamma\equiv P=\frac{dZ}{d\ln Q^2}\,Z^{-1}\ . \qquad (21)$$

The D-dimensional coefficient functions $\widetilde{C}_a$ include terms with ε^k, $k\geq 0$ in dimensional regularization with $D=4-2\varepsilon$. The transition functions Z collect all terms which are singular for $\varepsilon\to 0$. Inverting the second relation in Eq. (21) yields

$$Z\big|_{a_{\rm s}^n}\ =\ \frac{1}{\varepsilon^n}\frac{\gamma_0^n}{n!}+\ldots \qquad (22)$$
$$+\frac{1}{\varepsilon^2}\left(\frac{\gamma_0\gamma_{n-2}}{n(n-1)}+\frac{\gamma_{n-2}\gamma_0}{n}+\ldots\right)+\frac{1}{\varepsilon}\frac{\gamma_{n-1}}{n}\ .$$

At order $\alpha_{\rm s}^n$, the $\varepsilon^{-n}\ldots\varepsilon^{-2}$ contribution to T_a are given in terms of lower-order terms. The ε^{-n} and ε^0 coefficients include the n-loop splitting functions and (four-dimensional) coefficient functions C_a, respectively. Terms with ε^k, $0<k<l$ are required for the factorization at order $n+l$.

We now focus on the leading-logarithmic (LL) contributions to the off-diagonal quantities $T_{\phi,{\rm q}}$ and $T_{2,{\rm g}}$ and summarize the results of Ref. [24]. With $L\equiv\ln N$ these terms are of the form

$$T_{\phi,{\rm q}}^{(n)}/C_F\ \stackrel{\rm LL}{=}\ T_{2,{\rm g}}^{(n)}/n_f\ \stackrel{\rm LL}{=}\ \frac{L^{n-1}}{N\varepsilon^n}\sum_{k=0}^{\infty}(\varepsilon L)^k\mathcal{L}_{n,k}\,\cdot$$
$$\cdot\ \left(C_F^{n-1}+C_F^{n-2}C_A+\ldots+C_A^{n-1}\right)\ , \qquad (23)$$

i.e., the coefficients $\mathcal{L}_{n,k}$ are the same for both amplitudes and all combinations of C_F and C_A. Consequently an all-order relation for one colour structure of either amplitude is sufficient. The calculations of Ref. [2,21] imply such a relation,

$$T_{\phi,{\rm q}}\big|_{C_F\ {\rm only}}\ \stackrel{\rm LL}{=}\ T_{\phi,{\rm q}}^{(1)}\,\frac{\exp(a_{\rm s}T_{2,{\rm q}}^{(1)})-1}{T_{2,{\rm q}}^{(1)}}\ , \qquad (24)$$

in terms of the first-order expressions known to all powers of ε,

$$T_{\phi,{\rm q}}^{(1)}\ \stackrel{\rm LL}{=}\ -\frac{2C_F}{N}\,\frac{1}{\varepsilon}\,\exp(\varepsilon\ln N)\ , \qquad (25)$$
$$T_{2,{\rm q}}^{(1)}\ \stackrel{\rm LL}{=}\ 4C_F\,\frac{1}{\varepsilon^2}\,(\exp(\varepsilon\ln N)-1)\ . \qquad (26)$$

After carrying out the mass factorization to a very high order in $\alpha_{\rm s}$ (using FORM [25]), the all-order analytic expressions for the leading-logarithmic contributions to the splitting functions and coefficient functions have been derived. The former quantities are given by

$$P_{\rm qg}^{\rm LL}(N,\alpha_{\rm s})\ =\ \frac{n_f}{N}\,\frac{\alpha_{\rm s}}{2\pi}\,\mathcal{B}_0(\tilde{a}_{\rm s}) \qquad (27)$$

with

$$\mathcal{B}_0(x)\ =\ \sum_{n=0}^{\infty}\frac{B_n}{(n!)^2}\,x^n$$
$$=\ 1\ -\ \frac{x}{2}\ -\ \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!^2}\,|B_{2n}|\,x^{2n}\ , \qquad (28)$$

$$\tilde{a}_{\rm s}\ =\ \alpha_{\rm s}/\pi\ (C_A-C_F)\ln^2 N\ . \qquad (29)$$

The coefficients B_n in Eq. (28) are the Bernoulli numbers in the standard normalization of Ref. [26]. The corresponding result for $P_{\rm gq}$ is obtained by replacing n_f by C_F in Eq. (27), and exchanging C_A and C_F in Eq. (29).

Due to $B_{2n+1}=0$ for $n\geq 1$, the LL coefficients vanish at all even orders in $\alpha_{\rm s}$. Consequently the first lines of Eqs. (13) and (14) (recall the power counting (4)) are not at all accidental.

The corresponding all-order result for the coefficient function $C_{2,\mathrm{g}}$ reads

$$C_{2,\mathrm{g}}^{\mathrm{LL}}(N,\alpha_\mathrm{s}) \;=\; \frac{1}{2N\ln N}\,\frac{n_f}{C_A-C_F}\;\cdot \tag{30}$$
$$\cdot\left\{\exp\left(2\,C_F a_\mathsf{s}\ln^2 N\right)\mathcal{B}_0(\tilde{a}_\mathrm{s})-\exp\left(2\,C_A a_\mathsf{s}\ln^2 N\right)\right\}$$

$C_{\phi,q}^{\mathrm{LL}}$ can be obtained from this result by a simple substitution of colour factors.

Inserting Eqs. (5) (only the lowest-order terms with $A_q^{(0)} = 4\,C_F$ and $A_g^{(0)} = 4\,C_A$ contribute), (27), (30), and the well-known relation [27]

$$C_{2,\mathrm{q}} \;\stackrel{\mathrm{LL}}{=}\; \exp\left(2a_\mathsf{s}\,C_F\ln^2 N\right) \tag{31}$$

and its analogue for $C_{\phi,\mathrm{g}}$ into the physical kernel $K_{2\phi}$ in Eq. (11), one finds that the highest double logarithm indeed vanishes at all orders in α_s. The same is found for $K_{\phi 2}$. Hence the amplitude-based resummation verifies the conjecture made below Eq. (12) for $a \neq b$, if presently only for the leading double logarithm.

The function $\mathcal{B}_0(x)$ in Eq. (28) appears to be a new function. The relation between $|B_{2n}|$ in the second line and the even values of Riemann's ζ-function [26] implies that this series converges for all values of x. At positive x the even part of $\mathcal{B}_0$ compensates the odd $-x/2$ contribution up to an oscillation around zero, which persists, in an increasingly irregular manner, at very large (and possibly all) values of x [28].

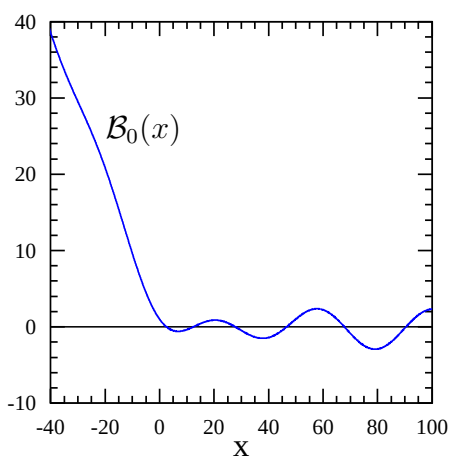

Figure 2. The function $\mathcal{B}_0(x)$ in Eq. (28), evaluated using its defining Taylor expansion.

6. Summary and Outlook

We have summarized the status of our large-x predictions of higher-order off-diagonal splitting functions and DIS coefficient functions. The coefficients of the highest three powers of $\ln(1-x)$ have been derived for the four-loop contributions to the splitting functions P_{qg} and P_{gq} from the three-loop coefficient functions and the single-logarithmic enhancement of the physical evolution kernel for the system $(F_2,\,F_\phi)$ of flavour-singlet structure functions at order α_s^4 [21]. In the present contribution we have employed these results to derive also the leading three large-x logarithms for the fourth-order gluon coefficient function $C_{L,\mathrm{g}}$ for the longitudinal structure function from the analogous kernel for $(F_2,\,F_L)$.

These results will become phenomenologically relevant, via effective x-space parametrizations analogous to, e.g., those of Ref. [29], once the next major step towards a full fourth-order calculation of deep-inelastic scattering, the extension of Ref. [30] to order α_s^4, has been taken.

The determination of flavour-singlet quantities from the physical kernels is neither rigorous, nor – unlike in flavour non-singlet cases [14,15] – can it be extended to all orders in α_s. We have presented first all-order leading-logarithmic results of a rigorous and more powerful approach, the prediction of the coefficients of the highest double logarithms from the D-dimensional structure of the unfactorized structure functions together with mass-factorization to all orders [24].

We expect that, similar to the non-singlet case, the all-order resummation of double-logarithmic large-x contributions to flavour-singlet quantities can be extended beyond parton evolution and inclusive DIS. For example, the leading-logarithmic results of Ref. [24] can be carried over directly to semi-inclusive electron-positron annihilation and Z- or Higgs-boson decay. On the other hand, we do not foresee an extension of our results to single-logarithmically enhanced large-x terms. It will be interesting to see whether such an extension can be achieved in alternative approaches such as the application of soft-collinear effective theory to large-x DIS [31] or the recent path-integral formulation of Ref. [32].

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